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THE ADJOINT SWITCHED CAPACITOR NETWORK AND ITS APPLICATION TO FREQUENCY, NOISE AND SENSITIVITY ANALYSIS

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SUMMARY

Given a linear network of capacitors, periodically controlled switches and sources we derive its adjoint network and show how it can be used to reduce the computational complexity of the frequency, noise and sensitivity analysis.

The construction of the adjoint switched capacitor circuit turns out to be quite simple, even for switched capacitor networks with more than two phases or with a continuous input–output path. Moreover this construction can be performed equivalently in the time domain or on the equivalent circuit.

In order to facilitate the implementation in a computer-aided-design program all derivations are performed in the modified nodal analysis formulation (MNA).

1. INTRODUCTION

Recently a lot of attention has been devoted to switched capacitor (SC) networks because they allow implementation of filters with low sensitivity in an integrated circuit. Since these circuits are often quite large, pen and paper techniques for time, frequency, noise and sensitivity analysis are of limited use. The widely used modified nodal analysis (MNA) formulation has allowed a simple implementation on computer[1,2]. However frequency, noise and sensitivity analysis require many network analyses. Hence it is an obvious question to wonder whether the adjoint network allows any simplification just as it did in the case of *RLC* networks. We show in this paper that an even greater reduction can be obtained for an SC network.

In Section 2 we review the basic MNA equations for time, frequency and z-domain analysis. Special attention is devoted to the maximal amount of information that can be derived from one network analysis. In Section 3 the adjoint switched capacitor network is constructed and it is shown to have a transfer matrix which is obtained by permuting the entries of the transfer matrix of the original network in a certain way. In Section 4 we derive equations for the frequency, noise and sensitivity analysis, which require a minimal number of analyses of the original SC network $\mathcal{N}$ and/or its adjoint $\tilde{\mathcal{N}}$ for some suitable excitations. We illustrate the equations with an example.

The adjoint network and the equations for frequency, noise and sensitivity analysis can be implemented in any SC network analysis program which is either based on time domain analysis and discrete Fourier transform[1–4] or on equivalent circuits.[2,5–6] We have realized the first approach by combining it with the general purpose SC network analysis program DIANA described in Reference 1. The results are excellent (see Reference 7 and the examples of Section 4). Using the sensitivities even network optimization can be performed.

All results and equations stated in this paper are very general in the sense that they apply to SC networks with many phases, arbitrary duty cycles and with or without continuous coupling between input and output. The linearity of controlled sources and capacitors however is always assumed. A preliminary version of some of the results can be found in Reference 11.

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2. THE MODIFIED NODAL ANALYSIS OF SWITCHED CAPACITOR NETWORKS

In this section we review the basic switched capacitor network analysis problems in the time, frequency and z-domain using the MNA framework. The equations have been derived in detail in Reference 2 and are interesting for practical circuit analysis programs[1] as well as for the derivation of the adjoint switched capacitor network in Section 3 as for frequency noise and sensitivity analysis (Section 4).

We consider arbitrary linear networks containing ideal switches, capacitors, independent voltage and charge sources (VS and QS) and four types of dependent sources (VCVS, QCVS, VCQS, QCQS). The switches are controlled by Boolean clock variables $\phi_i(t)=0$ or 1. $\phi_i(t)=0$ (resp., $\phi_i(t)=1$) corresponds to an open (resp., closed) switch at time t if this switch is driven by clock i. The time is partitioned into time slots $\Delta_k=(t_k, t_{k+1}]$ such that the clock signals do not vary in Δ_k i.e. $\phi_i(t)=\phi_{ik}$ for $t\in\Delta_k$. We assume that the clock signals are *T-periodic*, with N time slots (called N phases) in one period of duration T.

Theorem 1

If any above defined SC network is excited by a *piecewise-constant excitation* ($\mathbf{w}(t)=\mathbf{w}_m$, $\mathbf{u}(t)=\mathbf{u}_m$, $t\in\Delta_m$), then its response in the *time domain* is also piecewise-constant $\mathbf{v}(t)=\mathbf{v}_m$, $\mathbf{q}(t)=\mathbf{q}_m$ in Δ_m and given by:

$$\left[\begin{array}{c|c}\mathbf{A}_k & \mathbf{B}_k\\ \hline \mathbf{C}_k & \mathbf{D}_k\end{array}\right]\left[\begin{array}{c}\mathbf{v}_{k+lN}\\ \hline \mathbf{q}_{k+lN}\end{array}\right]=\left[\begin{array}{c}\mathbf{E}_k\mathbf{v}_{k+lN-1}\\ \hline \mathbf{0}\end{array}\right]+\left[\begin{array}{c}\mathbf{w}_{k+lN}\\ \hline \mathbf{u}_{k+lN}\end{array}\right] \tag{1}$$

where $\mathbf{v}(t)$ (resp., $\mathbf{u}(t)$) is the vector of the voltage responses at the nodes (resp., voltage sources in some selected branches) and where $\mathbf{q}(t)$ (resp., $\mathbf{w}(t)$) is the vector of the charges transferred in some selected branches (resp., injected by charge sources in the nodes) between the end t_{k+lN}^- of time slot Δ_{k+lN-1} and t for $t\in\Delta_{k+lN}$, and where the contributions to $\mathbf{A}_k$, $\mathbf{B}_k$, $\mathbf{C}_k$, $\mathbf{D}_k$ and $\mathbf{E}_k$ are given by the stamps of previous papers [Reference 2 (Figure 2), Reference 1 (Figure 2)].

By defining for one piecewise-constant signal $\{\mathbf{v}_1, \mathbf{v}_2 \ldots \mathbf{v}_N, \mathbf{v}_{N+1}, \ldots\}$ N z-transforms as:

$$\mathbf{V}_k(z)\triangleq\mathscr{Z}\{\mathbf{v}_{k+lN}\}=\sum_{l=0}^{\infty}\mathbf{v}_{k+lN}z^{-l}, \qquad k=1,2,\ldots N \tag{2}$$

and analogous definitions for $\mathbf{U}_k$, $\mathbf{Q}_k$ and $\mathbf{W}_k$ where $k=1,2,\ldots N$, the z-domain equations are:

$$\left[\begin{array}{cccc|cccc}
\mathbf{A}_1 & & & -\mathbf{E}_1z^{-1} & \mathbf{B}_1 & & & \\
-\mathbf{E}_2 & \mathbf{A}_2 & & & & \mathbf{B}_2 & & \\
 & -\mathbf{E}_3 & \mathbf{A}_3 & & & & \mathbf{B}_3 & \\
 & & \ddots & \ddots & & & & \ddots \\
 & & -\mathbf{E}_N & \mathbf{A}_N & & & & \mathbf{B}_N \\
\hline
\mathbf{C}_1 & & & & \mathbf{D}_1 & & & \\
 & \mathbf{C}_2 & & & & \mathbf{D}_2 & & \\
 & & \mathbf{C}_3 & & & & \mathbf{D}_3 & \\
 & & & \ddots & & & & \ddots \\
 & & & \mathbf{C}_N & & & & \mathbf{D}_N
\end{array}\right]
\left[\begin{array}{c}\mathbf{V}_1\\ \mathbf{V}_2\\ \mathbf{V}_3\\ \vdots\\ \mathbf{V}_N\\ \hline \mathbf{Q}_1\\ \mathbf{Q}_2\\ \mathbf{Q}_3\\ \vdots\\ \mathbf{Q}_N\end{array}\right]
=\left[\begin{array}{c}\mathbf{W}_1\\ \mathbf{W}_2\\ \mathbf{W}_3\\ \vdots\\ \mathbf{W}_N\\ \hline \mathbf{U}_1\\ \mathbf{U}_2\\ \mathbf{U}_3\\ \vdots\\ \mathbf{U}_N\end{array}\right] \tag{3}$$

where the missing entries are zero.

If the SC network is excited by an *arbitrary excitation* $\mathbf{w}(t)$, $\mathbf{u}(t)$, then the piecewise-constant part of the input and output $\mathbf{w}_m\triangleq\mathbf{w}(t_{m+1}^-)$, $\mathbf{u}_m\triangleq\mathbf{u}(t_{m+1}^-)$, $\mathbf{v}_m\triangleq\mathbf{v}(t_{m+1}^-)$ and $\mathbf{q}_m\triangleq\mathbf{q}(t_{m+m}^-)$ are determined by (1) and the remainder waveforms $\mathbf{w}^*(t)\triangleq\mathbf{w}(t)-\mathbf{w}_m$, $\mathbf{u}^*(t)\triangleq\mathbf{u}(t)-\mathbf{u}_m$, $\mathbf{v}^*(t)\triangleq\mathbf{v}(t)-\mathbf{v}_m$ and $\mathbf{q}^*(t)\triangleq\mathbf{q}(t)-\mathbf{q}_m$ for $t\in\Delta_m$ are related by:

$$\left[\begin{array}{c|c}\mathbf{A}_k & \mathbf{B}_k\\ \hline \mathbf{C}_k & \mathbf{D}_k\end{array}\right]\left[\begin{array}{c}\mathbf{v}^*(t)\\ \hline \mathbf{q}^*(t)\end{array}\right]=\left[\begin{array}{c}\mathbf{w}^*(t)\\ \hline \mathbf{u}^*(t)\end{array}\right] \quad \text{for } t\in\Delta_{k+lN} \tag{4}$$

Corollary 1

The z-domain inputs $\mathbf{W}_k, \mathbf{U}_k$ are related to the z-domain outputs $\mathbf{V}_k, \mathbf{Q}_k$ by the z-domain transfer matrix $\mathbf{M}$ of:

$$\left[\begin{array}{c} \mathbf{V}_1 \\ \mathbf{V}_2 \\ \cdots \\ \mathbf{V}_N \\ \hdashline \mathbf{Q}_1 \\ \mathbf{Q}_2 \\ \cdots \\ \mathbf{Q}_N \end{array}\right] = \underbrace{\left[\begin{array}{cccc|cccc} \mathbf{G}_{11} & \mathbf{G}_{12} & \cdots & \mathbf{G}_{1N} & \mathbf{H}_{11} & \mathbf{H}_{12} & \cdots & \mathbf{H}_{1N} \\ \mathbf{G}_{21} & \mathbf{G}_{22} & \cdots & \mathbf{G}_{2N} & \mathbf{H}_{21} & \mathbf{H}_{22} & \cdots & \mathbf{H}_{2N} \\ \cdots & \cdots & & & \cdots & \cdots & & \\ \mathbf{G}_{N1} & \mathbf{G}_{N2} & \cdots & \mathbf{G}_{NN} & \mathbf{H}_{N1} & \mathbf{H}_{N2} & \cdots & \mathbf{H}_{NN} \\ \hdashline \mathbf{K}_{11} & \mathbf{K}_{12} & \cdots & \mathbf{K}_{1N} & \mathbf{L}_{11} & \mathbf{L}_{12} & \cdots & \mathbf{L}_{1N} \\ \mathbf{K}_{21} & \mathbf{K}_{22} & \cdots & \mathbf{K}_{2N} & \mathbf{L}_{21} & \mathbf{L}_{22} & \cdots & \mathbf{L}_{2N} \\ \cdots & & & & & & & \\ \mathbf{K}_{N1} & \mathbf{K}_{N2} & \cdots & \mathbf{K}_{NN} & \mathbf{L}_{N1} & \mathbf{L}_{N2} & \cdots & \mathbf{L}_{NN} \end{array}\right]}_{\mathbf{M}} \left[\begin{array}{c} \mathbf{W}_1 \\ \mathbf{W}_2 \\ \cdots \\ \mathbf{W}_N \\ \hdashline \mathbf{U}_1 \\ \mathbf{U}_2 \\ \cdots \\ \mathbf{U}_N \end{array}\right] \tag{5}$$

In other words if only non-zero voltage sources are applied in time slots $l, l+N, l+2N, \ldots$ and if the output node voltages are only observed at the end of time slots k, $k+N$, $k+2N, \ldots$ then $\mathbf{H}_{kl}(z)$ relates the z-transform of this input sequence $\mathbf{U}_l(z)$ to this output sequence $\mathbf{V}_k(z)$.

Theorem 2

The above described linear T-periodic SC network generates for a voltage excitation $\mathbf{u}(t)$ with Fourier transform $\mathbf{U}(\omega)$ a voltage response $\mathbf{v}(t)$ with *Fourier transform* $\mathbf{V}(\omega)$ given by:

$$\mathbf{V}(\omega) = \sum_{n=-\infty}^{+\infty} \sum_{k=1}^{N} \mathbf{X}_k(\omega, \omega - n\omega_s)\mathbf{U}(\omega - n\omega_s) \tag{6}$$

where $\omega_s = 2\pi/T$, and the *transmission function* $\mathbf{X}_k\ (\omega, \Omega)$ for phase k is given by:

$$\mathbf{X}_k(\omega, \Omega) = \nu_k(\omega) \sum_{l=1}^{N} \mathbf{H}_{kl}(\mathrm{e}^{j\Omega T}) \exp\left[j(\Omega t_{l+1} - \omega t_{k+1})\right] + \left[\nu_k(\omega - \Omega) - \nu_k(\omega)\right] \exp\left[j(\Omega - \omega)t_{k+1}\right]\mathbf{H}_{kk}(\infty) \tag{7}$$

with $\mathbf{H}_{kl}(z)$ defined by (5) and

$$\nu_k(\rho) = 2\,\{\sin\left[\rho(t_{k+1} - t_k)/2\right] \exp\left[j\rho(t_{k+1} - t_k)/2\right]\}/T\rho \tag{8}$$

Often one is only interested in the spectrum of the voltage waveform $\hat{\mathbf{v}}_k(t)$, which is the voltage $\mathbf{v}(t)$ observed during phase k and zero outside. $\mathbf{V}_k(\omega)$ only contains those terms of (6) corresponding to the appropriate value k.

The proof of this theorem relies on standard Fourier transform techniques and is given in the appendix of Reference 2. Clearly in an SC network a sinusoidal excitation with pulsation ω generates an output with components at all pulsations $\omega + n\omega_s$. In practice the contribution for $n = 0$ is given by the frequency domain transfer functions $\mathbf{H}(\omega)$ (resp., $\mathbf{H}_k(\omega)$ which relates the phasor $\mathbf{U}$ of a sinusoidal excitation $\mathbf{u}(t) = \mathbf{U}\,\mathrm{e}^{j\omega T}$ to the phasor at the *same pulsation* ω in the output $\mathbf{v}(t)$ (resp., $\hat{\mathbf{v}}_k(t)$).

$$\mathbf{H}_k(\omega) = \nu_k(\omega) \sum_{l=1}^{N} \mathbf{H}_{kl}\,(\mathrm{e}^{j\omega T}) \exp\left[j\omega(t_{l+1} - t_{k+1})\right] + \left[(t_{k+1} - t_k)/T - \nu_k(\omega)\right]\mathbf{H}_{kk}(\infty) \tag{9a}$$

$$\mathbf{H}(\omega) = \sum_{k=1}^{N} \mathbf{H}_k(\omega) \tag{9b}$$

3. DERIVATION OF THE ADJOINT SWITCHED CAPACITOR NETWORK

In section 4 we will see that the number of computations needed in many analysis problems of switched capacitor networks can be greatly reduced by not analysing the original network $\mathcal{N}$, but its *adjoint network* $\tilde{\mathcal{N}}$. However an SC network is a linear T-periodic time-varying network and hence the usual definition and construction[8,9] for linear time-invariant networks should be extended. Another approach would be to derive a definition from the adjoint for non-linear time-varying networks.[10] It turns out to be more straightforward to describe first a systematic construction of the adjoint SC network and to prove afterwards that this SC network has indeed the useful properties. The derivation will be based on the MNA matrices but can be obtained also in the nodal or tableau analysis.

Construction of the adjoint SC network $\tilde{\mathcal{N}}$ of an SC network $\mathcal{N}$

(1) Construct a duplicate of $\mathcal{N}$ and denote the sources by $\tilde{\mathbf{u}}$ and $\tilde{\mathbf{w}}$.

(2) Modify the non-reciprocal elements (controlled sources) as follows. Interchange the controlling and controlled ports. Replace a VCVS by a QCQS and vice versa. Multiply the controlling coefficient by -1 in the case of a QCQS or VCVS.

(3) Reverse the time dependency of all time-varying elements in one period, i.e. the new clock signal $\tilde{\phi}_{rk}$ and the switching times $\tilde{t}_i$ satisfy

$$\tilde{\phi}_{rk} = \phi_{r\tilde{k}}, \qquad \tilde{k} = N - k + 1 \tag{10a}$$

$$\tilde{t}_{lN+k} = t_{lN+1} + t_{(l+1)N+1} - t_{(l+1)N+2-k} \tag{10b}$$

for $l = 0, 1, 2 \ldots$ and $k = 1, 2, \ldots N$.

For example, by applying this algorithm on the SC filter section of Figure 1(a) with clock signals (b) one obtains the adjoint SC network of Figure 1(c) with clock signals (d).

Observe that step 3 of the construction need not be executed if all clock signals are symmetric in one period. This is among others the case for clock signals with only two time slots of equal duration. Moreover, compared to the usual construction of the adjoint for linear time-invariant networks only step 3 is new and thus it takes into account that an SC network is a T-periodic network.

The useful property of the adjoint network of a linear time-invariant network is that its impedance matrix is the transpose of that of the original matrix. In the next theorem we prove that for an SC network one has to perform on the z-domain transfer matrix (5) additionally an operation which corresponds with the reversion of time in a period. Let the operation of *block row and column reversion* be defined and denoted as:

$$\left[\begin{array}{cccc:cccc}
\mathbf{G}_{11} & \mathbf{G}_{12} & \ldots & \mathbf{G}_{1N} & \mathbf{H}_{11} & \mathbf{H}_{12} & \ldots & \mathbf{H}_{1N} \\
\mathbf{G}_{21} & \mathbf{G}_{22} & \ldots & \mathbf{G}_{2N} & \mathbf{H}_{21} & \mathbf{H}_{22} & \ldots & \mathbf{H}_{2N} \\
\ldots & \ldots & & \ldots & \ldots & \ldots & & \ldots \\
\mathbf{G}_{N1} & \mathbf{G}_{N2} & & \mathbf{G}_{NN} & \mathbf{H}_{N1} & \mathbf{H}_{N2} & \ldots & \mathbf{H}_{NN} \\
\hdashline
\mathbf{K}_{11} & \mathbf{K}_{12} & \ldots & \mathbf{K}_{1N} & \mathbf{L}_{11} & \mathbf{L}_{12} & \ldots & \mathbf{L}_{1N} \\
\mathbf{K}_{21} & \mathbf{K}_{22} & \ldots & \mathbf{K}_{2N} & \mathbf{L}_{21} & \mathbf{L}_{22} & \ldots & \mathbf{L}_{2N} \\
\ldots & \ldots & & \ldots & \ldots & \ldots & & \ldots \\
\mathbf{K}_{N1} & \mathbf{K}_{N2} & \ldots & \mathbf{K}_{NN} & \mathbf{L}_{N1} & \mathbf{L}_{N2} & \ldots & \mathbf{L}_{NN}
\end{array}\right]^{\S}
=
\left[\begin{array}{cccc:cccc}
\mathbf{G}_{NN} & \ldots & \mathbf{G}_{N2} & \mathbf{G}_{N1} & \mathbf{H}_{NN} & \ldots & \mathbf{H}_{N2} & \mathbf{H}_{N1} \\
\ldots & & \ldots & \ldots & \ldots & & \ldots & \ldots \\
\mathbf{G}_{2N} & \ldots & \mathbf{G}_{22} & \mathbf{G}_{21} & \mathbf{H}_{2N} & \ldots & \mathbf{H}_{22} & \mathbf{H}_{21} \\
\mathbf{G}_{1N} & \ldots & \mathbf{G}_{12} & \mathbf{G}_{11} & \mathbf{H}_{1N} & \ldots & \mathbf{H}_{12} & \mathbf{H}_{11} \\
\hdashline
\mathbf{K}_{NN} & \ldots & \mathbf{K}_{N2} & \mathbf{K}_{N1} & \mathbf{L}_{NN} & \ldots & \mathbf{L}_{N2} & \mathbf{L}_{N1} \\
\ldots & & \ldots & \ldots & \ldots & & \ldots & \ldots \\
\mathbf{K}_{2N} & \ldots & \mathbf{K}_{22} & \mathbf{K}_{21} & \mathbf{L}_{2N} & \ldots & \mathbf{L}_{22} & \mathbf{L}_{21} \\
\mathbf{K}_{1N} & \ldots & \mathbf{K}_{12} & \mathbf{K}_{11} & \mathbf{L}_{1N} & \ldots & \mathbf{L}_{12} & \mathbf{L}_{11}
\end{array}\right] \tag{11}$$

Theorem 4

Given a switched capacitor network $\mathcal{N}$ with z-domain transfer matrix $\mathbf{M}$ and its adjoint network $\tilde{\mathcal{N}}$ with z-domain transfer matrix $\tilde{\mathbf{M}}$, then*

$$\tilde{\mathbf{M}} = \mathbf{M}^{\S T} \tag{12}$$

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* This result is consistent with the fact that $\mathbf{M}$ is a *hybrid* matrix and the property of the adjoint network in terms of the hybrid matrix, since the charge-source branches do not have associated reference directions.

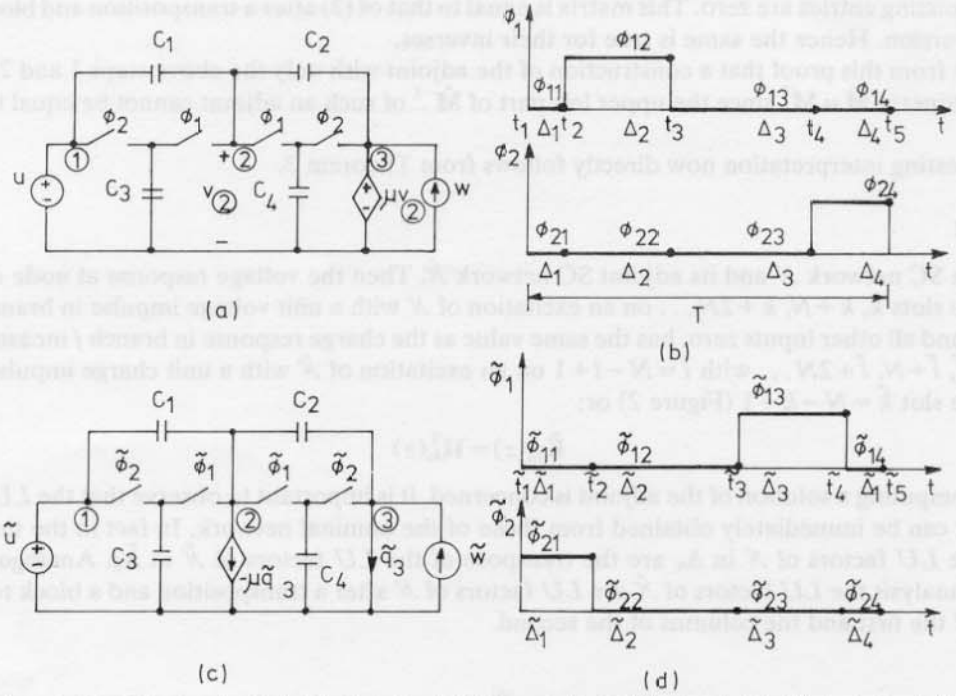

Figure 1. The SC network of (a) with clock signals (b) has an adjoint SC network (c) with clock signals (d)

Proof. The basic idea of the proof is to relate the matrices $\tilde{\mathbf{A}}_k \tilde{\mathbf{B}}_k \tilde{\mathbf{C}}_k \tilde{\mathbf{D}}_k$ and $\tilde{\mathbf{E}}_k$ of the time domain description

$$\left[\begin{array}{c|c} \tilde{\mathbf{A}}_k & \tilde{\mathbf{B}}_k \\ \hline \tilde{\mathbf{C}}_k & \tilde{\mathbf{D}}_k \end{array}\right] \left[\begin{array}{c} \tilde{\mathbf{v}}_{k+lN} \\ \hline \tilde{\mathbf{q}}_{k+lN} \end{array}\right] = \left[\begin{array}{c} \tilde{\mathbf{E}}_k \tilde{\mathbf{v}}_{k-1+lN} \\ \hline \mathbf{0} \end{array}\right] + \left[\begin{array}{c} \tilde{\mathbf{w}}_{k+lN} \\ \hline \tilde{\mathbf{u}}_{k+lN} \end{array}\right] \tag{13}$$

of $\tilde{\mathcal{N}}$ to the matrices $\mathbf{A}_k \mathbf{B}_k \mathbf{C}_k \mathbf{D}_k$ and $\mathbf{E}_k$ of $\mathcal{N}$. In fact we want to show that for $k = 1, 2, \ldots N$

$$\begin{aligned} &\tilde{\mathbf{A}}_k = \mathbf{A}^{\mathrm{T}}_{N-k+1}, \qquad \tilde{\mathbf{D}}_k = \mathbf{D}^{\mathrm{T}}_{N-k+1}, \\ &\tilde{\mathbf{B}}_k = \mathbf{C}^{\mathrm{T}}_{N-k+1}, \qquad \tilde{\mathbf{C}}_k = \mathbf{B}^{\mathrm{T}}_{N-k+1}, \\ &\tilde{\mathbf{E}}_k = \mathbf{E}^{\mathrm{T}}_{N-k'+1}, \quad \text{where } k' = k-1 \quad \text{for } k = 2, \ldots N \quad \text{and} \quad k' = N \quad \text{for} \quad k = 1. \end{aligned} \tag{14}$$

It follows now from the construction of the adjoint SC network and the stamps of Reference 2 (simple and composite branches) that the contributions of corresponding components of $\mathcal{N}$ and $\tilde{\mathcal{N}}$ to the matrices in (1) and (13) satisfy (14). Hence (14) is satisfied since the matrices are obtained by accumulating all these contributions.

From (14) we obtain the following z-domain transfer matrix of

$$\tilde{\mathbf{M}} = \left[\begin{array}{ccccc|cccc} \mathbf{A}^{\mathrm{T}}_N & & & & -\mathbf{E}^{\mathrm{T}}_1 z^{-1} & \mathbf{C}^{\mathrm{T}}_N & & & \\ -\mathbf{E}^{\mathrm{T}}_N & \mathbf{A}^{\mathrm{T}}_{N-1} & & & & & \mathbf{C}^{\mathrm{T}}_{N-1} & & \\ & -\mathbf{E}^{\mathrm{T}}_{N-1} & \mathbf{A}^{\mathrm{T}}_{N-2} & & & & & \mathbf{C}^{\mathrm{T}}_{N-2} & \\ & & \ddots & -\mathbf{E}^{\mathrm{T}}_2 & \ddots\ \mathbf{A}^{\mathrm{T}}_1 & & & \ddots & \mathbf{C}^{\mathrm{T}}_1 \\ \hline \mathbf{B}^{\mathrm{T}}_N & & & & & \mathbf{D}^{\mathrm{T}}_N & & & \\ & \mathbf{B}^{\mathrm{T}}_{N-1} & & & & & \mathbf{D}^{\mathrm{T}}_{N-1} & & \\ & & \mathbf{B}^{\mathrm{T}}_{N-2} & & & & & \mathbf{D}^{\mathrm{T}}_{N-2} & \\ & & & \ddots & & & & & \ddots \\ & & & & \mathbf{B}^{\mathrm{T}}_1 & & & & \mathbf{D}^{\mathrm{T}}_1 \end{array}\right] \tag{15}$$

where the missing entries are zero. This matrix is equal to that of (3) after a transposition and block row and column reversion. Hence the same is true for their inverses. □

It is clear from this proof that a construction of the adjoint with only the above steps 1 and 2 would not have the property $\tilde{\mathbf{M}} = \mathbf{M}^{\mathrm{T}}$ since the upper left part of $\tilde{\mathbf{M}}^{-1}$ of such an adjoint cannot be equal to that part of $\mathbf{M}^{-1\mathrm{T}}$.

An interesting interpretation now directly follows from Theorem 3.

Corollary 2

Given an SC network $\mathcal{N}$ and its adjoint SC network $\tilde{\mathcal{N}}$. Then the voltage response at node i measured during time slots k, $k+N$, $k+2N\ldots$ on an excitation of $\mathcal{N}$ with a unit voltage impulse in branch j during time slot l and all other inputs zero, has the same value as the charge response in branch j measured during time slots $\tilde{l}$, $\tilde{l}+N$, $\tilde{l}+2N\ldots$ with $\tilde{l} = N - l + 1$ on an excitation of $\tilde{\mathcal{N}}$ with a unit charge impulse in node i during time slot $\tilde{k} = N - k + 1$ (Figure 2) or:

$$\tilde{\mathbf{K}}_{\tilde{l}\tilde{k}}(z) = \mathbf{H}_{kl}^{\mathrm{T}}(z) \tag{16}$$

As far as computing a solution of the adjoint is concerned, it is important to observe that the LU factors of the adjoint can be immediately obtained from those of the nominal network. In fact in the time-domain analysis the LU factors of $\mathcal{N}$ in Δ_k are the transpose of the LU factors of $\tilde{\mathcal{N}}$ in $\tilde{\Delta}_{\tilde{k}}$. Analogously in the z-domain analysis the LU factors of $\tilde{\mathcal{N}}$ are LU factors of $\mathcal{N}$ after a transposition and a block reversion of the rows of the first and the columns of the second.

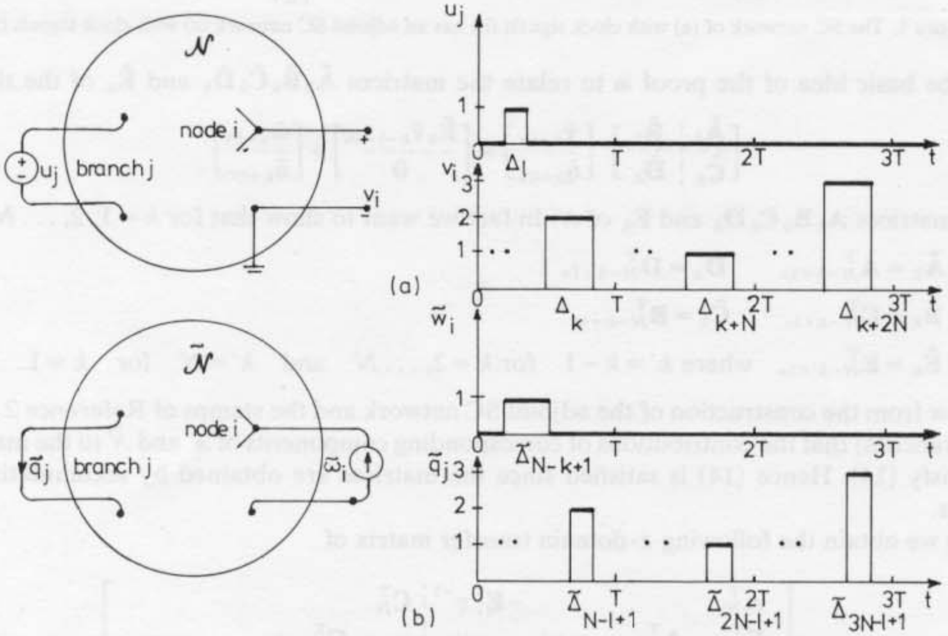

Figure 2. For a unit voltage excitation in branch j at phase l the voltage response at node i during phase k of the original network $\mathcal{N}$(a), is the same as the charge response in branch j at phase $N-l+1$ on a unit charge excitation at node i during phase $N-k+1$ in the adjoint network $\tilde{\mathcal{N}}$(b)

Given an SC network $\mathcal{N}$ and its adjoint SC network $\tilde{\mathcal{N}}$, we can apply the construction of an equivalent network of Reference 2 on both networks and obtain the equivalent network $\mathcal{N}_{\mathrm{e}}$ and the equivalent adjoint network $\tilde{\mathcal{N}}_{\mathrm{e}}$. For those designers who are more familiar with the equivalent SC networks, it can be shown that if the usual construction of the adjoint for linear time-invariant circuits is applied on $\mathcal{N}_{\mathrm{e}}$ and if also the N phases of any charge or voltage waveform are reversed then $\tilde{\mathcal{N}}_{\mathrm{e}}$ is obtained.

4. FREQUENCY, NOISE AND SENSITIVITY COMPUTATIONS USING THE ADJOINT SC NETWORK

It will appear that the computational advantages of the use of the adjoint switched capacitor network in the computation of frequency, noise and sensitivity properties are even more impressive than in the time-invariant case. As described in Reference 2 frequency-domain characteristics can be obtained via the solution of the time-domain equations (1) and discrete Fourier transform or via the solution of z-domain equations (3). Here we illustrate the advantages for the first approach as it is implemented in DIANA.[1] The same equations can be used in the second approach but recent investigations have shown that in this case further improvements are possible which are beyond the scope of this paper.

The key idea is to make optimal use of the information available from one or more analyses of a switched capacitor network and its adjoint under selected excitations. By computing the response on a pulse in phase l using (1) and by using DFT on these signals the value of one column of $\mathbf{M}$ for some values $z = e^{j\omega T}$ is known. This we call one network analysis and we compare the number of network analyses needed with and without the adjoint SC network.

4.1 *Computation of the frequency properties of SC networks*

The use of SC networks relies on their abilities to generate suitable frequency domain transfer function $H_k^{(ij)}(\omega)$ between the voltage source in branch j and the voltage $\hat{v}_k^{(i)}(t)$ at node i. If we use (9a) in order to compute $H_k^{(ij)}(\omega)$, the entries $H_{kl}^{(ij)}(e^{j\omega T})$ for $l = 1, \ldots N$ are needed, which are obtained by performing N network analyses of the original circuit. Using (16) of Theorem 3 we can convert (9a) into:

$$H_k^{(ij)}(\omega) = \nu_k(\omega) \sum_{l=1}^{N} \tilde{K}_{\tilde{l}\tilde{k}}^{(ji)}(e^{j\omega T}) \exp[j\omega(t_{l+1} - t_{k+1})] + [(t_{k+1} - t_k)/T - \nu_k(\omega)]\tilde{K}_{\tilde{l}\tilde{k}}^{(ji)}(\infty)$$

with $\tilde{l} = N - l + 1$ and $\tilde{k} = N - k + 1$. All $\tilde{K}_{\tilde{l}\tilde{k}}^{(ji)}(e^{j\omega T})$ for $\tilde{l} = 1, \ldots N$ can be obtained from *one network analysis* of the adjoint network.

The same substitution can be performed in the transmission function $\mathbf{X}_k(\omega, \Omega)$ (7) so that all effects of high frequency inputs on the baseband of the output (aliasing) can be studied using one network analysis of the adjoint network. Of course if one is interested in $H^{(ij)}(\omega)$ all N^2 entries $H_{kl}^{(ij)}(e^{j\omega T})$ for $k, l = 1, \ldots N$ are needed and N network analyses are needed if the original or the adjoint network is used. However it must be noted that in most of the practical SC networks one is only interested in one phase k because the output in the other phases is only a sample and hold version of phase k. Let us illustrate the computations of the transfer function and the aliasing effects with an example obtained using the DIANA program[1,7], which is adapted to handle arbitrary SC circuits.

Example. Consider the treble tone control filter of Figure 1(a) with $C_1 = 7{\cdot}353$ pF, $C_2 = 1{\cdot}739$ pF, $C_3 = C_4 = 1$pF and $\mu = -1{,}000$, and with clock signals of Figure 1(b) and $T = 10^{-5}$ s. The plot of the voltage amplification transfer function (amplitude in dB) between u and $\hat{v}_{24}$ (phase 1) for input frequencies F varying (logarithmically) from $F_s/1{,}024 = 1/1{,}024T$ to $F_s/2$ is given in Figure 3(a) (solid line). The folding effect A_m (aliasing) of input frequencies F (top) in u varying between $F_s/2$ and F_s onto the baseband frequencies $F_s - F$ (bottom) in $\hat{v}_{24}$ is represented by the dotted curve. Both are obtained by analysing the adjoint switched capacitor network *once* with a unit excitation $\hat{w}$ during phase 4, computing 4 Fourier transforms (FFT) and combining the results as described above.

4.2. *Noise analysis of SC networks*

Usually one is interested in the output noise voltage at one node i due to the noise in many (say p) components. We assume that the noise of each device can be modelled by a voltage source whose voltage is a stationary stochastic process. The vector $\mathbf{u}(t)$ of the input noise voltage sources is then characterized by a spectral density matrix $\mathbf{S}_{\mathbf{u}}(\omega)$ which depends on the technology and which we assume to be known. Since the circuit is time-varying the output voltage $v^{(i)}(t)$ is not a stationary stochastic process. By the same

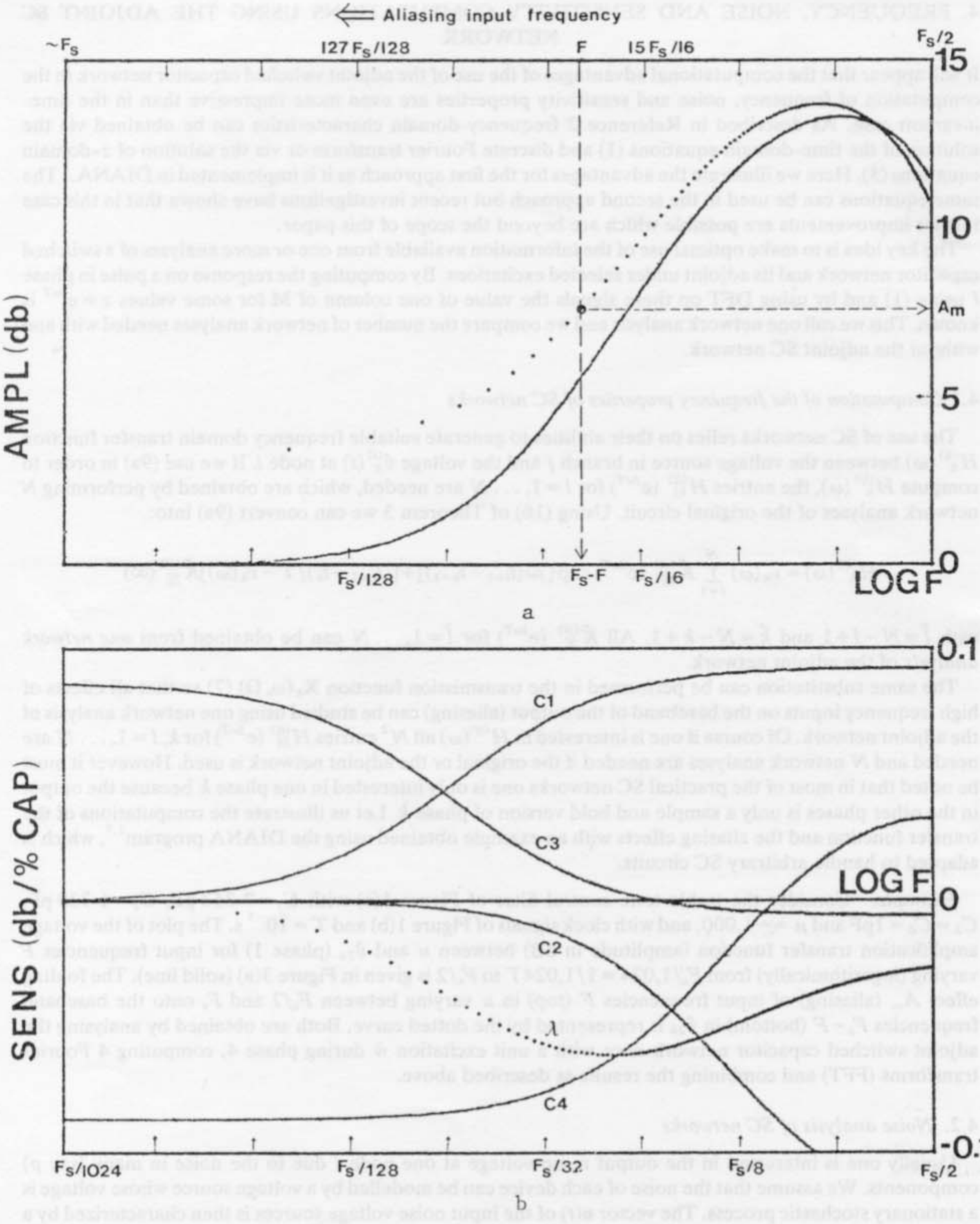

Figure 3. Computer analysis results of the SC network of Figure 1. (a) the amplitude of the transfer function and the aliasing effect, and (b) the sensitivities

computations as those of Reference 4 the average spectral density function is given by

$$S_{v^{(i)}}(\omega)=\sum_{n=-\infty}^{+\infty}\sum_{k=1}^{N}\mathbf{X}_k^{(i.)}\left(\omega,\omega-n\frac{2\pi}{T}\right)\mathbf{S}_{\mathbf{u}}\left(\omega-n\frac{2\pi}{T}\right)\sum_{j=1}^{N}\mathbf{X}_j^{(i.)*}\left(\omega,\omega-n\frac{2\pi}{T}\right) \tag{18}$$

where* denotes complex conjugate and transpose, and

$\mathbf{X}_k^{(i.)}(\omega,\Omega)$ is the ith row of $\mathbf{X}_k(\omega,\Omega)$ of (9)

If p is the number of noise voltage sources, then from (7) and (18), the computation of $S_v(i)$ requires N^2p different $H_{kl}^{(ij)}$ for $k, l=1,2,\ldots N$ and where j assumes p different values and hence it needs *Np different network analyses*. Using Theorem 3 the ith row of the transmission function is given by:

$$\begin{aligned}\mathbf{X}_k^{(i.)}(\omega,\Omega)=\nu_k(\omega)\sum_{l=1}^{N}\tilde{\mathbf{K}}_{\tilde{l}\tilde{k}}^{(.i)\mathrm{T}}(\mathrm{e}^{j\Omega T})\exp\left[j(\Omega t_{l+1}-\omega t_{k+1})\right]\\+\left[\nu_k(\omega-\Omega)-\nu_k(\omega)\right]\exp\left[j(\Omega-\omega)t_{k+1}\right]\tilde{\mathbf{K}}_{\tilde{k}\tilde{k}}^{(.i)}(\infty)^{\mathrm{T}}\end{aligned} \tag{19}$$

where $\tilde{l}=N-l+1$ and $\tilde{k}=N-k+1$. This computation requires only one adjoint network analysis, since all entries of a column of $\mathbf{M}$ can be obtained from one network analysis. If we use (19) *N adjoint network analyses* are needed. Since practical circuits have many noise sources, $(p>1)$, this implies that the use of the adjoint allows considerable savings in the computations.

4.3 *Sensitivity analysis of SC networks*

In sensitivity analysis one is basically interested in the variation in the input–output relation due to a variation in one or several of the internal parameters. First we obtain the derivative of an arbitrary entry $H_{kl}^{(ij)}(z)$ of the z-domain transfer function (5) with respect to a parameter and combine these equations then in the computation of the sensitivities of the frequency-domain transfer function $H_k^{(ij)}(\omega)$.

Let the MNA matrix in (1) have a size $s\times s$, then the transfer matrix $\mathbf{M}$ of (5), has a size $Ns\times Ns$. From (3) and the circuit realization one can determine the derivative of the matrix $\mathbf{M}^{-1}$ of (3) with respect to any parameter λ (e.g. a capacitance, or gain or temperature or a technological or geometric parameter). By differentiating $\mathbf{M}\mathbf{M}^{-1}=\mathbf{1}$ with respect to λ and left multiplication with $\mathbf{M}$, we obtain

$$\frac{\partial\mathbf{M}}{\partial\lambda}=-\mathbf{M}\frac{\partial\mathbf{M}^{-1}}{\partial\lambda}\mathbf{M} \tag{20}$$

Clearly the right hand side is easier to calculate since $\partial\mathbf{M}^{-1}/\partial\lambda$ is known from the design and $\mathbf{M}$ is known by inverting the matrix in (3) which implies Ns network analyses. If only one (or a few) entries of $\partial\mathbf{M}/\partial\lambda$ are needed, the number of network analyses can be greatly reduced by using the adjoint network. If one is interested in the derivative of $H_{kl}^{(ij)}(z)$, $z=\mathrm{e}^{j\omega T}$ (i.e. the z-transform transfer function from the voltage in branch j during phase l to the voltage at node i and during phase k) with respect to the parameter λ we will show that *one original and one adjoint network analysis* are sufficient. From

$$\frac{\partial\mathbf{M}}{\partial\lambda}=-\tilde{\mathbf{M}}^{\S\mathrm{T}}\frac{\partial\mathbf{M}^{-1}}{\partial\lambda}\mathbf{M} \tag{21}$$

and the contributions of the components to the matrix of (3) we obtain the following derivatives:

(a) For a capacitor C connecting node m to node n we have:

$$\begin{aligned}\frac{\partial H_{kl}^{(ij)}(z)}{\partial C}=-\sum_{r=1}^{N}(\tilde{G}_{\tilde{r}\tilde{k}}^{(mi)}-\tilde{G}_{\tilde{r}\tilde{k}}^{(ni)})(H_{rl}^{(mj)}-H_{rl}^{(nj)})+\sum_{r=1}^{N-1}(\tilde{G}_{\tilde{r}-1\tilde{k}}^{(mi)}-\tilde{G}_{\tilde{r}-1\tilde{k}}^{(ni)})(H_{rl}^{(mj)}-H_{rl}^{(nj)})\\+z^{-1}(\tilde{G}_{1\tilde{k}}^{(mi)}-\tilde{G}_{1\tilde{k}}^{(ni)})(H_{Nl}^{(mj)}-H_{Nl}^{(nj)})\end{aligned} \tag{22}$$

with $\tilde{r}=N-r+1$ and $\tilde{k}=N-k+1$.

(b) For a voltage controlled voltage source where the voltage in branch q is A times the voltage of node m with respect to node n we have:

$$\frac{\partial H_{kl}^{(ij)}(z)}{\partial A}=\sum_{r=1}^{N}\tilde{K}_{\tilde{r}\tilde{k}}^{(qi)}\,(H_{rl}^{(mj)}-H_{rl}^{(nj)}) \tag{23}$$

(c) If all capacitors are a known function of a parameter λ then the matrices $\partial\mathbf{A}_k/\partial\lambda$ are known from the design and:

$$\frac{\partial H_{kl}^{(ij)}(z)}{\partial\lambda}=-\sum_{r=1}^{N}\tilde{\mathbf{G}}_{\tilde{r}\tilde{k}}^{(.i)\mathrm{T}}\frac{\partial\mathbf{A}_r}{\partial\lambda}\mathbf{H}_{rl}^{(.j)}+\sum_{r=1}^{N-1}\tilde{\mathbf{G}}_{\tilde{r}-1\tilde{k}}^{(.i)\mathrm{T}}\frac{\partial\mathbf{A}_{r+1}}{\partial\lambda}\mathbf{H}_{rl}^{(.j)}+z^{-1}\tilde{\mathbf{G}}_{1k}^{(.i)\mathrm{T}}\frac{\partial\mathbf{A}_1}{\partial\lambda}\mathbf{H}_{Nl}^{(.j)} \tag{24}$$

where $\mathbf{H}_{rl}^{(.j)}$ is again the jth column of $\mathbf{H}_{rl}$.
(d) Later we will need another derivative:

$$\frac{\partial H_{kl}^{(ij)}(z)}{\partial z}=-z^{-2}\tilde{\mathbf{G}}_{1k}^{(.i)\mathrm{T}}\mathbf{A}_1\mathbf{H}_{Nl}^{(.j)} \tag{25}$$

If composite branches are used in the MNA equations, only those terms in the sums (22, 24) have to be taken which physically appear in that phase. The derivatives with respect to the gain of QCVS, VCQS and QCQS can be determined analogously as those for a VCVS but are of less practical value and are hence omitted.

In practice one is interested in the sensitivities of the frequency domain transfer functions (9). We present here the sensitivities of $H_k^{(ij)}$ (i.e. at phase k) in terms of the derivatives (22–25), which can be easily verified from (9a).

(a)–(c) For a parameter μ which can be a capacitance or an amplification or a parameter which affects all capacitors we have:

$$\begin{aligned}S_{\mu}^{H_k^{(ij)}}(\omega)&\triangleq\frac{\mu}{H_k^{(ij)}(\omega)}\frac{\partial H_k^{(ij)}(\omega)}{\partial\mu}\\&=\frac{\lambda}{H_k^{(ij)}(\omega)}\left\{\nu_k(\omega)\sum_{l=1}^{N}\frac{\partial H_{kl}^{(ij)}(\mathrm{e}^{j\omega T})}{\partial\mu}\exp\left[j\omega(t_{l+1}-t_{k+1})\right]+\left[(t_{k+1}-t_k)/T-\nu_k(\omega)\right]\frac{\partial H_{kk}^{(ij)}(\infty)}{\partial\mu}\right\}\end{aligned} \tag{26}$$

(d) We illustrate the effect of variation in the clock switching time t_{m+1} in each period for the case, $m=1,2,\ldots N$ and $m\neq k-1,k$

$$\frac{\partial H_k^{(ij)}(\omega)}{\partial t_{m+1}}=j\omega\nu_k(\omega)\exp\left[j\omega(t_{m+1}-t_{k+1})\right]H_{km}^{(ij)}(\mathrm{e}^{j\omega T}), \tag{27}$$

For a variation in the clock time scale τ we have:

$$\begin{aligned}\frac{\partial H_k^{(ij)}(\omega)}{\partial\tau}&\triangleq\sum_{m=1}^{N}\frac{t_{m+1}-t_1}{T}\frac{\partial H_k^{(ij)}(\omega)}{\partial t_{m+1}}+\frac{\partial H_k^{(ij)}(\omega)}{\partial T}\\&=j\omega\nu_k(\omega)\sum_{l=1}^{N}\exp\left[j\omega(t_{l+1}-t_{k+1})\right]\left.\frac{\partial H_{kl}^{(ij)}(z)}{\partial z}\right|_{z=\mathrm{e}^{j\omega T}}\\&\quad+\{\nu_k(\omega)[T-(j\omega T+1/\nu_k(\omega))](t_{k+1}-t_k)/T^2\}H_{kk}^{(ij)}(\infty)\\&\quad+\frac{1}{T}\sum_{l=1}^{N}H_{kl}^{(ij)}(\mathrm{e}^{j\omega T})\exp\left[j\omega(t_{l+1}-t_{k+1})\right]\\&\quad\times\{\nu_k(\omega)[j\omega(t_{l+1}-t_{k+1})-1]+\exp\left[j\omega(t_{k+1}-t_k)\right](t_{k+1}-t_k)/T\}\end{aligned} \tag{28}$$

and the sensitivity with respect to the clock time scale can then be obtained in terms of (28):

$$S_{\tau}^{H_k^{(ij)}}(\omega)\triangleq\frac{1}{H_k^{(ij)}(\omega)}\frac{\partial H_k^{(ij)}(\omega)}{\partial\tau} \tag{29}$$

(e) In order to compute the group delay and the slope of the amplitude frequency characteristic we need $\partial H_k^{(ij)}/\partial\omega$.

$$\frac{\partial H_k^{(ij)}(\omega)}{\partial\omega}=\frac{[\omega(t_{k+1}-t_k)+j]\exp[j\omega(t_{k+1}-t_k)]-j}{T\omega^2}\left\{\sum_{l=1}^{N}H_{kl}^{(ij)}(\mathrm{e}^{j\omega T})\exp[j\omega(t_{l+1}-t_{k+1})]-H_{kk}^{(ij)}(\infty)\right\}$$
$$+j\nu_k(\omega)\left\{\sum_{l=1}^{N}\exp[j\omega(t_{l+1}-t_{k+1})]\left[(t_{l+1}-t_{k+1})H_{kl}^{(ij)}(\mathrm{e}^{j\omega T})+T\,\mathrm{e}^{j\omega T}\left.\frac{\partial H_{kl}^{(ij)}(z)}{\partial z}\right|_{z=\mathrm{e}^{j\omega T}}\right]\right\} \quad (30)$$

where $\partial H_{kl}^{(ij)}/\partial z$ is given by (25). Set

$$\ln H_k^{(ij)}(\omega)\triangleq\alpha_k^{(ij)}(\omega)+j\phi_k^{(ij)}(\omega)$$

then the group delay in terms of (30) is:

$$T_k^{(ij)}(\omega)\triangleq\frac{\partial\phi_k^{(ij)}(\omega)}{\partial\omega}=\operatorname{Im}\left(\frac{1}{H_k^{(ij)}(\omega)}\frac{\partial H_k^{(ij)}(\omega)}{\partial\omega}\right) \quad (31)$$

and gain slope, which is of interest in stability analysis, is given by

$$\frac{\partial\alpha_k^{(ij)}(\omega)}{\partial\omega}=\operatorname{Re}\left(\frac{1}{H_k^{(ij)}(\omega)}\frac{\partial H_k^{(ij)}(\omega)}{\partial\omega}\right) \quad (32)$$

where Re and Im are respectively the real and imaginary part of a complex number.

Usually the practical quantities are the sensitivities of the gain and the phase, which can be easily derived from the sensitivities of (26)–(29).

Let us now count the minimal number of network analyses that is needed in the equations (26)–(32). Those sensitivities of $H_k^{(ij)}$ need the same *N nominal network analyses and 1 adjoint network analysis*, except for (27), where one adjoint network analysis is sufficient. If one is interested in the sensitivity of $H^{(ij)}$ analogous equations can be derived. These have not been included for the sake of brevity. For those computations *N nominal and N adjoint network analyses* are needed. This figure should be compared with the *Ns* network analyses if no adjoint was used. We conclude this section with an example.

Example. Consider again the treble tone control filter of Figure 1 with the values described above. We are interested in the sensitivity of the voltage amplification between u and v_{24} (amplitude) with respect to variations in C_1, C_2, C_3, C_4 and a variation of all capacitors with the same amount λ. Using DIANA we obtain Figure 3(b) in dB/per cent variation over the frequency range $F_s/1{,}024$ to $F_s/2$ in logarithmic scale by taking 0·086858 times the real value of (26). All these sensitivities only require 4 nominal and 1 adjoint network analyses.

5. CONCLUSIONS

A simple and straightforward construction is described for the adjoint SC network. Beyond the well-known construction of an adjoint linear time-invariant circuit only a reversion of all time dependencies in one period have to be performed. This construction can also be obtained as a special case of a more general class of adjoint networks.

The computational savings in the frequency, noise and sensitivity analysis are impressive and even greater than for linear time-invariant networks.

We have derived here all practical equations for frequency, (baseband and aliasing), noise and sensitivity analysis in the MNA framework. They can be easily implemented in any analysis program based on time domain analysis or on equivalent circuits in the z-domain. Our experience with the implementation[7] in DIANA (time-domain) is that the additional programming effort, needed for the adjoint, is certainly worth the computational savings.

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