Interpolation-Based Maximum Likelihood Channel Estimation Using OFDM Pilot Symbols

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Abstract

An interpolation-based maximum likelihood channel estimation scheme using OFDM pilot symbols is proposed. Instead of direction estimation of the frequency response on each subchannel, an interpolation filter is used on the pilot symbols to estimate a smaller set of coefficients that are sufficient to characterize the multipath channel. The actual frequency responses on the subchannels are then computed through inverse filtering of these coefficients. Because the same amount signal energy is used to estimated a reduced set of unknowns, the estimation accuracy is improved. The scheme is well suited for packet-based communication systems where pilot symbols instead of pilot tones are usually used at the beginning of packet for fast synchronization and channel estimation.

1 Introduction

Orthogonal Frequency Division Multiplexing (OFDM) has gained considerable interest in recent years[1, 2]. In OFDM system, data are modulated on frequency domain subchannels and is scaled by different subchannel frequency response coefficients after passing through the multipath channel. For coherent detection, these subchannel frequency responses must be estimated through the use of pilots.

Most of the literature on OFDM channel estimation have been focused on using pilot tones to interpolate the channel response [3, 4, 5, 6, 7]. Pilot tones usually distributed in both frequency and time directions to estimate time varying channel response. Such scheme is suited for continuous transmission systems such as digital video broadcasting where the steady state channel estimation results rather than the convergence speed towards them matter.

For packet-based communication system, the situation is different. The training is usually done at the very beginning of a packet in the form of pilot symbols to allow rapid and accurate estimation of the channel. If good channel estimation is not available before data decoding, some of the data may be lost, possibly triggering a packet retransmission. In addition, packets are usually short enough to warrant a constant channel response for the duration of packet and the channel estimation needs only to be done once at the beginning of a packet.

This paper intends to show a theoretically optimal approach for channel estimation given a number of pilot symbols. As an example of such pilot symbols, consider the IEEE 802.11a standard, where a long pilot symbol is provided as part of the packet preamble for both frequency offset estimation and initial channel estimation.

The paper is organized as following. In Section 2, we describe the direct channel estimation approach. In Section 3, we discuss the interpolation-based channel estimation approach. Section 4 shows the construction of the interpolation filter. In Section 5, the performance of the interpolation-based channel estimation is analyzed. Section 6 discusses channel estimation using multiple pilot symbols. The simulation comparison between the interpolation-based approach and direct estimation approach is shown in Section 7. The last section is the conclusion.

2 Direct Channel Estimation

In an OFDM receiver, channel estimation is performed in frequency-domain on the signal output from the FFT block. The channel equation is

$$Y(k) = C(k)X(k) + Z(k) \qquad (1)$$

where k is the subchannel (or subcarrier) index, $Y(k)$ is the signal output from the FFT, $C(k)$ is the channel frequency response coefficient, and $Z(k)$ is the noise. If the

FFT input noise is white, the output noise $Z(k)$ is also white. The $X(k)$s are known pilots with unit amplitude, the channel response is estimated as

$$\hat{C}(k) = X^*(k)Y(k) \tag{2}$$

In the direct estimation approach (2), channel response coefficients are estimated separately as if they are independent. However, in a practical OFDM system, the channel frequency response is usually oversampled by the subcarriers and the coefficients $C(k)$s are correlated. Correlation brings redundancy which can be used to reduce noise and improve estimation accuracy. In the following, we propose a maximum likelihood channel estimation scheme. Notice that for the theoretical derivation, we assume the total number of subcarriers is infinite.

3 Interpolation-Based Maximum Likelihood Channel Estimation

Instead of directly estimating the channel response coefficients $C(k)$s, consider express them as

$$C(k) = \sum_{n=-\infty}^{\infty} c(n)W(k - Qn)$$

and estimate the new set of coefficients $c(n)$s. Here $W(k-Qn)$ is the interpolation filter and Q is called the oversampling factor, which should be an integer no less than 1. We will define these terms and show how to find the interpolation filter in the next section. Since that the ratio between the total number of $C(k)$s and the total number of $c(n)$s is Q, using the same pilot symbols to estimate the $c(n)$s improves the estimation accuracy.

We require the interpolation filter satisfy the orthogonality condition

$$\sum_{k=-\infty}^{\infty} W(k - Qn)W^*(k - Qm) = \delta(n - m) \tag{3}$$

Thus $C(k)$ and $c(n)$ form a transform pair through $W(k-Qn)$ as

$$\begin{cases} C(k) = \displaystyle\sum_{n=-\infty}^{\infty} c(n)W(k - Qn) \\ c(n) = \displaystyle\sum_{k=-\infty}^{\infty} C(k)W^*(k - Qn) \end{cases} \tag{4}$$

Referring to the subcarrier channel equation (1), since the noise $Z(k)$s are independent Gaussians with same variance, the maximum likelihood channel estimation finds the set of $C(k)$s that maximize the cost function

$$\Psi = \sum_{k=-\infty}^{\infty} 2\mathrm{Re}\left[C(k)X(k)Y^*(k)\right] - \sum_{k=-\infty}^{\infty} |C(k)X(k)|^2 \tag{5}$$

If we express the $C(k)$s in terms of $c(n)$s, the cost function (5) becomes

$$\begin{aligned} \Psi &= \sum_{n=-\infty}^{\infty} \left\{2\mathrm{Re}\left[c(n)w^*(n)\right] - |c(n)|^2\right\} \qquad (6) \\ &= \sum_{n=-\infty}^{\infty} \left(-|c(n) - w(n)|^2 + |w(n)|^2\right) \end{aligned}$$

with

$$w^*(n) = \sum_{k=-\infty}^{\infty} X(k)Y^*(k)W(k - Qn)$$

and is maximized when

$$c(n) = w(n)$$

Thus, the interpolation-base maximum likelihood channel estimation estimates a set of coefficients

$$\hat{c}(n) = \sum_{k=-\infty}^{\infty} X^*(k)Y(k)W^*(k - Qn) \tag{7}$$

The original channel response coefficients are found through

$$\hat{C}(k) = \sum_{n=-\infty}^{\infty} \hat{c}(n)W(k - Qn) \tag{8}$$

4 Interpolation Filter

The general method to construct interpolation filters is through Fourier transform of certain time domain windows of channel impulse response. We should assume the channel impulse response is time-limited. In practice, the time span of the channel impulse response may be considered as the range over which the majority of the multipath energy is captured. The time domain window must be flat over the time span of the channel impulse response so that the impulse response can be masked out undistorted using the window.

Since the time span varies with the channel and is generally not known in practice, a worst case time span is used instead. Referring to Figure 1, timing synchronization often aligns the receiver time origin to the energy peak of the channel impulse response. However, the precursor delay spread T_- and post-cursor delay spread T_+

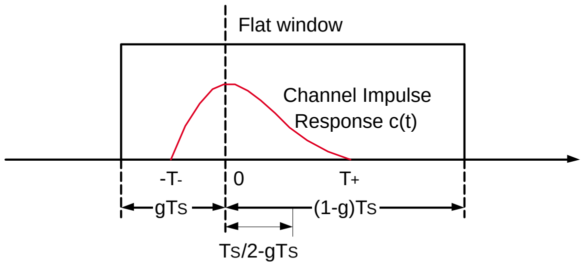

Figure 1: Illustration of time-domain windowing on multipath delay spread pro£le.

of the impulse response are generally not known. The worst case time span T_S satisfies

$$\left\{ \begin{array}{rcl} gT_S & = & T_-|_{\max} \\ (1-g)T_S & = & T_+|_{\max} \end{array} \right. \tag{9}$$

where the maximizations are taken over all the multipath channels in a target propagation environment. For example, a system designed for worst case pre-cursor spread of $100\mathrm{ns}$ and post-cursor spread of $200\mathrm{ns}$ has $T_S = 300\mathrm{ns}$ and $g = 1/3$.

To align the center of the worst case time span to the receiver time origin, the channel impulse response must be shifted to the left by $T_S/2 - gT_S$. Or equivalently, a phase factor is multiplied to the channel frequency response, i.e.

$$c(t + T_S/2 - gT_S) \leftrightarrow C(k)e^{j2\pi k \frac{T_S/2 - gT_S}{T}}$$

where T is the FFT symbol period. The above is achieved by multiplying the factor

$$e^{j2\pi k \frac{T_S/2 - gT_S}{T}}$$

to the frequency domain signal $Y(k)$ in (1).

For simplicity, we still denote the shifted channel impulse response $c(t)$ which is now time limited to $[-T_S/2, T_S/2]$. Using a window $w(t)$ that is flat over $[-T_S/2, T_S/2]$ and is time limited to $[-T_Q/2, T_Q/2]$ where $T_Q \geq T_S$, we can write

$$c(t) = w(t)c_{T_Q}(t) \tag{10}$$

where $c_{T_Q}(t)$ has periodicity T_Q, whose waveform in $[-T_Q/2, T_Q/2]$ coincide with that of $c(t)$. Because $c_{T_Q}(t)$ is periodic, it can be expressed as

$$c_{T_Q}(t) = \sum_{n=-\infty}^{\infty} c(n)e^{j2\pi n \frac{t}{T_Q}}$$

or in Fourier domain

$$C_{T_Q}(f) = \sum_{n=-\infty}^{\infty} c(n)\delta\left(f - \frac{n}{T_Q}\right)$$

Fourier transform (10) yields

$$C(f) = \sum_{n=-\infty}^{\infty} c(n)W\left(f - \frac{n}{T_Q}\right)$$

which sampled at $f = k/T$ gives

$$C\left(\frac{k}{T}\right) = \sum_{n=-\infty}^{\infty} c(n)W\left(\frac{f}{T} - \frac{n}{T_Q}\right)$$

Expressed in the units of $1/T$, the above is denoted as

$$C(k) = \sum_{n=-\infty}^{\infty} c(n)W(k - Qn)$$

The over sampling factor Q is defined as

$$Q = \frac{T}{T_Q} \tag{11}$$

To find a class of orthogonal filters that satisfies (3), we write the filter coefficient sequence $W(k)$ in terms of its discrete time Fourier transform (DTFT) $w(x)$, i.e.

$$W(k) = T\int_{-\frac{1}{2T}}^{\frac{1}{2T}} w(x)e^{j2\pi xTk}dx$$

Using Parseval's theorem, the orthogonality condition (3) becomes

$$\begin{aligned} & \sum_{k=-\infty}^{\infty} W(k-Qn)W^*(k-Qm) \\ = \; & T\int_{-\frac{1}{2T}}^{\frac{1}{2T}} |w(x)|^2 e^{j2\pi xTQ(m-n)}dx \end{aligned}$$

The above is $\delta(n - m)$ if $w(x)$ is a square root raised cosine window with roll off factor β, amplitude $\sqrt{Q}$, and width $1/(QT)$ and if the condition

$$Q > 1 + \beta \tag{12}$$

is satisfied. The orthogonal filter

$$\begin{aligned} W(k) = & \frac{1}{\sqrt{Q}}\left[\frac{\sin \pi(1-\beta)\frac{k}{Q}}{\pi \frac{k}{Q}} + \frac{4\beta}{\pi}\cos \pi(1+\beta)\frac{k}{Q}\right] \\ & \times \frac{1}{1 - 16\beta^2 \frac{k^2}{Q^2}} \end{aligned} \tag{13}$$

is obtained by inverse DTFT on $w(x)$.

5 Performance Analysis

Assuming the white noise $Z(k)$ is normalized, referring to (1), the direct channel estimation normalized error variance is expressed as

$$\frac{E\left[\left|\hat{C}(k) - C(k)\right|^2\right]}{|C(k)|^2} = \frac{1}{|C(k)|^2} \tag{14}$$

To find the noise in an interpolation-based estimation approach, we expand (7) as

$$\hat{c}(n) = \sum_{k=-\infty}^{\infty} C(k)W^*(k-Qn) + z(n) \tag{15}$$

where

$$z(n) = \sum_{k=-\infty}^{\infty} Z(k)X^*(k)W^*(k-Qn) \tag{16}$$

Since the interpolation filter satisfies the orthogonality and the data $X(k)$s are normalized, the noise $z(n)$s are independent Gaussians with unit power. Equation (16) basically shows that the noise $z(n)$ is the original noise $Z(k)$ passed through a low pass filter that has $1/Q$ bandwidth of the original noise spectrum. Thus, the total noise power is reduced by a factor of Q in the interpolation-based channel estimation.

The original channel response coefficients are obtained by substituting (15) into (8), i.e.

$$\hat{C}(k) = C(k) + \hat{Z}(k) \tag{17}$$

where the noise

$$\hat{Z}(k) = \sum_{n=-\infty}^{\infty} z(n)W(k-Qn) \tag{18}$$

is now colored and the noise power at different subcarrier is now different, i.e.

$$E\left[|\hat{Z}(k)|^2\right] = \sum_{n=-\infty}^{\infty} |W(k-Qn)|^2 \tag{19}$$

Referring to (17), the interpolation-based channel estimation normalized error variance is then

$$\frac{\sum_{n=-\infty}^{\infty} |W(k-Qn)|^2}{|C(k)|^2} \tag{20}$$

6 Multiple Pilot Symbols

When there are multiple pilot symbols and the multipath channel response is unchanged during these symbols, the channel equation (1) is expressed as

$$Y(m,k) = C(k)X(m,k) + Z(m,k)$$

where m is the symbol index and the noise $Z(m,k)$ is uncorrelated across symbol and subcarrier. For the direct estimation, the channel is estimated as

$$\hat{C}(k) = \frac{1}{M}\sum_m X^*(m,k)Y(m,k) \tag{21}$$

where M is the total number of pilot symbols. For interpolation-based estimation, the channel frequency response coefficients are calculated using (8) with $\hat{c}(n)$ estimated as

$$\hat{c}(n) = \sum_{k=-\infty}^{\infty} \left[\frac{1}{M}\sum_m X^*(m,k)Y(m,k)\right] W^*(k-Qn) \tag{22}$$

In both cases, the normalized estimation error variances, i.e. (14) and (20), are reduced by a factor M.

7 Simulation Result

The simulation is performed for an OFDM system with $N = 64$ subcarriers, FFT symbol period of 3.2μs (i.e. subcarrier spacing 0.3125MHz), and carrier frequency of 2.44GHz. The oversampling factor Q is chosen to be 8 and the interpolation filter time span according to (11) is 400ns. The square root raised cosine filter (13) has roll-off factor $\beta = 1/4$. The target worst case multipath impulse response time span T_S in (9) is 300ns with $g = 1/3$. For the simulation, $M = 2$ pilot symbols are used for channel estimation.

Figure 2 shows a multipath channel impulse response generated from the simulation of the Berkeley Wireless Research Center (BWRC) using a ray-tracing simulator BWRSim [8]. Figure 3 shows the corresponding frequency response. Figure 4 plots the normalized channel estimation error STD for the two channel estimation schemes at different SNR. At each SNR, a total of 5000 simulations with different noise seeds are used to compute the error STD curves. The 1dB interpolation-based channel estimation performs almost as good as 10dB direct estimation. The graph also shows there is edge effect–the estimation error increases at edge subcarriers–due to finite number of subcarriers are used.

8 Conclusion

An interpolation-based maximum likelihood channel estimation scheme using OFDM pilot symbols is proposed. Instead of directly estimating the channel frequency response coefficients on the subchannels, it estimates a reduced set of channel coefficients that are sufficient to characterize the multipath channel. An orthogonal filter transformation links the original set of channel coefficients with the new set of channel coefficients and the filter is found to be square root raised cosine. The performance of the interpolation-based channel estimation is analyzed. Simulation comparison in a practical system shows it achieves an order of magnitude improvement in estimation accuracy.

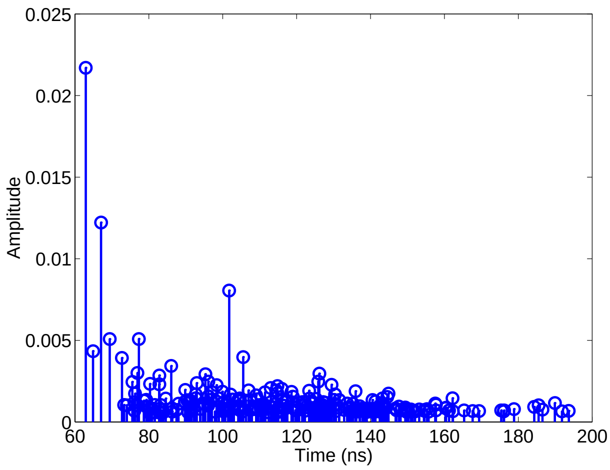

Figure 2: Simulated channel impulse response.

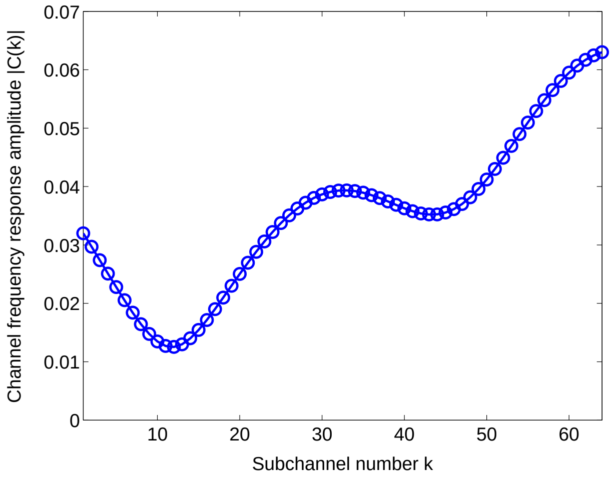

Figure 3: Channel frequency response amplitude.

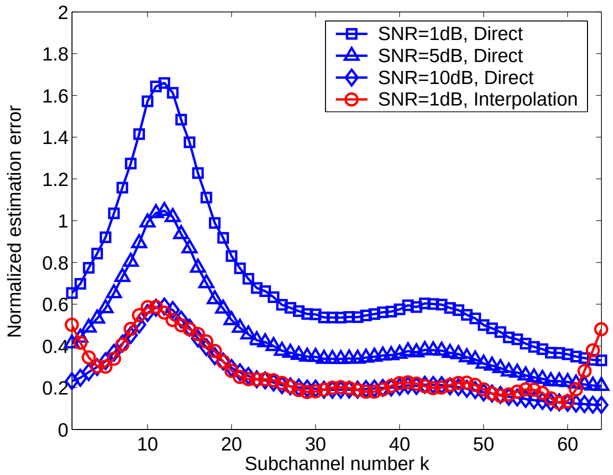

Figure 4: Normalized channel estimation error comparison.

References

[1] J. A. C. Bingham, "Multicarrier modulation for data transmission: An idea whose time has come," *IEEE Commun. Mag.*, vol. 28, pp. 5–14, May 1990.

[2] R. van Nee and R. Prasad, *OFDM for Wireless Multimedia Communications*. Artech House, 2000.

[3] O. Edfors, M. Sandell, J.-J. van de Beek, S. K. Wilson, and P. O. Böjesson, "Ofdm channel estimation by single value decomposition," *IEEE Transactions on Communications*, vol. 46, pp. 931–939, July 1998.

[4] R. Negi and J. Cioff£, "Pilot tone selection for channel estimation in a mobile ofdm system," *IEEE Transactions on Consumer Electronics*, vol. 44, pp. 1122–1128, August 1998.

[5] Y. Li, L. J. Cimini, and N. R. Sollenberger, "Robust channel estimation for ofdm systems with rapid dispersive fading channels," *IEEE Transactions on Communications*, vol. 46, pp. 902–915, July 1998.

[6] O. Edfors, M. Sandell, J.-J. van de Beek, S. K. Wilson, and P. O. Börjesson, "Analysis of dft-based channel estimators for ofdm," *Wireless Personal Communications*, vol. 12, pp. 55–70, 2000.

[7] J.-J. van de Beek, O. Edfors, M. Sandell, S. K. Wilson, and P. O. Börjesson, "On channel estimation in ofdm systems," *Proceedings of VTC'95*, pp. 715–719, July 1995.

[8] http://bwrc.eecs.berkeley.edu