SOLUTIONS

University of California, Berkeley
EECS 142

Fall 2006
Prof. A. Niknejad

Midterm Exam (closed book)
Thursday, October 17, 2006

Guidelines: Closed book. You may use a calculator. Do not unstaple the exam. *Warning:* Illustrations not to scale.

Common two-port equation:

$$Y_{in} = Y_{11} - \frac{Y_{12}Y_{21}}{Y_L + Y_{22}}$$

$$Y_{out} = Y_{22} - \frac{Y_{12}Y_{21}}{Y_S + Y_{11}}$$

$$G_p = \frac{P_L}{P_{in}} = \frac{|Y_{21}|^2}{|Y_L + Y_{22}|^2}\frac{\Re(Y_L)}{\Re(Y_{in})}$$

Simple trigonometric identify:

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

$$\sin(x+y) = \cos(y)\sin(x) + \cos(x)\sin(y)$$

$$2\cos(x)\cos(y) = \cos(x+y) + \cos(x-y)$$

Distortion equations: $s_o = a_1 s_i + a_2 s_i^2 + a_3 s_i^3 + \cdots$

$$IM_2 = 2HD_2 = \frac{a_2}{a_1}S_i$$

$$IM_3 = 3HD_3 = \frac{3}{4}\frac{a_3}{a_1}S_i^2$$

Series inversion: If, $s_i = a_1 s_o + a_2 s_o^2 + a_3 s_o^3 + \cdots$, then $s_o = b_1 s_i + b_2 s_i^2 + b_3 s_i^3 + \ldots$ where

$$b_1 = \frac{1}{a_1}$$

$$b_2 = -\frac{a_2}{a_1^3}$$

$$b_3 = \frac{2a_2^2}{a_1^5} - \frac{a_3}{a_1^4}$$

Cascade of two power series:

$$c_1 = a_1 b_1$$

$$c_2 = b_1 a_2 + b_2 a_1^2$$

$$c_3 = b_1 a_3 + 2b_2 a_1 a_2 + b_3 a_1^3$$

Effect of feedback on distortion (not required for this test but useful)

$$b_1 = \frac{a_1}{1 + a_1 f} = \frac{a_1}{1 + T}$$

$$b_2 = \frac{a_2}{(1+T)^3}$$

$$b_3 = \frac{a_3(1+T) - 2a_2^2 f}{(1+T)^5}$$

Taylor Series Expansion about $x = 0$

$$f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots$$

Binomial Theorem

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$$

1. (*35 points*) Answer the following questions succinctly.

   (a) (*9 points*) A system is described by the following non-linear equation

$$S_o = a_1 S_i + a_5 S_i^5$$

   Calculate the apparent gain for a sinusoidal input signal.

$$\left(\frac{e^{x}+e^{-x}}{2}\right)^5 = \sum_{k=0}^{5} \binom{5}{k} \frac{e^{kx}\, e^{-x(5-k)}}{2^5} \qquad x \triangleq \omega t$$

$$= \sum_{k=0}^{5} \binom{5}{k} e^{2kx-5x}$$

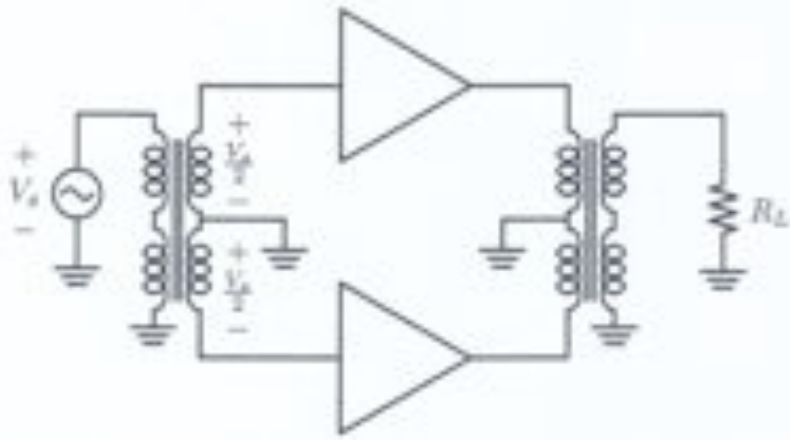
$$k=2 \qquad \binom{5}{2} e^{-x} = \frac{5\cdot 4}{2} = 10$$

$$k=3 \qquad \binom{5}{3} e^{x} = 10$$

$$\frac{a_5\,10}{2^5}\left(e^{x}+e^{-x}\right) = \frac{a_5\,10}{2^4} \cos \omega t$$

$$\text{GAIN} = \frac{a_1 S_1 \cos\omega t + a_5 \cdot \frac{10}{16} \cos\omega t \cdot S_1^5}{S_1 \cos\omega t}$$

$$\text{GAIN}(S_1) = a_1\Big(\underset{\substack{\uparrow \\ \text{LINEAR GAIN} \\ \text{FOR } S_1 \ll 1}}{1} + \frac{a_5}{a_1}\frac{5}{8}\underset{\substack{\uparrow \\ \text{SIGNAL} \\ \text{DEPENDENT} \\ \text{GAIN } \propto S_1^4}}{S_1^4}\Big)$$



(b) (*9 points*) Calculate the iIP_2 and iIP_3 of the overall amplifier. Each individual amplifier has an $iIP_2 = -7\,\text{dBm}$ and an $iIP_3 = +2\,\text{dBm}$. Assume the transformer is ideal and the windings are 1 : 1.

SINCE THE AMP IS BALANCED, $iIP_2 = \infty$

FOR EACH AMP:

$v_i = \frac{V_S}{2}$ SO WE SHOULD DOUBLE THE INPUT TO PRODUCE THE SAME DISTORTION.

$iIP_3 = +2\,\text{dBm} + 6\,\text{dBm}$

$= +8\,\text{dBm}$

(c) (*8 points*) Given a power amplifier with $P_{-1\,dB} = +33\,dBm$, a power gain of 22 dB, find the required input power "back-off" in order to operate with a signal-to-distortion (SDR) ratio of at least 40 dB. Assume a two-tone test with two equal power carriers. Your answer should be a power level at the input of the amplifier that results in intermodulation produces 40 dB lower than the signal tones.

$P_{-1dB} = 33\,dBm$ ASSUME AT OUTPUT

$P_{OIP_3} \simeq P_{-1dB} + 10\,dBm = 43\,dBm$

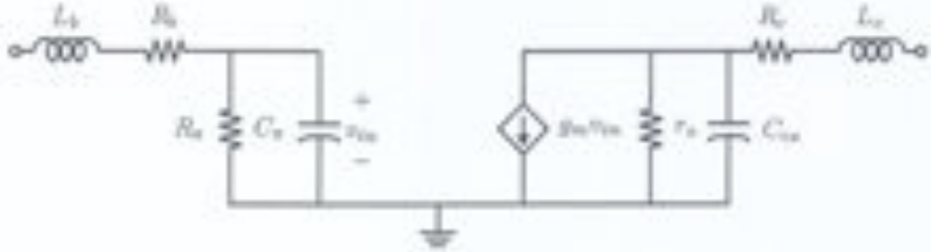
FOR $SDR = 40\,dB \Rightarrow$ BACK-OFF 20dB

(2dB PER 1dB BACKOFF)

$P^o_{Back} = P_{OIP_3} - 20\,dBm = 23\,dBm$

$P_{i_{3,Back}} = P^o_{Back} - G = 23\,dBm - 22\,dB$

$\hat{=} +1\,dBm$



(d) (*9 points*) For the transistor shown above, calculate the maximum power gain (in dB) at 2.5 GHz. Use the following transistor parameters: $f_T = 25\,\text{GHz}$, $I_C = 10\,\text{mA}$, $\beta_0 = 250$, $V_A = 5\,\text{V}$, $R_b = 5\,\Omega$, $R_c = 15\,\Omega$, $L_b = 3\,\text{nH}$, $L_c = 2\,\text{nH}$, $C_{jc} = 0\,\text{F}$, $C_{cs} = 0.1C_\pi$.

$$Z_{11} = Z_1 + Z_\pi = j\omega L_b + R_b + \frac{R_\pi}{1 + j\omega C_\pi R_\pi}$$

$$= j\omega L_b + R_b + \frac{R_\pi (1 - j\omega C_\pi R_\pi)}{1 + \omega^2 C_\pi^2 R_\pi^2}$$

$$Re(Z_{11}) = R_b + \frac{R_\pi}{1 + \omega^2 R_\pi^2 C_\pi^2} \qquad Z_{12} = 0$$

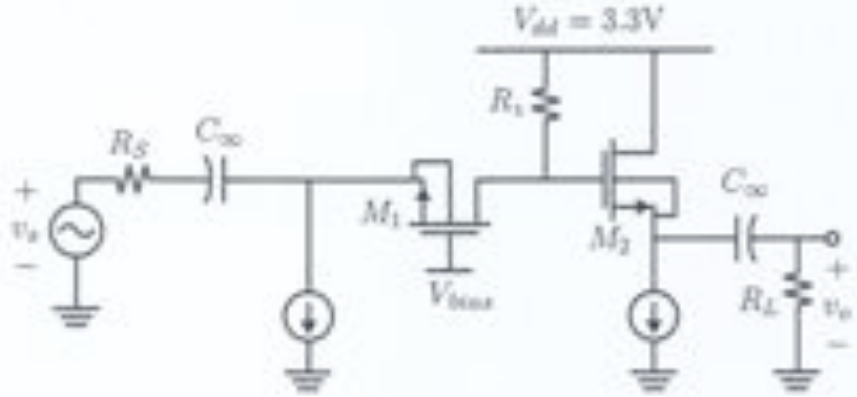
$$Re(Z_{22}) = R_c + \frac{r_o}{1 + \omega^2 C_o^2 R_o^2}$$

$$V_2 = Z_{21} I_1 + Z_{22} \cancel{I_2}^{\,0}$$

$$= \underbrace{-g_m Z_o Z_\pi}_{Z_{21}} \; I_1$$

$$Z_{21} = \frac{-g_m R_\pi}{1 + j\omega C_\pi R_\pi} \; \frac{R_o}{1 + j\omega C_o R_o}$$

$$G_{max} = \frac{1}{4} \frac{|Z_{21}|^2}{Re(Z_{11})\, Re(Z_{22})}$$



2. (*65 points*) Consider the common-gate/source-follower two-stage amplifier shown above. Assume the maximum transistor $f_T = 40\,\text{GHz}$ ($V_{GS} = V_{dd}$), $V_{dd} = 3.3\,\text{V}$, $V_t = 0.3\,\text{V}$, $C_{gd} = 0\,\text{F}$, and $C_{db} = 0\,\text{F}$. Assume the amplifier is driven by a voltage source with source impedance of $50\,\Omega$ and load impedance $R_L = 500\,\Omega$.

   (a) (*14 points*) Find the maximum possible gain under an input and output impedance match in order to meet a 1 GHz bandwidth. Assume the maximum DC voltage drop across R_1 is 3 V. Under this voltage gain, estimate the bandwidth of the amplifier. (In this part you are adjusting g_{m1} and g_{m2} to satisfy an input/output match.)

$$\text{GAIN} = \frac{1}{2} \cdot g_m R_1 \cdot \frac{1}{2} = \frac{1}{4} \frac{2 I_D}{V_{sat}} R_1$$

BUT $\quad I_D R_1 \le 3\,V \qquad \text{GAIN} = \frac{1}{2} \frac{3V}{V_{sat}}$

MAKE V_{sat} AS SMALL AS POSSIBLE

$$f_T = \frac{1}{2\pi} \frac{3}{2} \frac{\mu (V_{GS} - V_t)}{L^2}$$

$$\omega_{T,max} \propto (V_{DD} - V_T) \qquad f_{-3dB} \simeq \frac{f_T}{2}$$

$$\omega_T = \frac{(V_{GS} - V_t)}{(V_{DD} - V_t)} \omega_{T,max} \Rightarrow 40\,\text{GHz} \cdot \frac{V_{sat}}{3} = 2\,\text{GHz}$$

$$V_{sat} = \frac{6}{40} = 150\,mV \qquad \text{GAIN} = \frac{1}{2} \frac{3 \cdot 40}{6} = 10$$

?

(b) (*14 points*) For this problem, the $V_{dsat} = 250\,\text{mV}$ and $I_{ds,1} = 15\,\text{mA}$, and $R_1 = 200\,\Omega$. Design a narrowband input impedance matching network (center frequency 1 GHz) using inductors and/or capacitors. Draw the complete schematic of the matching network and label the component values. Specify the bandwidth of the amplifier (selectivity). (*Hint:* Calculate the device f_T at this bias point to determine the input capacitance.)

$$g_{m1} = \frac{2I_D}{V_{sat}} \qquad \frac{1}{g_{m1}} = \frac{50}{6}\,\Omega$$

IF $R_S = 50\Omega$, $m = 6$ $\quad Q = \sqrt{m-1} = \sqrt{5}$

$$\frac{g_{m1}}{C_{gs1}} = \omega_T = \omega_{T,max}\frac{V_{sat}}{V_{DD}-V_t} = 2\pi \cdot 40\,\text{GHz} \cdot \frac{0.25}{3}$$

$C_{gs1} = 5.7\,pF$

RESONATE at $C_{gs1} \Rightarrow$ $\quad L_{gs} = \frac{1}{\omega_0^2 C_{gs1}} = 4.4\,nH$

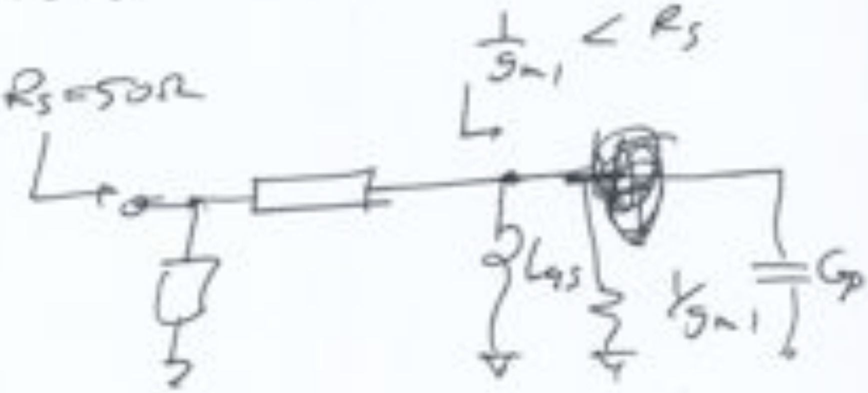
→ FOR MATCHING NETWORK

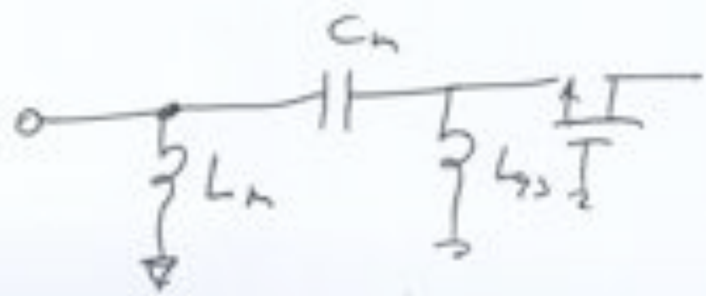
$X_m = Q \cdot \frac{1}{g_m} = \frac{\sqrt{5} \cdot 50}{6}$

$X_m' = (1+Q^{-1})X_m$

$C_m = 8.54\,pF$

$L_m = 4.27\,nH$





(c) (*8 points*) What is the power gain of the amplifier under matched conditions? Include the input matching network in your answer and assume that M2 is biased for an impedance match at the output.

UNDER UNILATERAL ASSUMPTION:

$$G_{max} = \frac{|Y_{21}|^2}{4\,\mathrm{Re}(Y_{22})\,\mathrm{Re}(Y_{11})}$$

THIS OCCURS UNDER MATCHED CONDITIONS.

$$|Y_{21}| = |g_{m1} \cdot R_1 \cdot g_{m2}|$$

$$\mathrm{Re}(Y_{11}) = g_{m1}$$

$$\mathrm{Re}(Y_{22}) = g_{m2}$$

$$G_{max} = \frac{1}{4}\,\frac{g_{m1}^2 R_1^2 g_{m2}^2}{g_{m1} \cdot g_{m2}}$$

$$= \frac{1}{4}\, g_{m1} R_1 \cdot g_{m2} R_1$$

$$= \frac{1}{\cancel{4}}\, \frac{6}{50\,\Omega}\, 200\,\Omega \cdot \frac{1}{5\,\Omega} \cdot \frac{\cancel{200}}{4}$$

$$= 24$$

(d) (*5 points*) Calculate the overall power series for this amplifier. Assume $R_S = 0\,\Omega$ and a square law device and neglect the device channel-length modulation. Neglect the second stage in the analysis.

$$v_o = G_{m1}(v_s) \cdot R_1 \, G_2$$

CONSTANT SAY $= 1/2$
FOR MATCHED

$$= \frac{1}{2} R_1 \left(g_{m1} v_s + g_{m2} v_s^2 \right)$$

SQUARE
LAW

$$I_D = \frac{\mu C_{ox}}{2}\left(\frac{W}{L}\right)(V_{GS} - V_t)^2$$

$$g_{m1} = \mu C_{ox}\left(\frac{W}{L}\right)(V_{GS} - V_t) \quad \left(= \frac{\partial I_D}{\partial V_{GS}} \right)$$

$$g_{m2} = \frac{1}{2} \frac{\mu C_{ox}}{2}\left(\frac{W}{L}\right) \quad \left(= \frac{1}{2!} \frac{\partial^2 I_D}{\partial V_{GS}^2} \right)$$

(e) (*12 points*) Calculate the overall power series for this amplifier but now include the source resistance. Assume the device is matched to the source. Neglect the second stage in the analysis.

USE FB ANALYSIS :

$$T = g_{m1} R_S = 1 \quad (\text{MATCHED})$$

$$g_{m1}' = \frac{g_{m1}}{1+T} = \frac{g_{m1}}{2}$$

$$g_{m2}' = \frac{g_{m2}}{(1+T)^3} = \frac{g_{m2}}{8}$$

$$g_{m3}' = -\frac{2 g_2^2 f}{(1+T)^5} = \frac{-2 g_{m2}^2 R_S}{2^5}$$

$$= -\frac{g_{m2}^2 R_S}{16}$$

(f) (*7 points*) Compute the IIP_3 of the amplifier in terms of the device V_{dsat} under impedance matched conditions. Neglect the second stage in the analysis and use your results from the pervious question.

$$V_{IIP_3} = \sqrt{\frac{4}{3}\left|\frac{a_1}{a_3}\right|}$$

$$= \sqrt{\frac{4}{3} \cdot \frac{g_{m1}}{2} \cdot \frac{16}{g_{m1}^2 \cdot R_S}}$$

$$= \sqrt{\frac{4}{3} \quad \frac{\cancel{\mu C_{ox}\left(\frac{W}{L}\right)} V_{sat}}{2} \cdot \frac{16}{\left(\frac{1}{4} \cdot \mu C_{ox} \frac{W}{L}\right)^2 R_S}}$$

$$= \sqrt{\frac{4}{6} \cdot \frac{V_{sat} \cdot 16^2}{\mu C_{ox}\left(\frac{W}{L}\right) \cdot R_S}}$$

$$= \frac{16 \cdot 2}{\sqrt{6}} \sqrt{\frac{V_{sat}/R_S}{\mu C_{ox}\left(\frac{W}{L}\right)}}$$

(g) (*5 points*) Why were you told to neglect the distortion of the second stage? Under what conditions is this assumption valid?

IF $R_L = \infty$, SINCE BODY/SOURCE ARE TIED TOGETHER, THERE IS NO V_T VARIATION $\Rightarrow$ LINEAR O/P STAGE (V_{GS} CONSTANT)

IF $R_L \neq \infty$, THE O/P CURRENT VARIES $\Rightarrow$ V_{GS} CHANGES $\Rightarrow$ DISTORTION.

NEED HIGH R_L $\rightarrow$ HIGH SOURCE DEGENERATION.