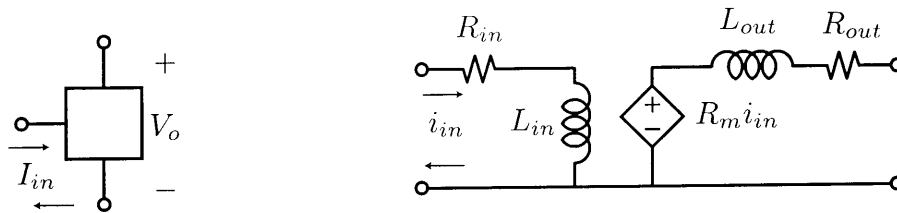


Midterm Exam (closed book)
Tuesday, October 19, 2004

Guidelines: Closed book. You may use a calculator. Do not unstaple the exam. *Warning:* Illustrations not to scale.

1. A new device has been invented with the following I-V characteristic:

$$V_o = -K_v \frac{1}{1 + \left(\frac{I_m}{I_p}\right)}$$



This relation holds for $I_{in} > 0$ and for any passive termination. The device parameters are as follows: $I_p = 750 \mu\text{A}$, $K_v = .6 \text{ V}$. A small-signal model including layout parasitics is shown above.

- (a) (6 points) Calculate an expression for the maximum gain of the amplifier at 10 GHz. Note:

$$G_{Tu,max} = \frac{|z_{21}|^2}{4\Re(z_{11})\Re(z_{22})}$$

By inspection

$$z_{11} = R_{in} + j\omega L_{in}$$

$$z_{22} = R_{out} + j\omega L_{out}$$

$$z_{21} = R_m$$

$$G_{T,max} = \frac{1}{4} \frac{R_m^2}{R_{in} R_{out}}$$

- (b) (10 points) Design a matching network to achieve an input match with $R_S = 50 \Omega$. Assume that $R_{in} = 5 \Omega$ and $L_{in} = 3 \text{ nH}$. Draw the complete input network to the amplifier and specify the bandwidth of the match. Note that $f_0 = 10 \text{ GHz}$.

$$Q = \sqrt{\frac{50}{5} - 1} = 3$$

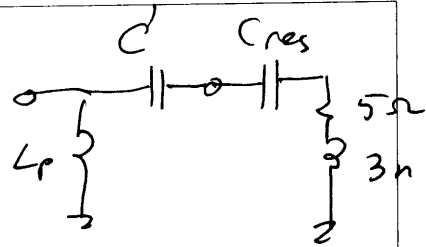
$$X_S = 3 \times 5 = 15$$

$$R_p = (1 + Q^2) \cdot 5$$

$$X_p = (1 + Q^{-2}) X_S = 16.67 \Omega \quad C = 954.7 \text{ fF}$$

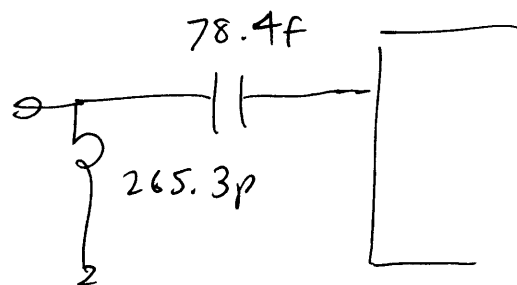
$$C = 78.4 \text{ fF} \quad (\text{series combination})$$

$$L_p = 265.3 \text{ pH}$$

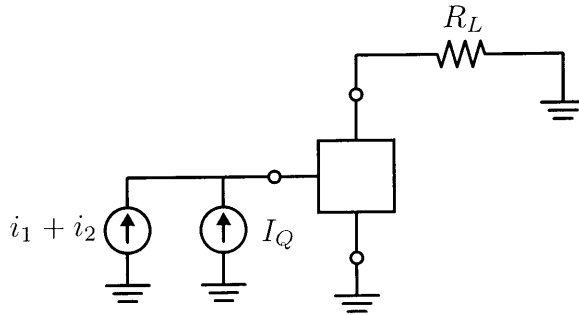


$$j\omega L_{in} = 188 \Omega \quad \text{we cap to resonate}$$

$$C_{res} = 84.66 \text{ fF}$$



$$B = \frac{f_0}{Q} = \frac{10 \text{ GHz}}{3} = 3.33 \text{ GHz}$$



(c) (10 points) For the circuit shown above, calculate the IM_3 at low frequency for two current input signals at 100 MHz and 101 MHz of magnitude $100 \mu\text{A}$. The circuit is biased as shown with $I_Q = 2 \text{ mA}$.

$$V_o = -K_v \frac{1}{1 + \frac{i_m + I_Q}{I_P}}$$

$$= -k_v \frac{1}{\left(1 + \frac{i_m}{I_P}\right) \left(1 + \frac{I_Q}{I_P}\right)}$$

$$I_x = \frac{1}{I_P \left(1 + \frac{I_Q}{I_P}\right)}$$

$$\boxed{I_x = \frac{1}{I_P + I_Q}}$$

$$V_o = -\frac{k_v}{1 + \frac{I_Q}{I_P}} \left(1 - \frac{i_m}{I_x} + \frac{i_m^2}{I_x^2} - \frac{i_m^3}{I_x^3} + \dots\right)$$

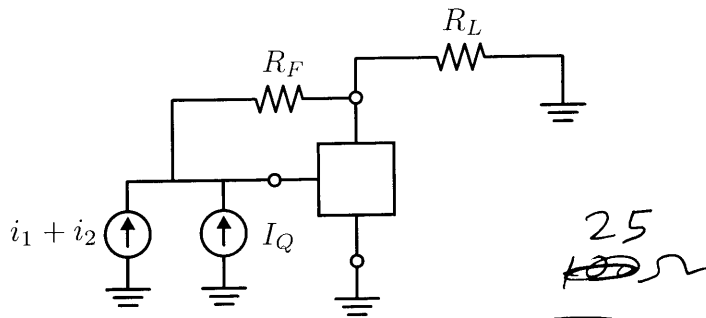
$$IM_3 = \frac{3}{4} \frac{a_3}{a_1} S_i^2 = \frac{3}{4} \frac{1}{I_x^2} \cdot I_x S_i^2$$

$$= \frac{3}{4} \left(\frac{i_m}{I_x}\right)^2$$

$$I_x = 2 \text{ mA} + 0.75 \text{ mA}$$

$$i_m = 100 \mu\text{A}$$

$$= -60 \text{ dBc}$$



(d) (10 points) Assume a shunt feedback resistor of value $R_F = 1\text{ k}\Omega$ is added to the circuit. Calculate HD_2 under this condition. Assume $R_F \gg R_{in}$. with $V_{out} = 100\text{ mV}$

$$i_F \approx \frac{V_o}{R_F} \quad f = \frac{1}{R_F}$$

$$T = a_1 f = \frac{R_m}{R_F}$$

$$R_m = \frac{k_v}{1 + \frac{I_Q}{I_P}} \cdot \frac{1}{I_X} = \left(\frac{0.6}{1 + \frac{2\text{m}}{0.75}} \right) \frac{1}{2.75\text{m}}$$

$$= 59.5 \Omega$$

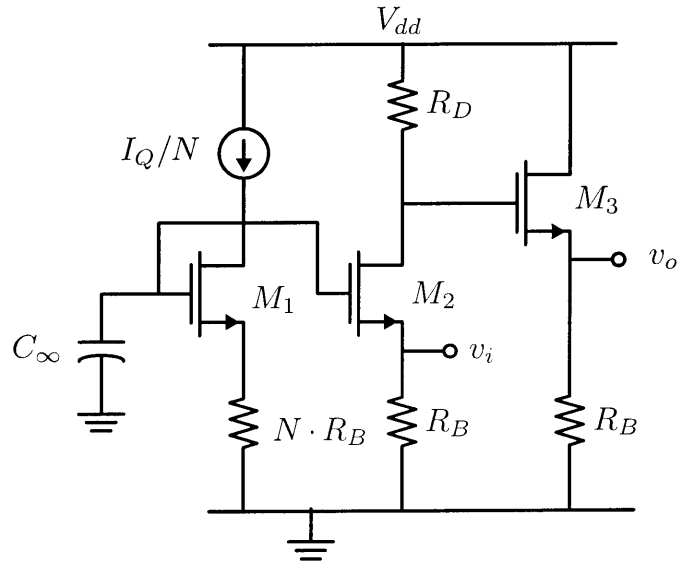
$$T = \frac{59.5}{125} = 2.38$$

$$HD_2 = \frac{1}{2} \frac{a_2}{a_1^2} \frac{1}{1+T} \text{ Som}$$

$$= \frac{1}{2} \frac{1}{I_X^2} \frac{I_X^2}{(1+T)} \text{ Som} \quad X$$

$$\left(\frac{1 + \frac{I_Q}{I_P}}{k_v} \right)$$

$$HD_2 = \frac{1}{2} \left(\frac{1 + \frac{I_Q}{I_P}}{k_v} \right) \left(\frac{1}{1+T} \right) \text{ Som} = -21 \text{ dBc}$$



2. The amplifier shown above is designed in a triple-well process (body and source are shorted). Assume that the resistor $R_B \gg \frac{1}{g_m}$ and $R_D \ll r_o$.

(a) (10 points) Calculate the bandwidth of the above amplifier. Assume the amplifier is designed for an input and output match.

DRAIN OF M_2

$$C_{tot} = C_{db2} + C_{\mu 2} + \frac{C_{gs3}}{1 + g_{m3}R_B} + C_{\mu 2}$$

$$R_{tot} \approx R_D$$

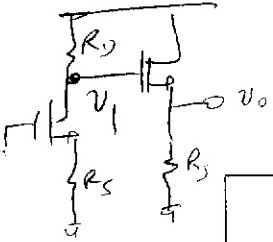
$$\tau = R_D C_{tot}$$

$$f = \frac{1}{2\tau}$$

(b) (10 points) Calculate $G_{T,max}$ for this amplifier. Explain your assumptions.

$$\begin{aligned} \operatorname{Re}(y_{11}) &\approx g_{m2} & y_{21} &= \frac{i_2}{v_1} \Big|_{v_2=0} = g_{m2} R_D g_{m3} \\ \operatorname{Re}(y_{22}) &\approx g_{m3} \\ \frac{|y_{21}|^2}{4 \operatorname{Re}(y_{11}) \operatorname{Re}(y_{22})} &= \frac{1}{4} \frac{g_{m2}^2 R_D^2 g_{m3}^2}{g_{m2} g_{m3}} = \frac{1}{4} \left(\frac{R_D}{R_S} \right)^2 \end{aligned}$$

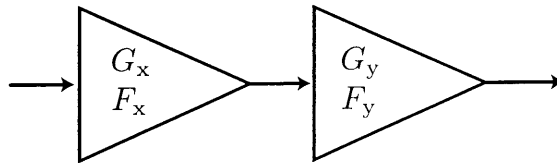
(c) (16 points) Calculate the noise figure for the amplifier. Assume the amplifier is driven by a source with resistance R_S and loaded by a matched load $R_L = R_S$. Assume the amplifier is designed for an input and output match. Ignore gate noise.



$$\begin{aligned} v_1 &= v_{R_D} + i_{d1} \left(\frac{1}{1+g_{m3}R_S} \right) R_D + i_s \left(\frac{g_{m3}R_S}{1+g_{m3}R_S} \right) R_D \\ g_{m3}R_S &= 1 \\ \overline{v_1^2} &= \overline{v_{R_D}^2} + \frac{1}{4} \overline{i_{d1}^2} R_D^2 + \frac{\overline{i_s^2} R_D^2}{4} \\ \overline{v_o^2} &= \overline{v_1^2} \left(\frac{g_{m2}R_S}{1+g_{m2}R_S} \right)^2 + \overline{i_{d2}^2} \frac{R_S^2}{(1+g_{m2}R_S)^2} \\ &= \overline{v_1^2} \frac{1}{4} + \overline{i_{d2}^2} \frac{R_S^2}{4} \\ \overline{v_{OT}^2} &= \left(\overline{v_{R_D}^2} + \frac{\overline{i_{d1}^2} R_D^2}{4} + \frac{\overline{i_s^2} R_D^2}{4} \right) \frac{1}{4} + \overline{i_{d2}^2} \frac{R_S^2}{4} \\ F &= 1 + \frac{4 \overline{v_{R_D}^2}}{\overline{i_s^2} R_D^2} + \frac{\overline{i_{d1}^2}}{\overline{i_s^2}} + \frac{4 \overline{i_{d2}^2} R_S^2}{\overline{i_s^2} R_D^2} \end{aligned}$$

$$= 1 + \frac{4R_S}{R_D} + \left(\frac{\sigma}{\alpha} \right) g_{m2} R_S + 4 \left(\frac{R_S}{R_D} \right)^3$$

3. You would like to design an amplifier with a power gain of 50 dB from two amplifiers, each with the following characteristics:



$$G_1 = 30 \text{ dB}$$

$$NF_1 = 1.7 \text{ dB} = 1.479$$

$$G_2 = 20 \text{ dB}$$

$$NF_2 = 1.3 \text{ dB} = 1.349$$

- (a) (8 points) Find the optimal ordering of the amplifiers in cascade to achieve the lowest possible noise figure.

$$F_{12} = F_1 + \frac{F_2 - 1}{G_1} = 1.479$$

$$F_{21} = 1.349 + \frac{0.479}{100} = 1.354 \checkmark$$

ORDER 2-1 is BETTER

$$M_1 = \frac{G_1 F_1 - 1}{G_1 - 1} = 1.479$$

$$M_2 = 1.353 \checkmark$$

- (b) (10 points) Find the lowest possible signal power (minimum detectable signal) to maintain an $SNR_o > 10$ dB. Assume the input noise is from a source resistance $R_s = 50 \Omega$ and the communication bandwidth is 1 MHz.

$$SNR_o = \frac{SNR_i}{F} > 10$$

$$SNR_i > 10 \cdot F = 13.54$$

$$P_i > 13.54 \cdot P_{noise} = 13.54 \cdot kT \cdot B$$

$$= 13.54 \cdot 4 \times 10^{-21} \text{ W} \cdot 10^6 = 5.42 \times 10^{-14} \text{ W}$$

$$= -103 \text{ dBm}$$

- (c) (10 points) Assume that the system is designed to work with a signal as large as -10 dBm. What's the required IIP_3 for the entire system to maintain a signal-to-distortion $SDR > 10$ dB.

$$SDR > 10$$

$$\frac{P_i}{P_d} > 10 \Rightarrow P_d < \frac{P_i}{10} = \frac{0.1 \text{ mW}}{10} = 0.01 \text{ mW}$$

IM_3 DEGRADES BY 2 dB/1 dB INPUT

INPUT INCREASES BY 5 dB $\Rightarrow IM_3 = 0$ dB

$$IIP_3 = -10 \text{ dBm} + 5 \text{ dB} = -5 \text{ dBm}$$