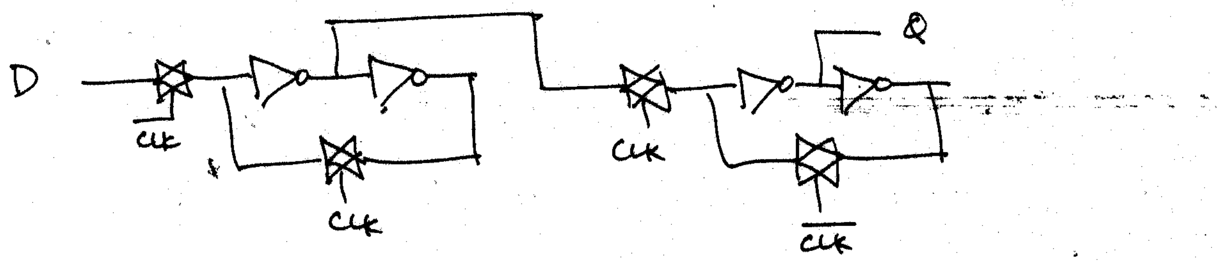
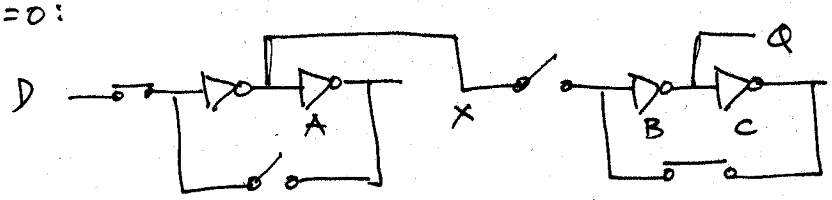
# EE141 HW 7 Solutions Spring 2008

## Part 1:

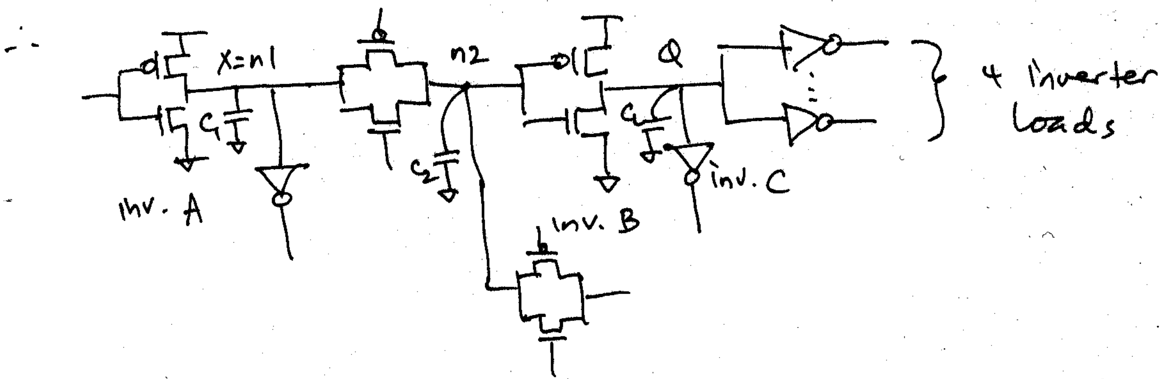



CLK = 0:




D is sampled into node X and the previous value is held in Q.

At the rising edge of the clock (CLK), X is isolated from the input D, and is stored in the slave latch. Thus, $t_{CLK-Q}$ is the time it takes for X to propagate to Q.




Since X (or n1) is already charged to the input value, the only capacitances we need to consider are $C_2$ and $C_L$.

$C_2 = 0.8\,fF + 2(0.52\,fF) = 1.84\,fF$

$C_L = 0.52\,fF + 5(0.8\,fF) = 4.52\,fF$

note: $C_{int\,inv} = 0.52\,fF$, $C_{G\,inv} = 0.8\,fF$, $R_{eq\,ave} = 15\,k\Omega$ (min nmos)

$$t_{p_{LH}} = t_{p_{HL}} = t_{clk\to Q\,ave} = \ln(2)\left[R + \frac{R}{2}\right]C_2 + \ln(2)\,RC_L$$

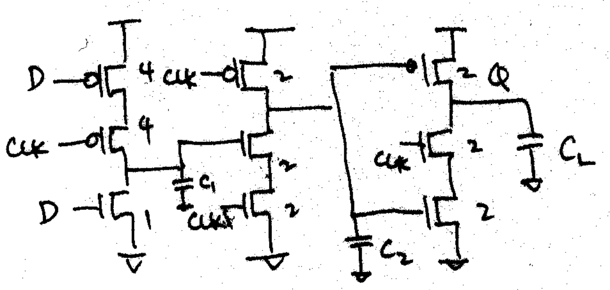
$$= \ln(2)\,\frac{3}{2}(15\,k\Omega)(1.84\,fF) + \ln(2)\,15\,k\Omega\,(4.52\,fF)$$

$$= 76\,ps$$

From simulations: $t_{clk-Q_{LH}}$ = ~~[illegible]~~ 67 ps, $t_{clk-Q_{HL}}$ = ~~[illegible]~~ 69 ps

$t_{clk-Q\,ave}$ = ~~[illegible]~~ 68 ps

## Part 2:




note:

$C_{jn} = 0.17\ fF \quad (w=1)$

$C_{jp} = 0.35\ fF \quad (w=2)$

~~$C_{gn} = 0.27\ fF \quad (w=1)$~~

$C_{gp} = 0.53\ fF \quad (w=2)$

$R_{eq} = 15K\Omega \quad (n,\ w=1)$

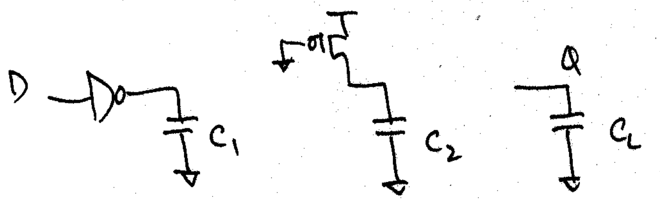
node capacitances:

$C_1 = 2(0.35)(2) + 2(0.53) = 2.46\ fF$

$C_2 = 2(0.17) + 0.35 + 0.53 + 2(0.27) = 1.76\ fF$

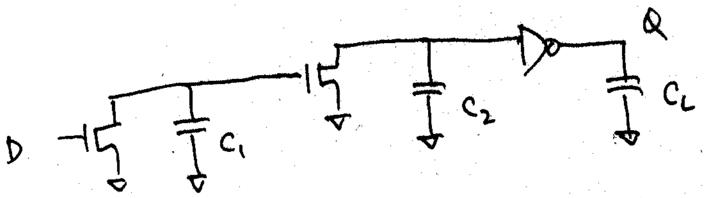
$C_L = 0.35 + 2(0.17) + 4(0.8) = 3.89\ fF$

when clk = 0:




D is sampled onto $C_1$
$C_2$ is precharged to VDD
$C_L$ maintains previous value of Q

when clk = 0 → 1:




Depending on the value of $C_1$, the output Q changes accordingly.

the clock-to-Q delay is the time needed to charge / discharge $C_2$ and $C_L$ (low to high transition) or $C_L$ (high to low transition)

$t_{clk-Q_{LH}} = (\ln 2)(15K\Omega)(1.76fF) + (\ln 2)(15K\Omega)(3.89fF) = 58.7\ ps$

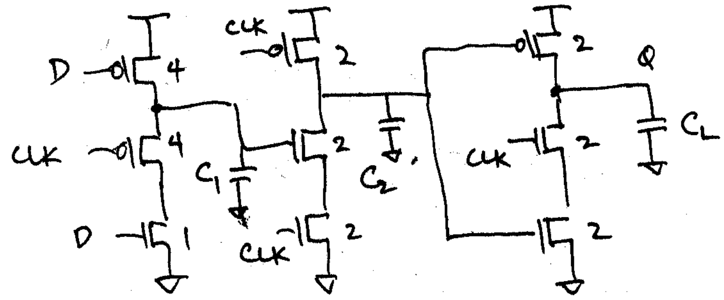
$t_{clk-Q_{HL}} = (\ln 2)(15K\Omega)(3.89\ fF) = 40.4\ ps$

<u>Simulation results</u>:

$t_{clk-Q_{LH}} = 58.16\ ps$

$t_{clk-Q_{HL}} = 24.35\ ps$

## Part 2: (alternate solution if you used the ckt. below)




note:
$C_{jn\,unit} = \frac{0.52}{3} = 0.17$ fF
$C_{jp\,unit} = 0.35$ fF
~~$C_{gn\,unit} = \frac{0.8}{3} = 0.27$ fF~~
$C_{gp\,unit} = 0.53$ fF
unit nmos = 1/2 unit pmos size
$R_{eq\,unit} = 15K\Omega$

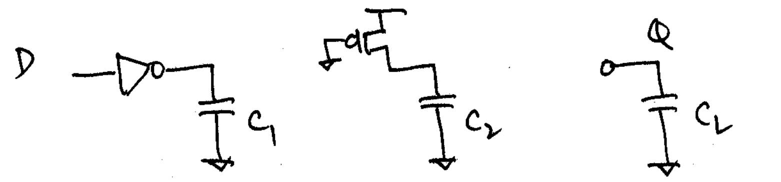
Estimate the node capacitances:

$C_1 = 2(0.35)2 + 2(0.53) = 2.46$ fF

$C_2 = 2(0.17) + 0.35 + 0.53 + 2(0.27) = 1.76$ fF

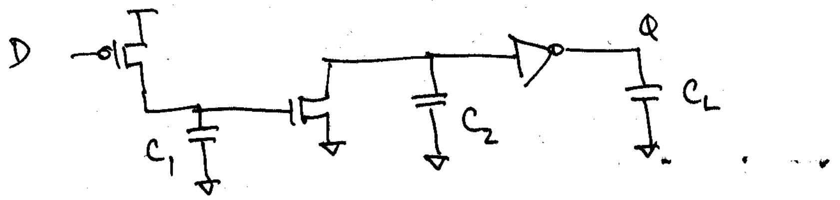
$C_L = 0.35 + 2(0.17) + 4(0.8) = 3.89$ fF

when CLK = 0:




D is sampled on $C_1$,
$C_2$ is pre-charged to $V_{DD}$,
$C_L$ maintains previous value of Q.

when CLK: 0 → 1:




Depending on the value stored on $C_1$, the output Q changes.

the clock-to-Q delay is the time needed to charge/discharge $C_2$ and $C_L$ (low to high transition) or $C_L$ (high to low transition).

$t_{clk-Q_{LH}} = \ln(2)(15K\Omega)(1.76\text{ fF}) + \ln(2)(15K\Omega)(3.89\text{ fF}) = 58.7$ ps

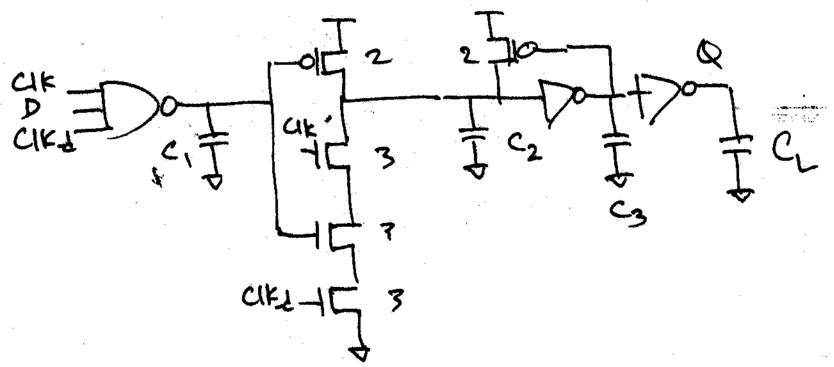
$t_{clk-Q_{HL}} = \ln(2)(15K\Omega)(3.89\text{ fF}) = 40.4$ ps

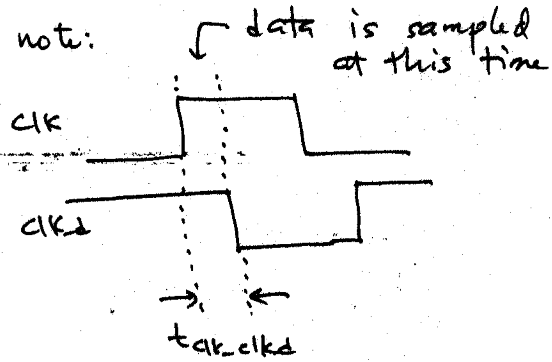
Simulation results:

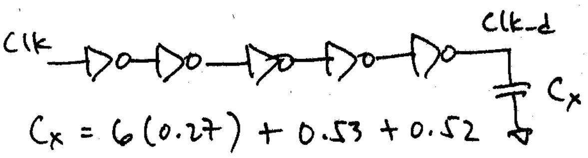
$t_{clk-Q_{LH}} = 58.16$ ps
$t_{clk-Q_{HL}} = 24.35$ ps

## Part 3:

Simplified schematic:



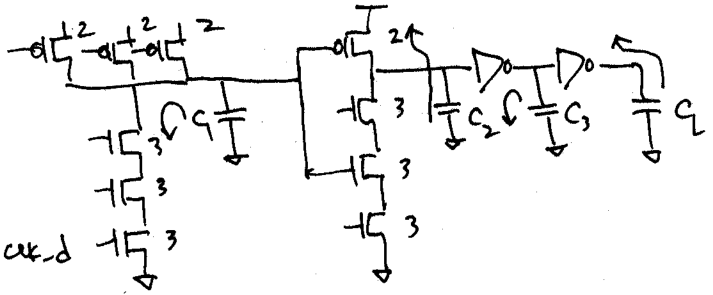




$C_x = 6(0.27) + 0.53 + 0.52$
$= 2.67 \text{ fF}$

$t_{clk\_clk\_d} = \ln 2 (15k\Omega)(2.67 \text{ fF}) + 4 \ln 2 (15k)(1.32 \text{ fF})$
$= 82.6 \text{ ps}$

For the L→H Q transition:



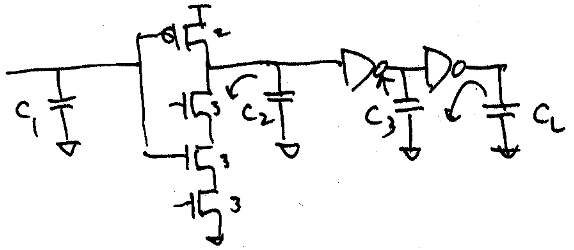

$C_1 = 3(0.17) + 3(0.35) + 0.53 + 3(0.27) = 2.9 \text{ fF}$
$C_2 = 3(0.17) + 0.35 + 0.17 + 0.8 = 1.83 \text{ fF}$
$C_3 = 0.8 + 0.52 + 0.27 = 1.59 \text{ fF}$
$C_L = 0.52 + 4(0.8) = 3.72 \text{ fF}$

$t_{clk\_d \text{ to } Q} = (\ln 2)(15k\Omega)[2.9 \text{ fF} + 1.83 \text{ fF} + 1.59 \text{ fF} + 3.72 \text{ fF}] = 104.4 \text{ ps}$
$t_{clk\text{-}Q} = 104.4 \text{ ps} + 82.6 \text{ ps} = 187 \text{ ps}$

Simulation results

$t_{clk\text{-}Q_{LH}} = 102 \text{ ps}$
$t_{clk\text{-}Q_{HL}} = 65 \text{ ps}$

For the H→L Q transition:




$t_{clk\_d \text{ to } Q} = (\ln 2)(15k\Omega)[1.83 + 1.59 + 3.72] = (\ln 2)(15k\Omega)(7.14 \text{ fF}) = 74.2 \text{ ps}$
$t_{clk\text{-}Q} = 74.2 \text{ ps} + 82.6 \text{ ps} = 156.8 \text{ ps}$

## Simulation Results

MS FF (part 1) :

$t_{su}$ = 51.4 ps

$t_{hold}$ = −4.95 ps

TSPC (part 2)

$t_{su}$ = 37.5 ps

$t_{hold}$ = 20.6 ps

Pulse-based FF (part 3)

$t_{su}$ = 8.2 ps

$t_{hold}$ = 43.4 ps