SYSTOLIC ARRAYS — HT KUNG

- SMALL GRAINED — SIMD
- MOST SUCCESS IN NUMERICAL CALC. AND IMAGE PROCESSING

BASIC PRINCIPLE:

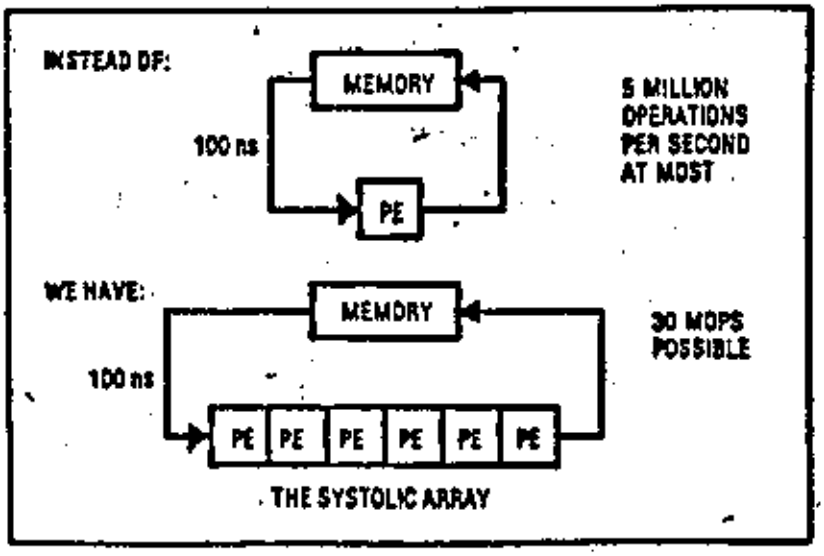


Figure 1. Basic principle of a systolic system.

(OTHERWISE KNOWN AS PIPELINING)

ARCHITECTURAL ISSUES:

1) REGULAR STRUCTURE
   - LOCAL INTERCONNECTION
   - MODULAR
   - EXTENDABLE (SCALEABLE)
2) TEMPORAL LOCALITY
   - ONE TIME UNIT BETWEEN EACH NODE
     - IMPLIES SYNCHRONIZATION
   - PIPELINED

EXAMPLE :

CONVOLUTION (MATRIX MULT.)

GIVEN K WEIGHTS $\{w_1, w_2, \ldots, w_k\}$ (FIXED)

& INPUT SEQUENCE $\{x_1, x_2, \ldots\}$

⇓

COMPUTE THE OUTPUT SEQUENCE $\{y_1, y_2 \ldots\}$

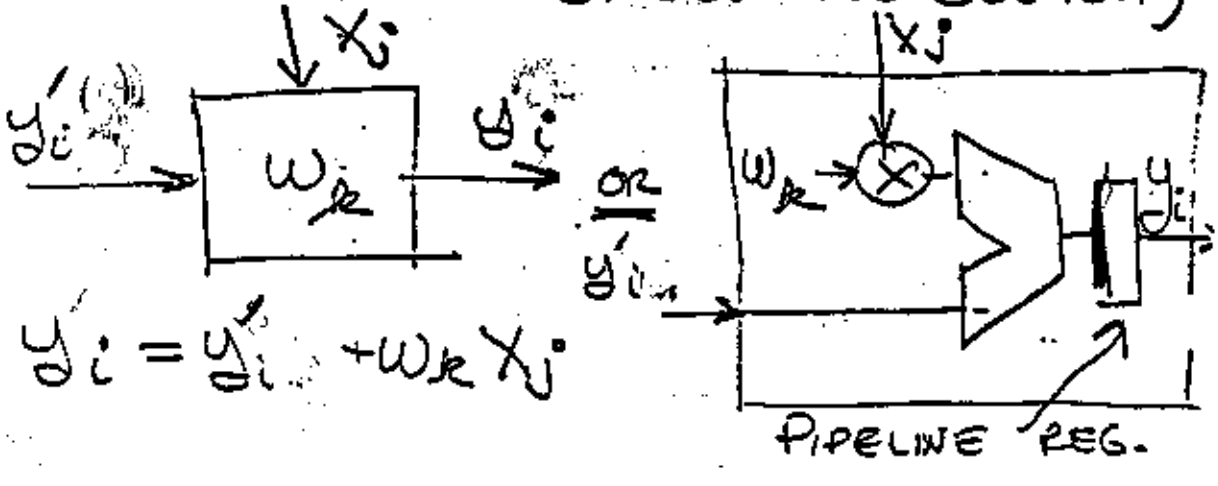
WHERE

$$y_i = w_1 x_i + w_2 x_{i+1} + \ldots + w_k x_{i+k-1}$$
$$= \sum_{j=1}^{k} w_j x_{i+j-1}$$

$$y_1 = w_1 x_1 + w_2 x_2 + \ldots$$
$$y_2 = w_1 x_2 + w_2 x_3 + \ldots$$

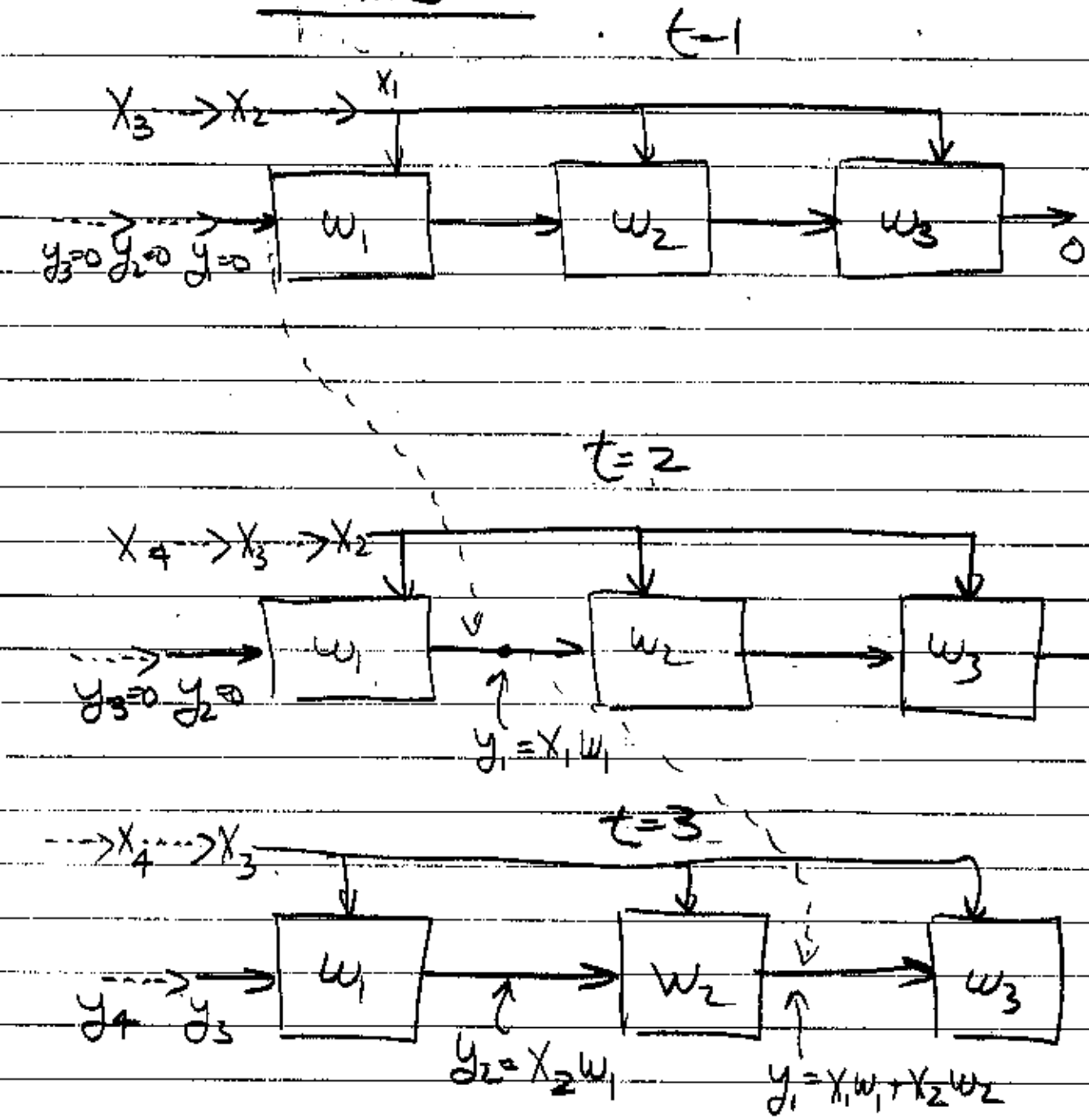
METHOD 1H. – ACTUALLY SEMI-SYSTOLIC WHICH MEANS IT HAS BROADCAST

BASIC PE (PROCESSING ELEMENT)

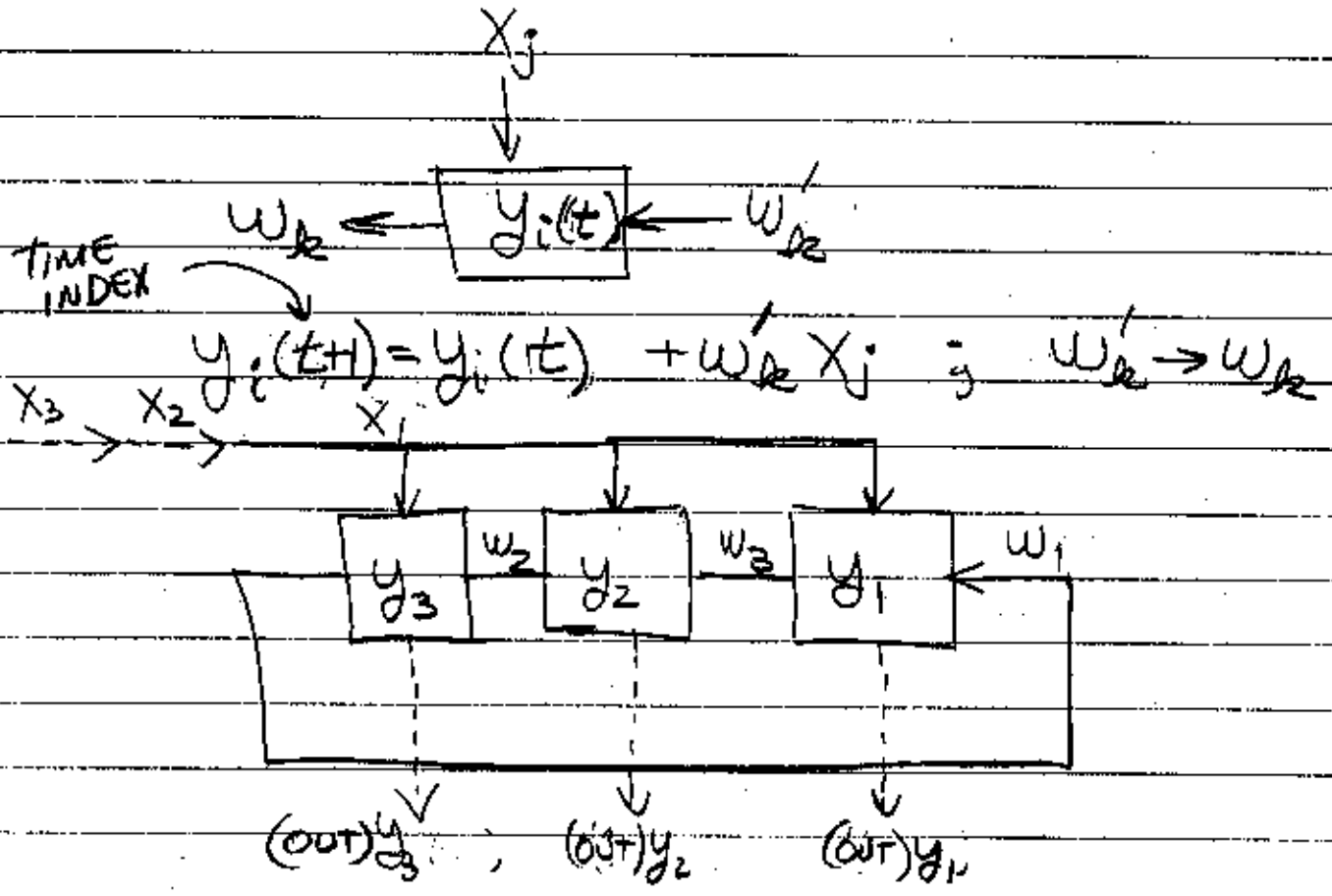


$$y_i = y'_i + w_k x_j$$

METHOD 1


t=1
X3 → X2 → X1
w1
w2
w3
0
y3=0 y2=0 y1=0
t=2
X4 → X3 → X2
w1
w2
w3
v
y3=0 y2=0
y1=X1w1
t=3
→X4 → X3
w1
w2
w3
v
y4 y3
y2=X2w1
y1=X1w1+X2w2

METHOD 2 $t=0$



$t=0$ $y_1 \Rightarrow$ OUT, $y_1 = w_1 X_1$

$t=1$ $y_1 = w_1 X_1 + w_2 X_2$

$t=2$ $y_1 = w_1 X_1 + w_2 X_2 + w_3 X_3$

$t=3$ $y_1 \Rightarrow$ OUT $y_p = w_1 X_4$

$t=0$

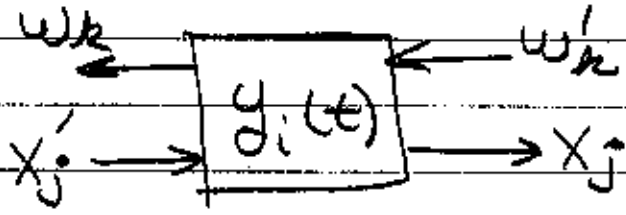
$t=1$ $y_2 \Rightarrow$ OUT, $y_2 = w_1 X_2$

$t=2$ $y_2 = w_1 X_2 + w_2 X_3$

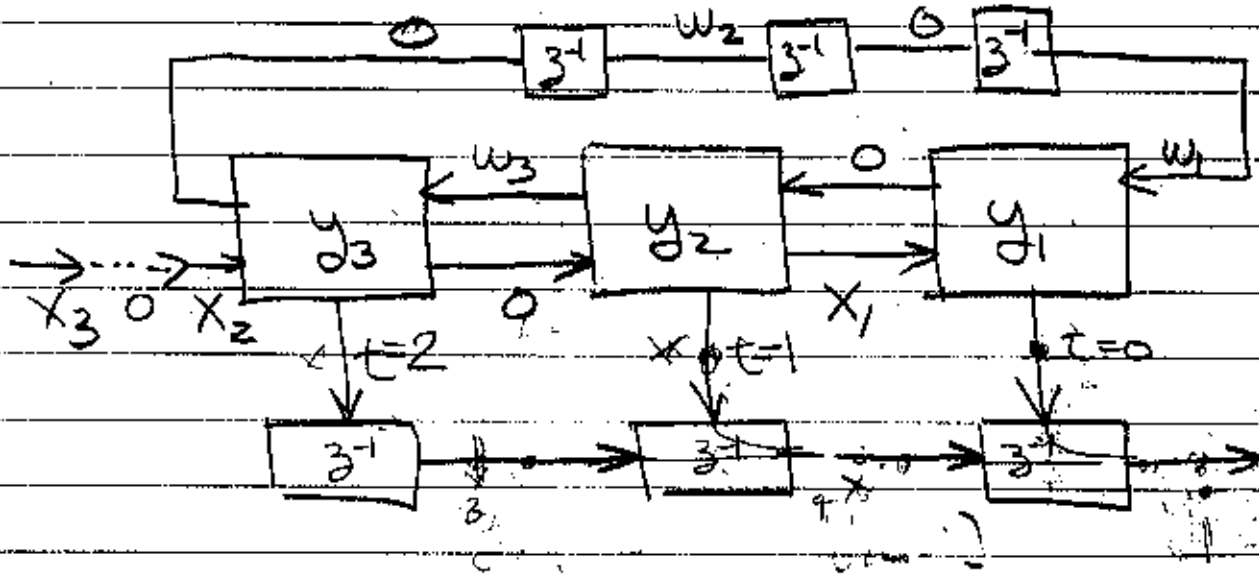
$t=3$ $y_2 = w_1 X_2 + w_2 X_3 + w_3 X_4$

$t=4$ $y_2 \Rightarrow$ OUT $y_2 = w_1 X_5$

METHOD 3



$$y_i(t+1) = y_i(t) + w'_k x'_j$$



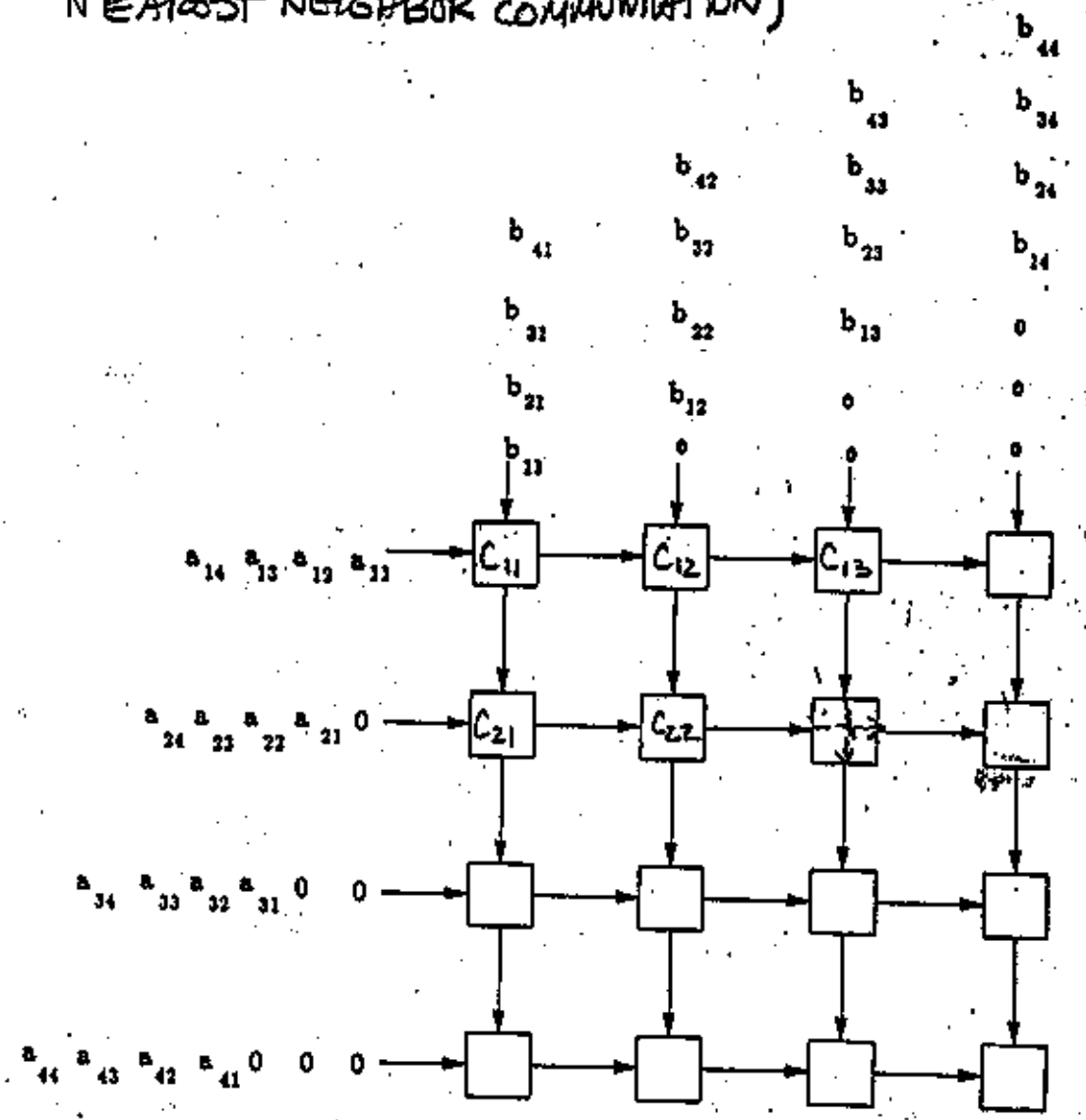
t=0	$y_1 \Rightarrow$ OUT	$y_1 = w_1 x_1$	y_2
=1		$y_1 = w_1 x_1$	$y_2 \Rightarrow$ $y_2 = w_1 x_2$
2		$y_1 = w_1 x_1 + w_2 x_2$	$y_2 = w_1 x_2$
3		$y_1 = w_1 x_1 + w_2 x_2$	$y_2 = w_1 x_2 + w_2 x_3$
4		$y_1 = w_1 x_1 + w_2 x_2 + w_3 x_3$	
5		$y_2 = w$	
6			

FULLY SYSTOLIC – ALL I/O'S AT BOUNDARY ⇒ SCALEABLE

2-D SYSTOLIC ARRAYS

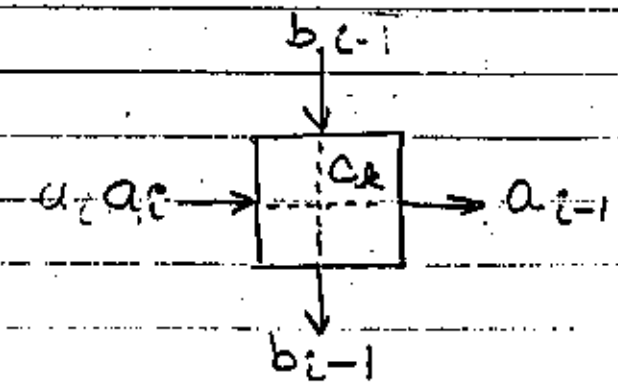
- MATRIX MULTIPLICATION

(1 OF 27 IMPLEMENTATIONS WITH NEAREST NEIGHBOR COMMUNICATION)

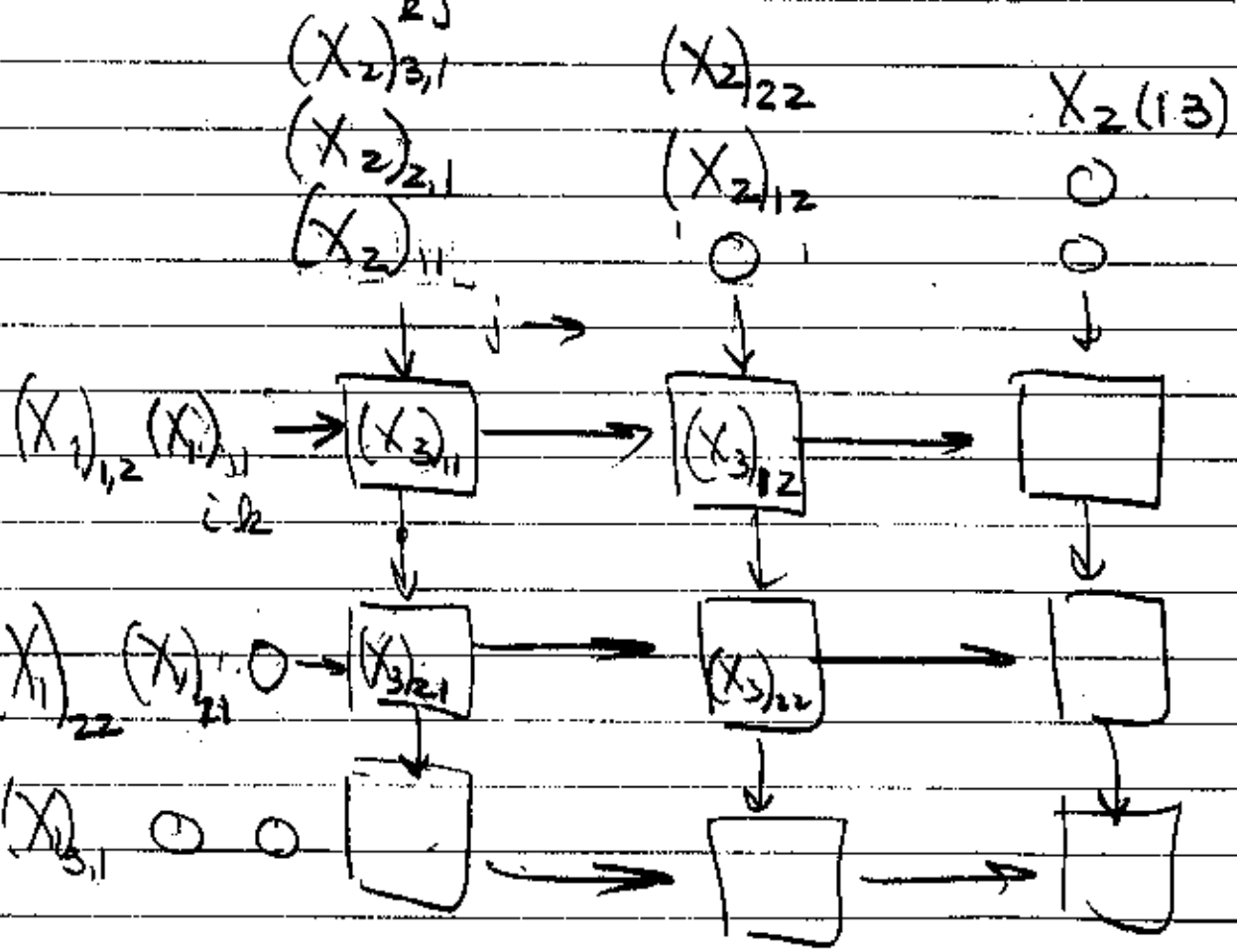


$$C_{ij} = \sum_{k=1}^{h} a_{ik} b_{kj}$$

$$C_k = C_{k-1} + a_i b_i$$



2D Systolic Array.



$$(X_3)_{ij} = \sum_{k=1}^{N} (X_1)_{ik} (X_2)_{kj}$$