Appendix I (extracted from Peimin Chi's MS thesis)

Preamble -- MLSR Sequences

The most widely chosen preambles are maximum length shift register (MLSR) sequences, or m-sequence for short [7]. An m-sequence is generated by an m-stage shift register with linear feedback as illustrated in Figure 2.2. The name maximum length comes from the fact that the maximum number of non-zero configurations for the shift registers is 2^m-1 and the output sequence possesses a period of 2^m-1.

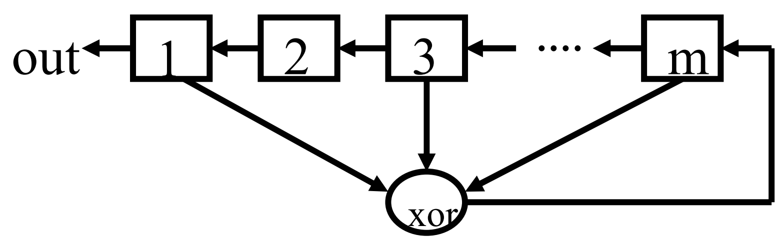

Figure 2.2 Shift Register Structure for m-sequence Generation

2.2.1 Generation of m-sequences

It is clear from Figure 2.2, that once an m is chosen and the content of the registers are initialized, the only factor that affects the output sequence is how the feedback connections are made. The feedback connections can be specified mathematically by the use of characteristic polynomial [7]. (For a detailed discussion about characteristic polynomials, please see **Appendix A2.1**) A characteristic polynomial f(x) is a degree m polynomial of the following form

$$f(x) = 1 + \sum_{i=1}^{m} c_i x^i$$

where $c_i \in \{0,1\}$. The coefficient c_i is associated with the $(m+1-i)^{th}$ register, if there is a feedback connection from $(m+1-i)^{th}$ register, then $c_i = 1$, otherwise $c_i = 0$. Therefore, for the particular feedback connection give in Figure 2.2, $c_1 = c_{m-2} = c_m = 1$, and all other coefficients are zero. It should be noted that characteristic polynomials are not unique for a fix m, that is different feedback connections can lead to sequences with maximum period. Table 2.1 lists some characteristic polynomials taken from [7] for different m-sequences.

m	Characteristic polynomial, f(x)
3	$1 + x + x^3$
4	$1 + x + x^4$
5	$1 + x^2 + x^5$
6	$1 + x + x^6$

Table 2.1 Characteristic Polynomials for Different m-sequences

2.2.2 Properties of m-sequences

An m-sequence has the following properties:

1) it has a length of $N=2^m-1$ bits
2) it is periodic with period N

3) each period has 2^{m-1} +1s and $2^{m-1}-1$ –1s (It is customary to represent the binary 1 with the decimal number +1 and the binary 0 with –1.)

The most important characteristic of an m-sequence is its autocorrelation property. Since m-sequence is periodic with period N, its autocorrelation function is also periodic with period N. It is known that the autocorrelation function R(k) is

$$R(k) = \sum_{i=0}^{N-1} p[i]p^*[i+k] = \begin{cases} N & \text{if } k = 0 \\ -1 & \text{if } 1 \le k \le N-1 \end{cases}$$

From the above expression, we see when two identical m-sequences are exactly aligned, the autocorrelation reaches the peak value of N; with any other offset (circular shift is implicitly assumed), autocorrelation decreases dramatically to –1. The autocorrelation function R(k) looks very much like a scaled version of the discrete delta function. This autocorrelation property of m-sequences is almost ideal viewed from a practical point of view.