Adaptive Frequency Domain Equalization for OFDM

(Orthogonal Frequency Division Multiplex)

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outline

part 1: introduction

- **OFDM - basics**
- **fixed radio access scenario and channel models**

part 2: adaptive frequency domain equalization for OFDM

- **linear MIMO detectors for OFDM**
- **LMS- and RLS-algorithms**
- **modified RLS-algorithms (MRLS)**
- **differential coding**

part 3: implementation

- **novel QRD-based square-root MRLS-algorithm**
- **complexity**

part 4: summary and outlook

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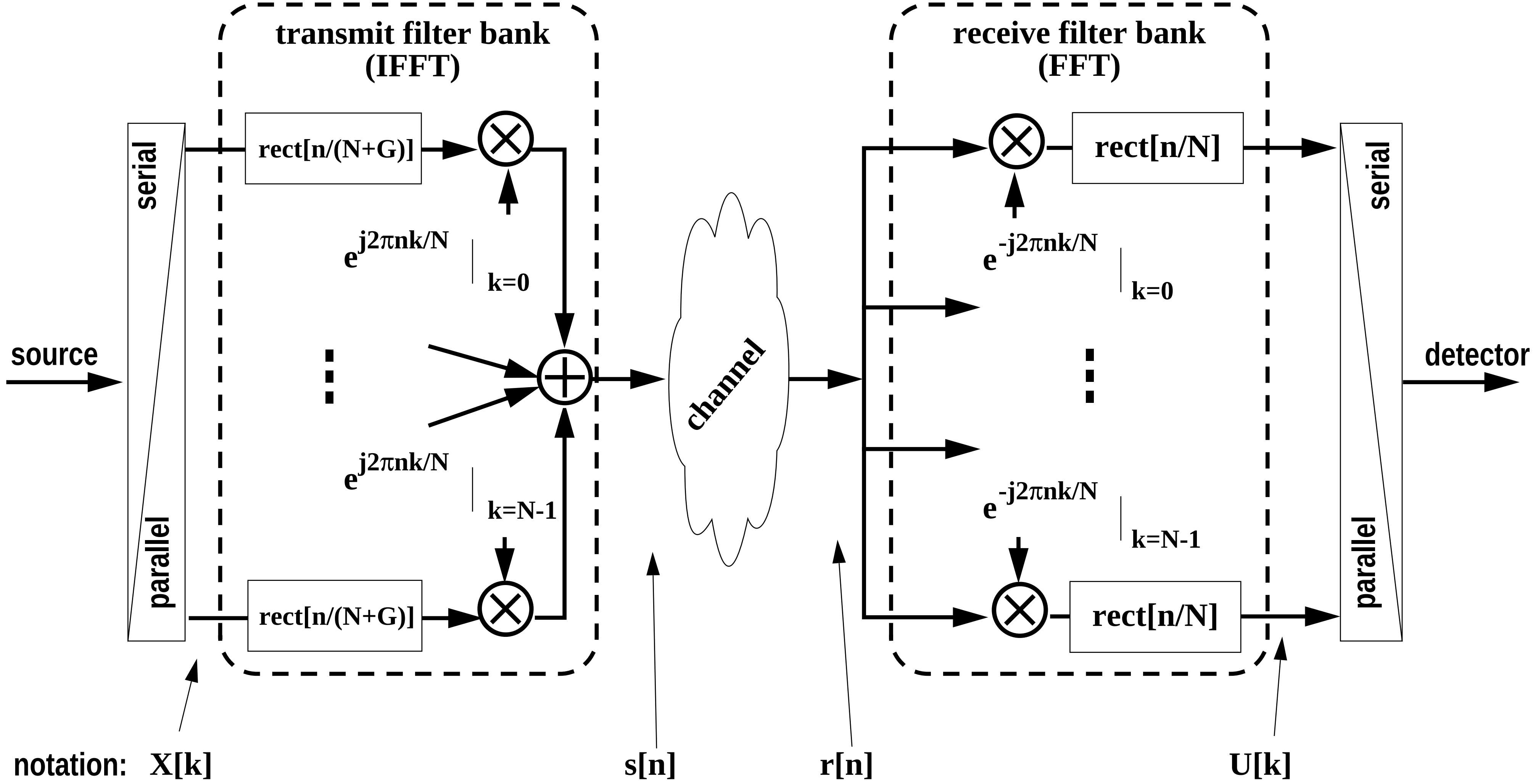

k = 0 ... N-1 subcarrier index, n = 0 ... N-1 sample index within one OFDM symbol, G = guard interval

(I)FFT (inverse) fast Fourier transform *rect[n/N] rectangular window function of n with length N*

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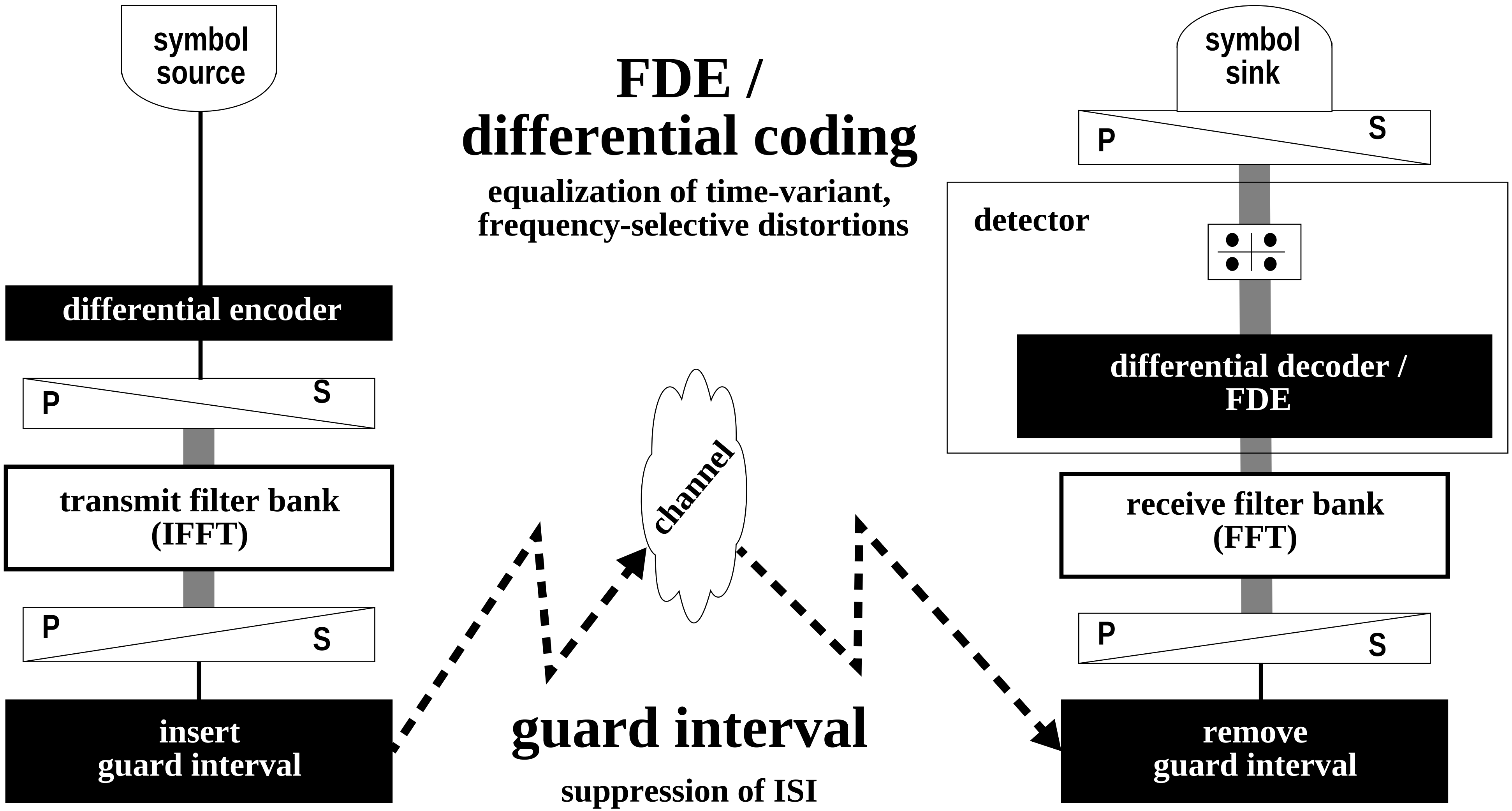
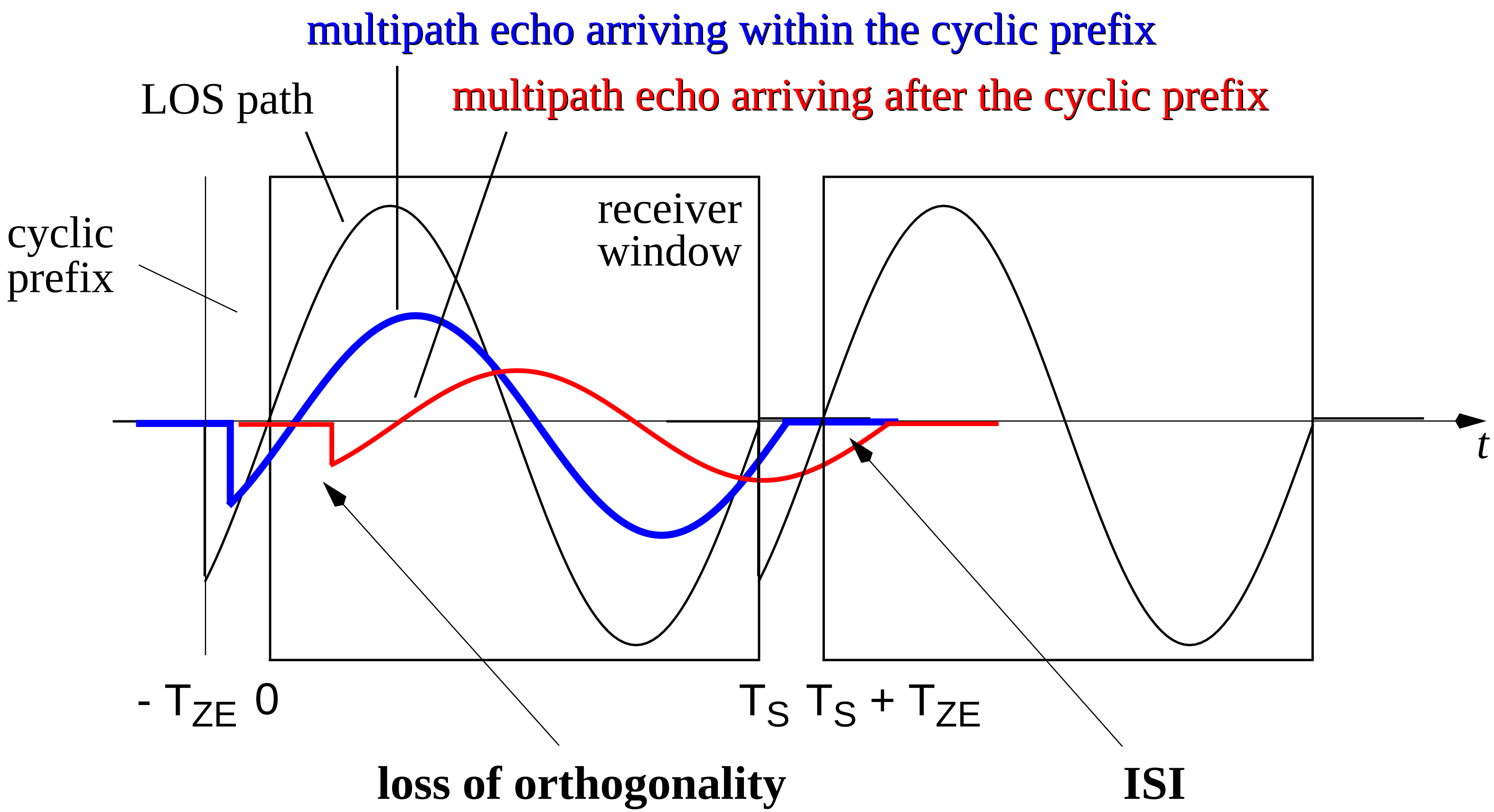
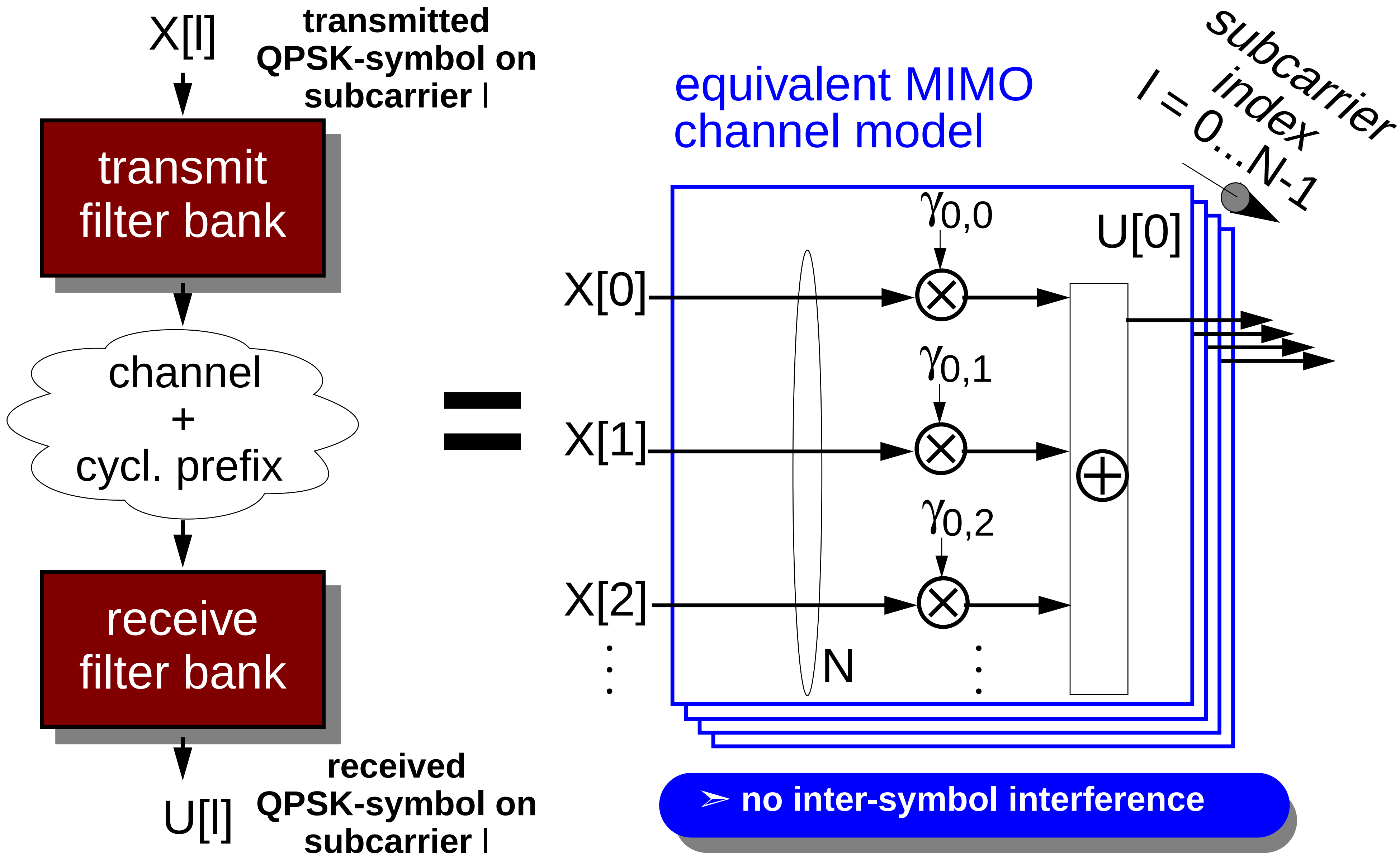


(I)FFT (inverse) fast Fourier transform *P/S parallel / serial conversion* *ISI inter-symbol interference* *FDE frequency domain equalization*



LOS — line of sight
ISI — inter-symbol interference
AWGN — additive white gaussian noise

A perfect cyclic prefix transforms a stationary ISI-channel into N parallel AWGN-channels.



QPSK quaternary phase shift keying

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- **no inter-symbol interference (ISI)**

  the transmission of a single OFDM-symbol can be considered

- **inter-carrier interference (ICI)**
  - **phase noise**
  - **synchronization errors**
  - **time-variant fading channels**
  - **...**

- **individual distortions on each subcarrier**

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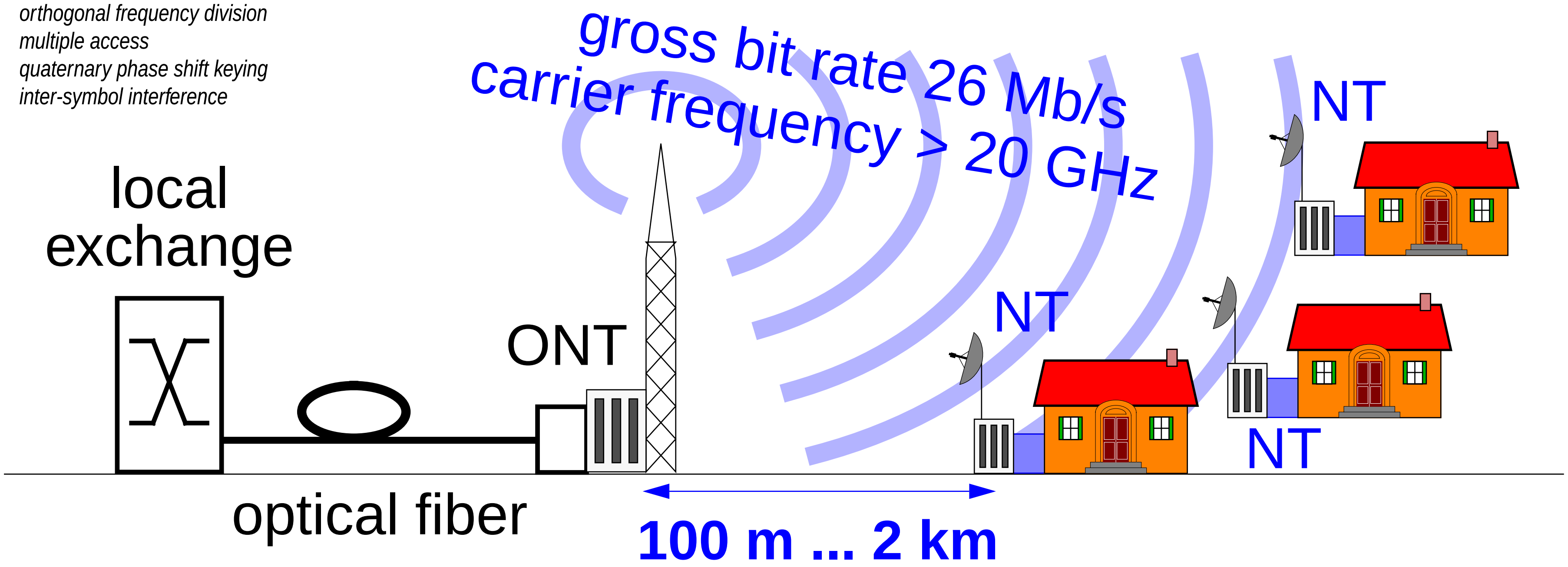
NT	*network termination*
ONT	*optical network termination*
OFDMA	*orthogonal frequency division multiple access*
QPSK	*quaternary phase shift keying*
ISI	*inter-symbol interference*



OFDM / OFDMA proposal with N = 512 **subcarriers and QPSK:**

- **short channel impulse responses with respect to the OFDM symbol duration**
- **negligible capacity loss introduced by perfect guard intervals**
- **all ISI are completely eliminated**
- **each OFDM-symbol is considered seperately**

worst case: omnidirectional antennas

- **rician line of sight**
- **multipath propagation**
  - **90 % of energy are received within 150 ns**
  - **echoes attenuated less than 15 dB are received within 200 ns**
- **reflections on moving cars are strongly attenuated**
  - **Rayleigh distributed Doppler components**
  - **80 km/h -> maximum Doppler frequency 2 kHz @ 28 GHz**

Sources: WALES, et al. (1993), GROND, et al. (1990): measurements in Bristol, UK
KARTHAUS, et al. (1997 - 1998): measurements within the *ATMmobil* [1] project

[1] supported by the German government

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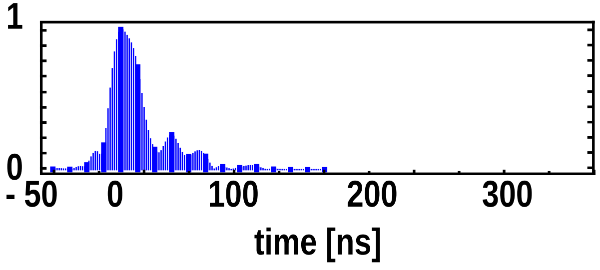
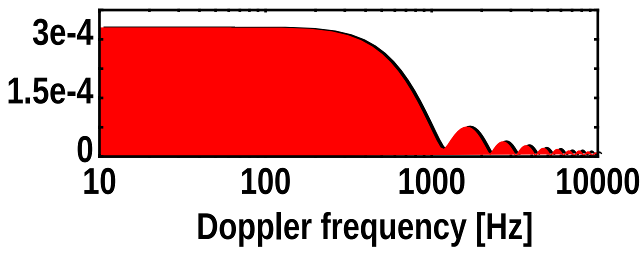
fading channel: omnidirectional antenna at the subscriber, sectorized antenna at the base station

normalized impulse response

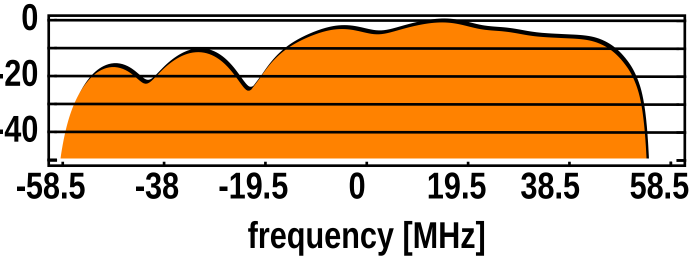


a) stationary

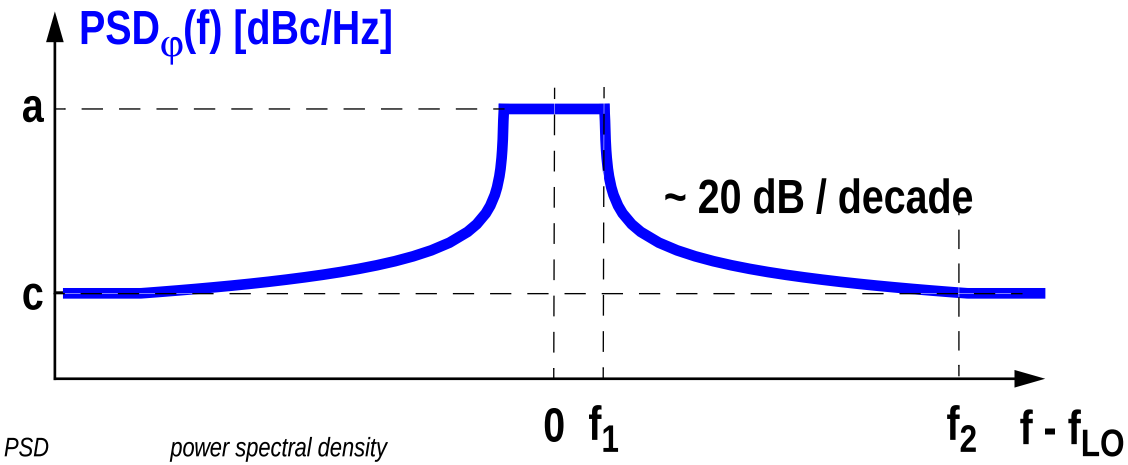
b) normalized Doppler spectrum



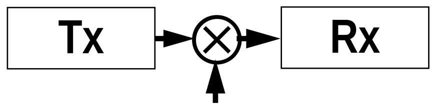
[dB] | normalized transfer function |^2



oscillator phase noise: f_{LO} = 38 GHz, f_1 = 20 kHz, a = -65 dBc/Hz, c = -125 dBc/Hz



PSD *power spectral density*



$$p[n] = e^{j\varphi[n]} \approx 1 + j\varphi[n]$$

$\varphi[n]$ zero mean gaussian process

$\varphi[n] \ll 1$ rad

degradations due to phase noise

- common phase error (CPE)
  - identical for all subcarriers during one OFDM-symbol
  - easy to compensate
  - $\rightarrow$ 0 with increasing bandwidth of phase noise
  - $\rightarrow$ 0 with increasing number of subcarriers

- inter-carrier interference (ICI)
  - nearly gaussian for high numbers of subcarriers
  - $=$ 0 for stationary phase noise
  - ICI-power increases with the number of subcarriers

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OFDM transmission with cyclic prefix is optimum in ISI-channels in terms of Shannon´s capacity, provided:

- **the number of subcarriers tends to infinity**
- **the transmit power is distributed among all subcarriers in an optimum way (loading algorithms)**

ISI *inter-symbol interference*

Time Division Multiple Access (TDMA) + Adaptive Decision Feedback Equalizer

Radio Channel:

multipath delay: 20 ... 300 ns

coherence time: T_C = 720 µs

Uplink:

minimal data-rate: 32 kb/s

separation of two ATM-cells: 12 ms >> T_C

Fast converging algorithms have to be used (RLS)

→ high complexity

(100,000 - 200,000 gates with a 0.35 µm standard cell library)

→ Does *multicarrier transmission* reduce the implementation cost ?

ATM *asynchronous transfer mode* *RLS* *recursive least squares*

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detectors for parallel ISI & ICI channels

- **optimum**
  - **MLSE**
    - high complexity increasing with number of subcarriers
- **suboptimum** (--- sequence estimation → MLSE)
  - **nonlinear MIMO**
    - **DFE, RNN**
      - when ISI occur (short guard interval)
  - **linear MIMO**
    - **channel estimation**
      - european DVB-T standard
      - many pilot symbols
      - fixed transmission frames
    - **adaptive**
      - **ZF**
        - noise enhancement
      - **MMSE**
        - **LMS, RLS, MRLS**
  - **differential MIMO**
    - **over time / frequency**
      - european DAB standard
      - affected by double ICI-power
      - simple receivers

ISI	*inter-symbol interference*
ICI	*inter-carrier interference*
MLSE	*maximum likelihood sequence estimation*
MIMO	*multiple-input multiple-output*
DFE	*decision feedback equalizer*
RNN	*recurrent neural network*
DAB	*digital audio broadcast*
DVB-T	*terrestrial digital video broadcast*
ZF	*zero forcing*
MMSE	*minimum mean squared error*
LMS	*least mean squares*
(M)RLS	*(modified) recursive least squares*

Since we want to analyze <u>adaptive equalization</u> ...

... we express the time-variance of the channel coefficients by means of

the time-variance of the optimum equalizer coefficients !

optimum coefficient vector at time m → $\hat{\mathbf{c}}_m$

process noise vector → $\mathbf{v}_{P,m}$

$$\hat{\mathbf{c}}_{m+1} = \hat{\mathbf{c}}_m + \mathbf{v}_{P,m}$$

dynamic first order Markoff model

MIMO *multiple-input multiple-output*

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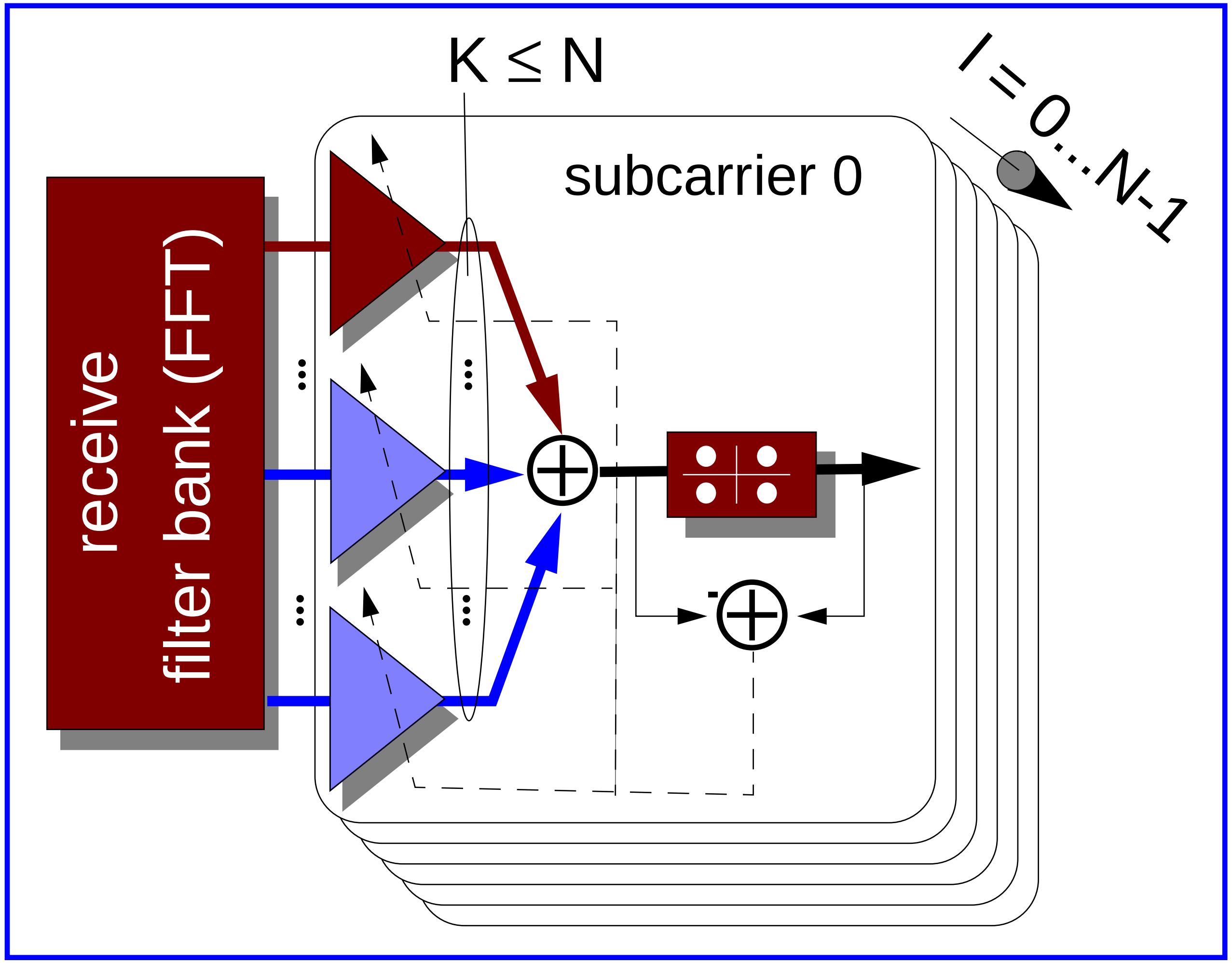


- linear equalization & ICI-compensation
- decoupling
  equalizer coefficients of each subcarrier can be adapted independently
- parallel multiple-input single-output (MISO) structures
- ideal guard interval
  → 1 complex valued coefficient per branch

- ICI of adjacent subcarriers dominate
- K << N branches per MISO-equalizer

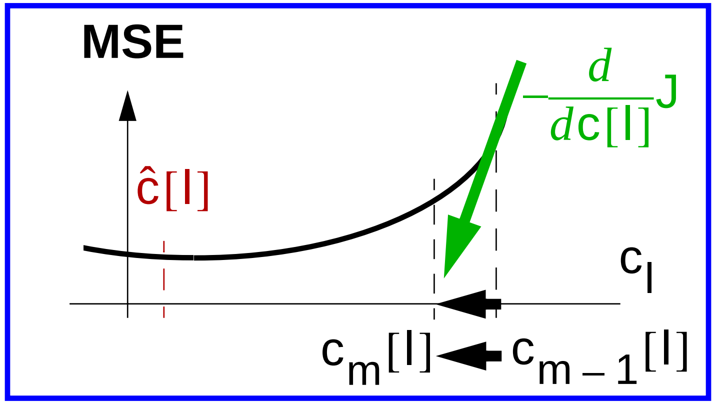
FFT	*fast Fourier transform*
ICI	*inter-carrier interference*
DFE	*decision feedback equalizer*
MIMO	*multiple-input multiple-output*

least mean squares (LMS) algorithm

cost function

mean squared error (MSE) on all subcarriers l

$$J = \sum_{l=0}^{N-1} E\left\{ \left| e_m[l] \right|^2 \right\}$$



low complexity ...

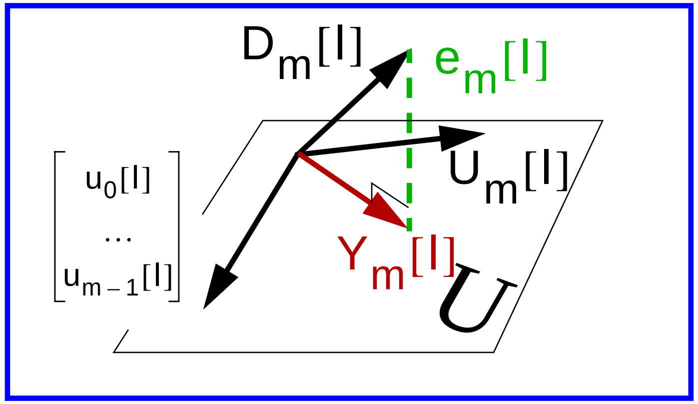
but slow convergence

recursive least squares (RLS) algorithm

cost function

sum of all squared errors observed at time 0 ... m **on all subcarriers l**

$$J = \sum_{l=0}^{N-1} \sum_{i=0}^{m} \lambda^{m-i} \cdot \left| e_m[l] \right|^2$$



fast convergence ...

but high complexity & numerical problems

the RLS-algorithm is the special case of the Kalman filter for a stationary state-space model !

➢ **suboptimum tracking properties**

$$\hat{\mathbf{c}}_{m+1} = \hat{\mathbf{c}}_m$$

$$D_m = \hat{\mathbf{c}}_m \cdot \mathbf{u}_m + v_{M,m}$$

desired response

measurement noise

equalizer input vector

optimum coefficient vector at time *m*

➢ **design modified RLS-algorithms with better tracking properties from a dynamic state-space model**

(HAYKIN, et al. 1995, 1997)

first order Markoff model

weight factor

process noise

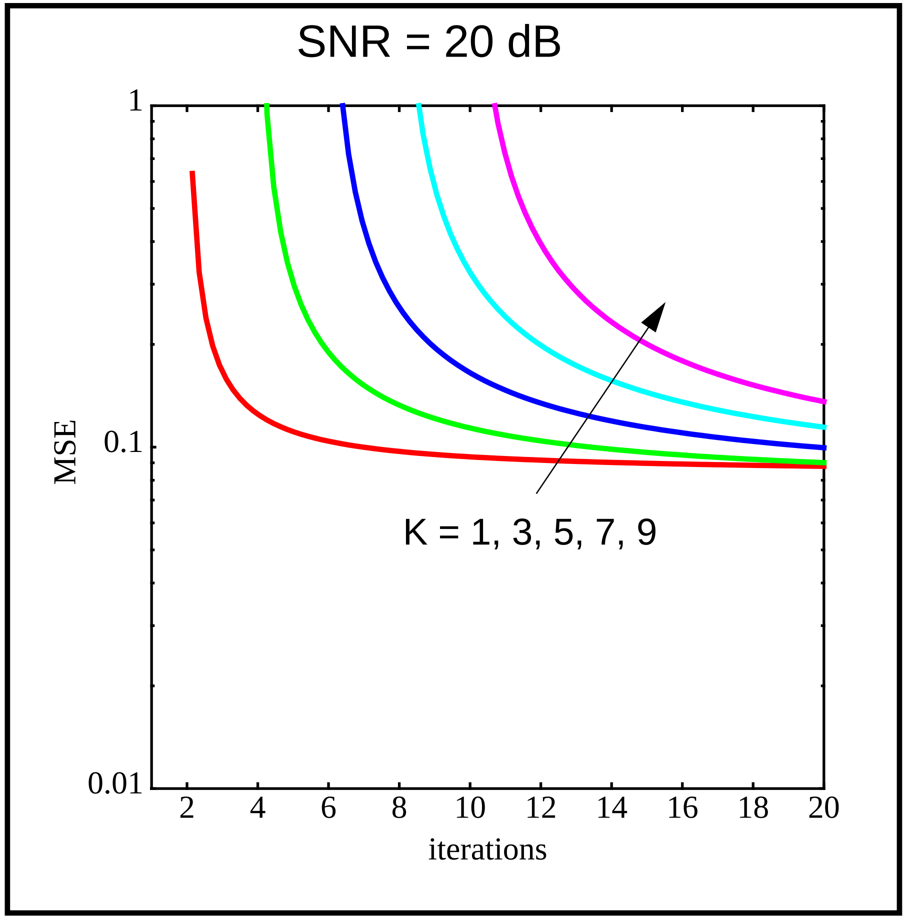
$$\hat{\mathbf{c}}_{m+1} = \eta \cdot \hat{\mathbf{c}}_m + \mathbf{v}_{P,m}$$

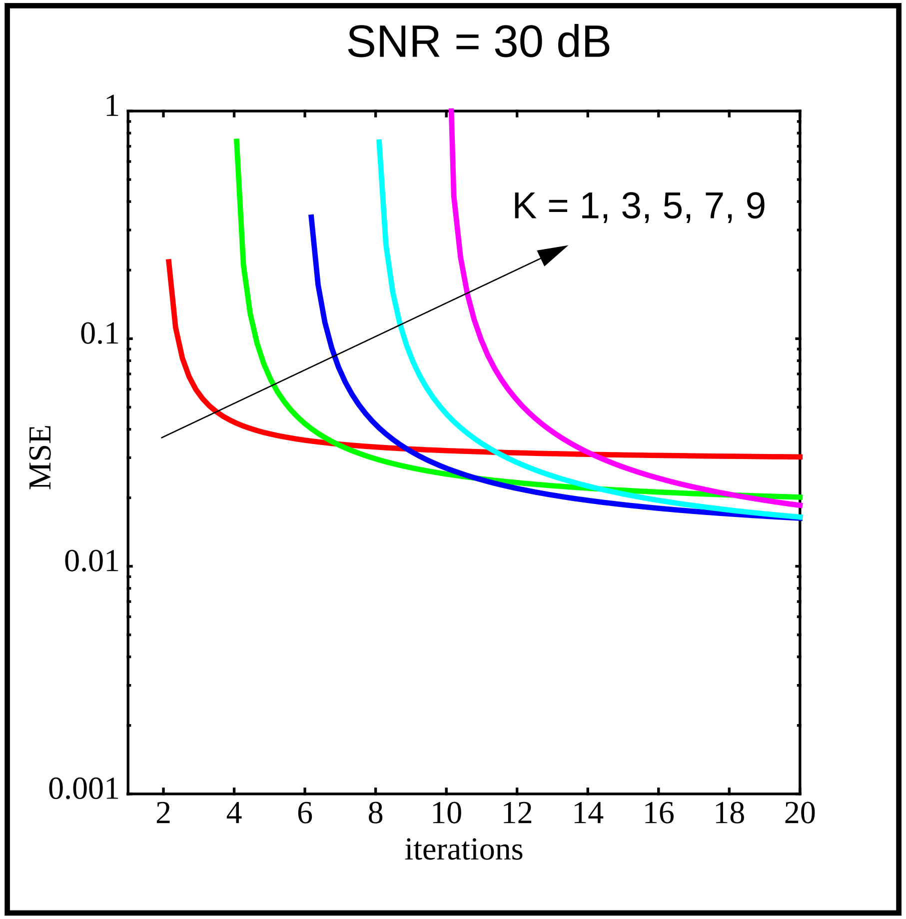
$$D_m = \hat{\mathbf{c}}_m \cdot \mathbf{u}_m + v_{M,m}$$

➢ **invoke the modification automatically only when the transmission channel is essentially changing**

(PARK & JUN, 1992 / JIANG & COOK, 1992)

RLS *recursive least squares*





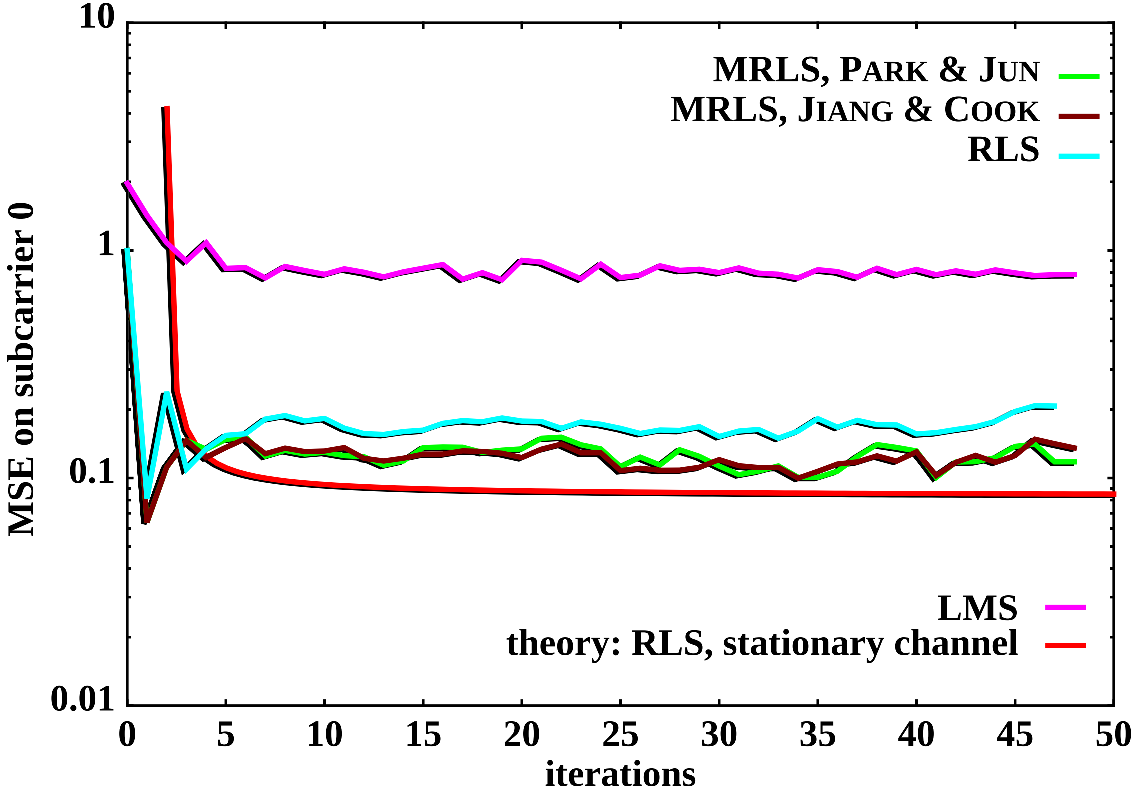
SNR	*signal to noise ratio*
MSE	*mean squared error*
RLS	*recursive least squares*
MIMO	*multiple-input multiple-output*

- ❑ theoretical convergence in a stationary equivalent MIMO channel
- ❑ stationary frequency selective ICI channel

ICI negligible vs. AWGN

➢ K = 1

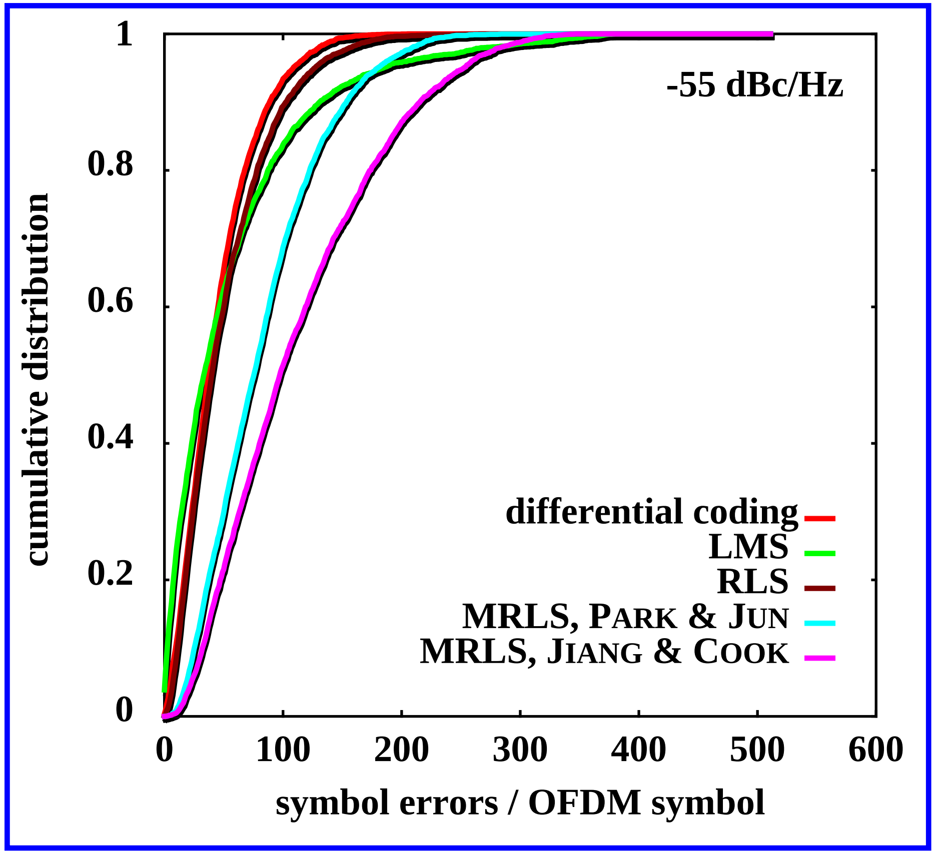
time-variant, frequency-selective fading channel
SNR = 24 dB

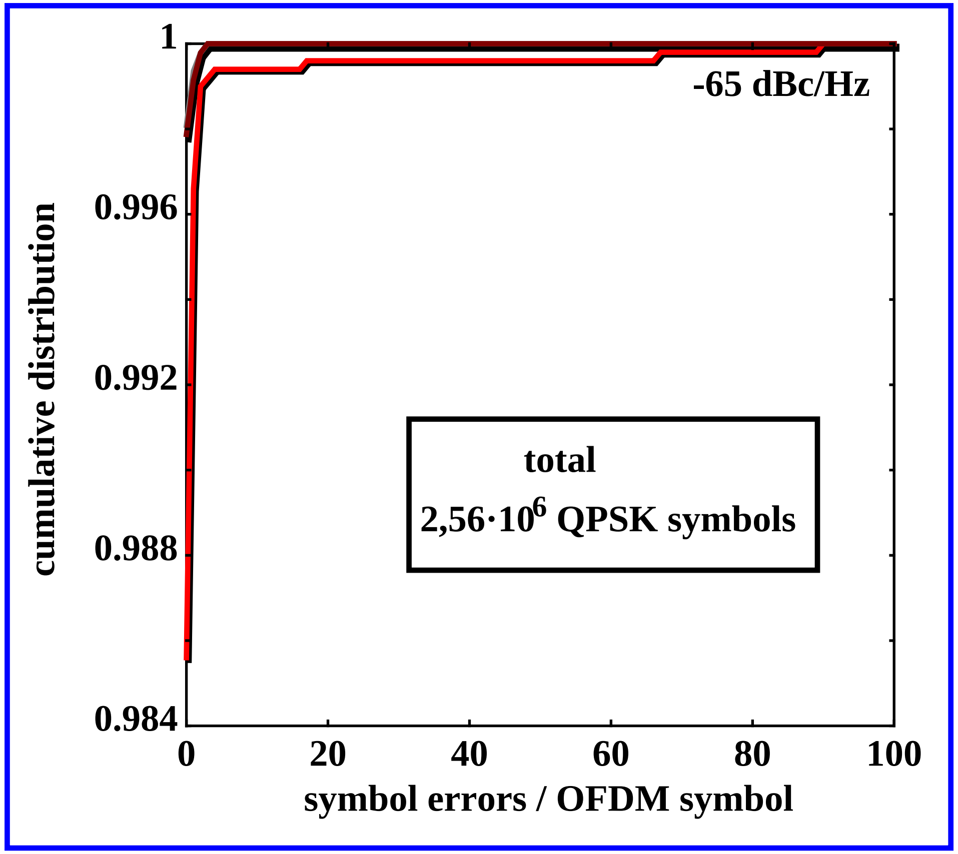


MSE	mean squared error
MISO	multiple-input single-output
AWGN	additive white gaussian noise
ICI	inter-carrier interference
SNR	signal to noise ratio
LMS	least mean squares
(M)RLS	(modified) recursive least squares

Kalman filter based modifications improve the tracking properties of the RLS algorithm.

stationary frequency-selective fading channel + phase noise





symbol error rate

d.c.	LMS	RLS	P&J	J&C
0.089	0.10	0.098	0.16	0.22

symbol error rate

d.c.	LMS	RLS	P&J	J&C
$1.0 \cdot 10^{-4}$	$1.2 \cdot 10^{-5}$	$5.4 \cdot 10^{-6}$	$3.8 \cdot 10^{-5}$	$6.2 \cdot 10^{-6}$

- **The capacity loss due to ideal cyclically extended guard intervals is negligible in fixed radio access scenarios.**
- **Phase noise introduces severe ICI.**
- **Linear adaptive FDE in combination with ideal guard intervals provides more flexibility than coherent channel estimation and is less sensitive with respect to ICI than differential coding.**
- **Modified RLS algorithms with better tracking capabilities can be designed from dynamic state-space models.**
- **Neither the RLS, LMS or MRLS algorithms, nor differential coding, can mitigate the requirements on the purity of oscillators in OFDM-systems significantly.**

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RLS equalizers in their conventional form have several drawbacks:

high complexity

- $O(\mathrm{N}^2)$ multiplications and additions, and $O(\mathrm{N})$ divisions per iteration
- fast RLS algorithms can not be applied to OFDM due to independent input vectors

numerical problems

- the inverse correlation matrix is computed as the difference of 2 positive definite matrices and may not remain meaningful in a finite precision environment

large wordlengths required

- the RLS algorithm operates on the square of the input vectors

These drawbacks can be tackeled by square-root RLS algorithms !

RLS *recursive least squares*

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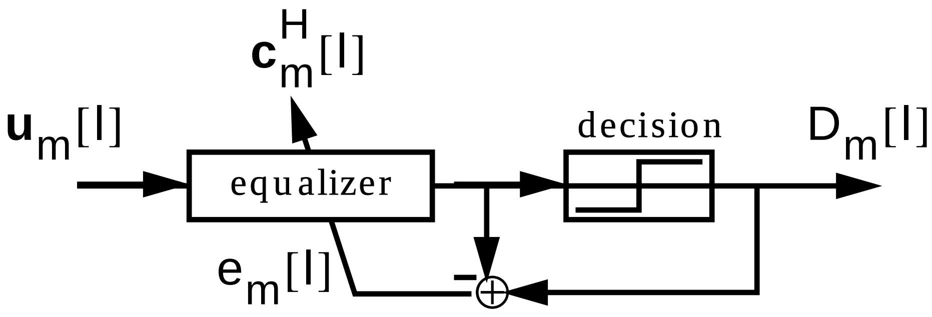
RLS *recursive least squares*

applying the QR decomposition to the matrix of input vectors

$$\mathbf{U}_m = \left[\sqrt{\lambda}^n \mathbf{u}_0 \;\; \sqrt{\lambda}^{n-1} \mathbf{u}_1 \; \ldots \; \mathbf{u}_m \right]^H$$



QR decomposition $\quad Q_m U_m = \begin{bmatrix} \mathbf{R}_m \\ \mathbf{0} \end{bmatrix}$

(Q ... unitary, R ... upper triangular matrix)

$$\mathbf{D}_m = \begin{bmatrix} \sqrt{\lambda}^n D_0 \\ \sqrt{\lambda}^{n-1} D_1 \\ \ldots \\ D_m \end{bmatrix} \qquad \mathbf{e}_m = \begin{bmatrix} \sqrt{\lambda}^n e_0 \\ \sqrt{\lambda}^{n-1} e_1 \\ \ldots \\ e_m \end{bmatrix}$$

multiplying the error vector by an unitary matrix does not change the cost function $|e_m|^2$

$$\mathbf{e}_m = D_m - \mathbf{U}_m \hat{\mathbf{c}}_m$$

$$Q_m \mathbf{e}_m = Q_m D_m - \begin{bmatrix} \mathbf{R}_m \\ \mathbf{0} \end{bmatrix} \hat{\mathbf{c}}_m$$

$\Rightarrow$ $\hat{\mathbf{c}}_m = \mathbf{R}_m^{-1} \mathbf{p}_m$

advantages

- ❑ **reduced dynamic range**
- ❑ **good numerical properties**
- ❑ **implementation with a systolic array**

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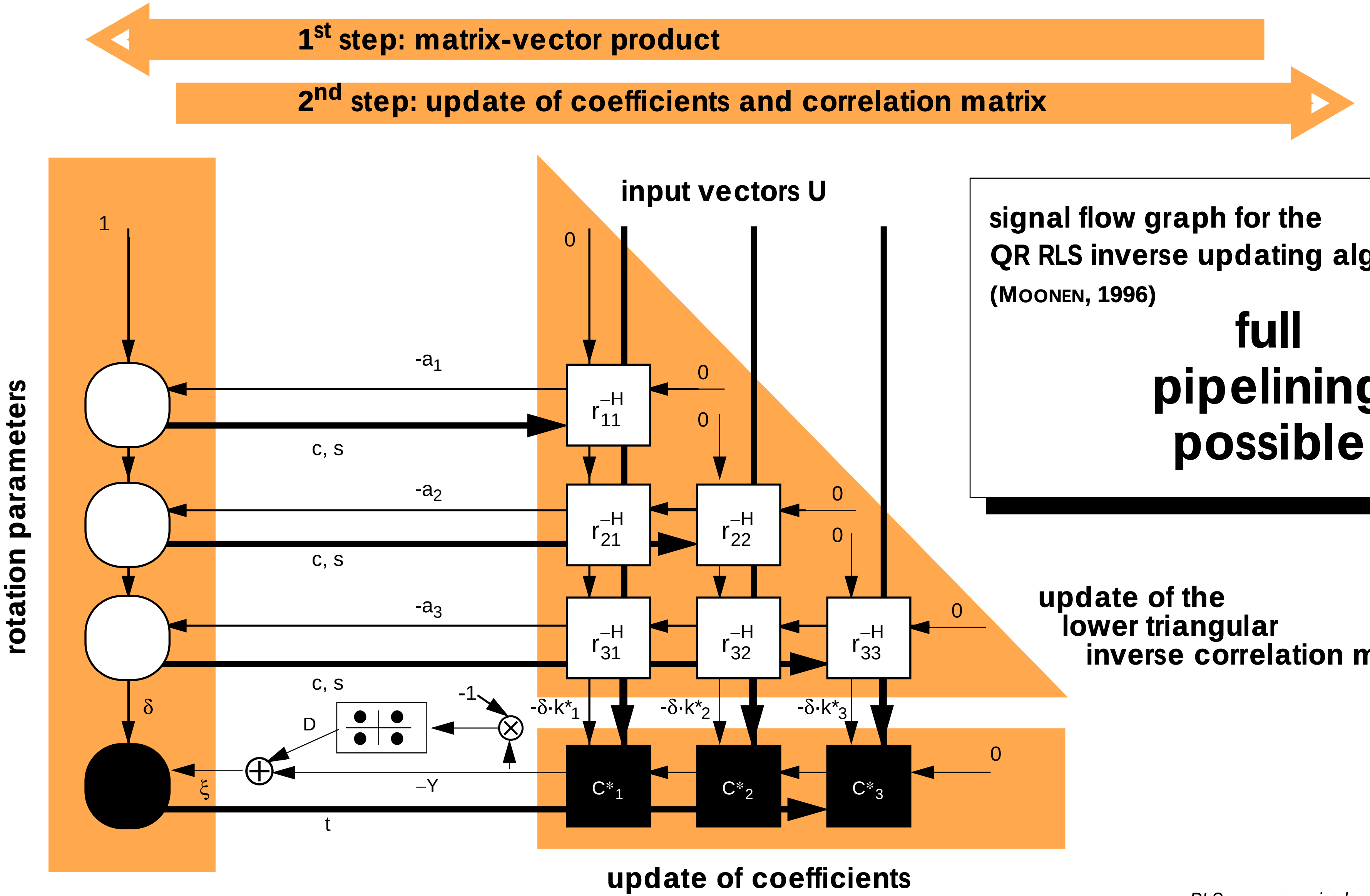

signal flow graph for the
QR RLS inverse updating algorithm
(Moonen, 1996)

full pipelining possible

RLS *recursive least squares*

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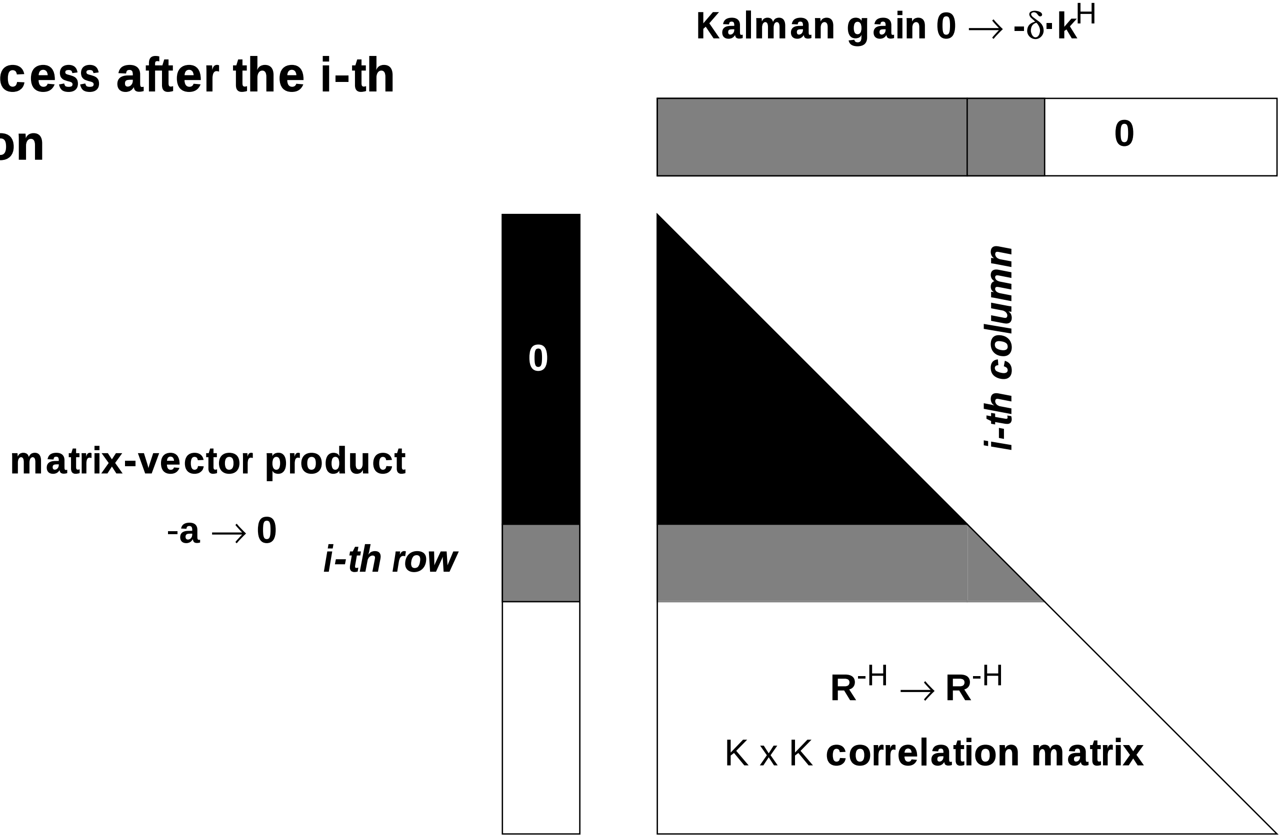
➢ each iteration, K Givens rotation parameters have to be computed to transform the matrix-vector product into a zero vector to update the correlation matrix

updating process after the i-th Givens rotation



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novel QRD based architecture for MRLS algorithms

express the recursive update equation for the modified correlation matrix as a matrix product

$$\mathbf{P}_{m+1,m}[l] = (\mathbf{P}_{m,m-1}[l] - \zeta_m[l]\mathbf{k}_m^H[l]) + q_m\mathbf{1} = \mathbf{P}_m[l] + q_m[l]\mathbf{1}$$

$$= \begin{bmatrix} \sqrt{q_m[l]}\mathbf{1} \\ \mathbf{R}_m^{-H}[l] \end{bmatrix}^H \begin{bmatrix} \sqrt{q_m[l]}\mathbf{1} \\ \mathbf{R}_m^{-H}[l] \end{bmatrix}$$

R^{-H} ... lower triangular matrix

new method:

- ❑ **Cholesky-factorization of the modified correlation matrix**
- ❑ **compute the Cholesky-factorization by means of Givens-rotations**

$$\prod_{j=1}^{J} \Theta_{j,m} \cdot \begin{bmatrix} \sqrt{q_m[l]}\mathbf{1} \\ \mathbf{R}_m^{-H}[l] \end{bmatrix} = \begin{bmatrix} 0 \\ \mathbf{R}_{m+1,m}^{-H}[l] \end{bmatrix}$$

MRLS	*modified recursive least squares*
QRD	*QR decomposition*

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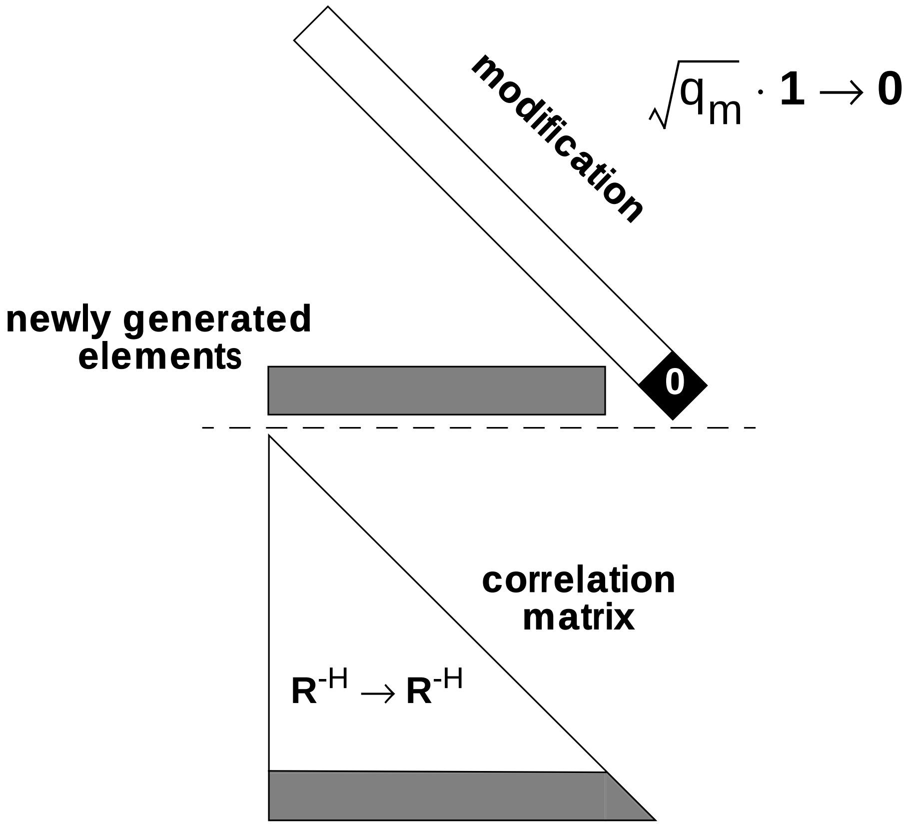
Novel Square-Root MRLS Algorithm - II

- ➢ **for the modification, a diagonal matrix has to be rotated into the lower triangular correlation matrix**
- ➢ **this operation is similar to the correlation matrix update of the conventional QR RLS algorithm (GENTLEMAN & KUNG, 1981)**



modification process after the first Givens rotation

MRLS modified recursive least squares

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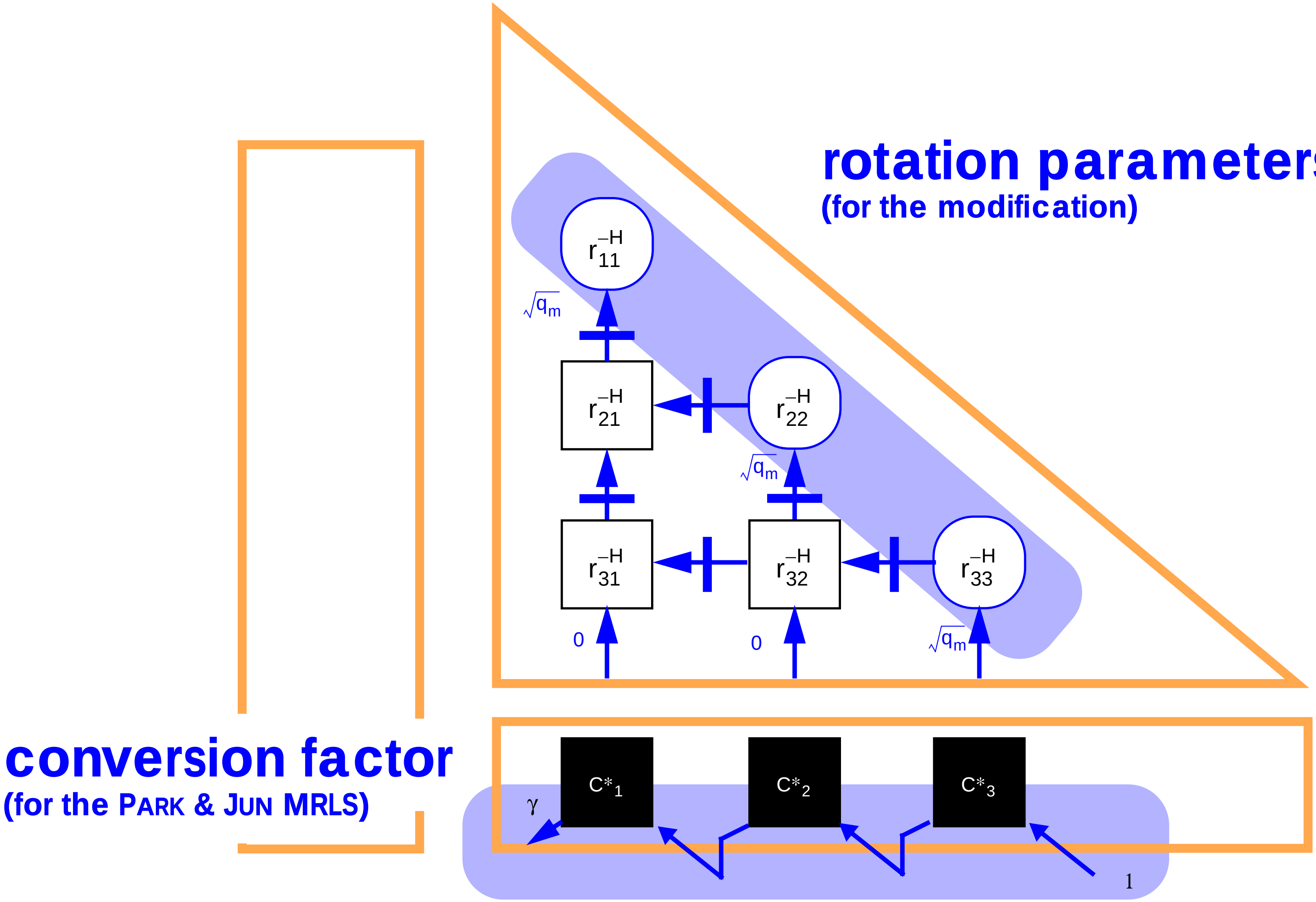
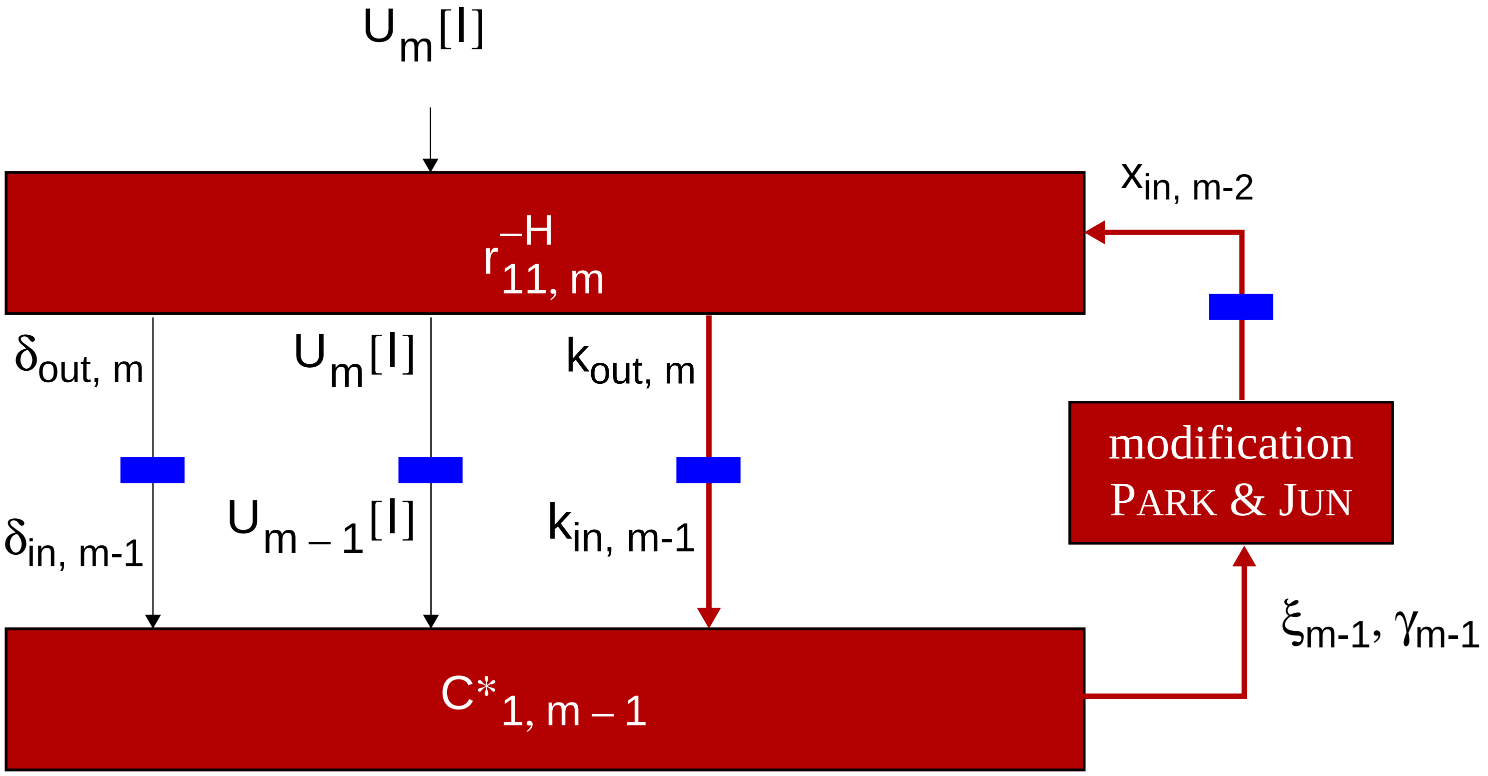
add the operations for the modification to MOONEN´s array



The critical path contains the feedback loop for the modification.

- ➢ **delay relaxation**
- → **fully pipelined MRLS (PARK & JUN) for K = 1**
- → **small degradation when the modification changes slowly with time**



SER degradation due to finite precision

		$\langle W_U \rangle$	$\langle W_{max} \rangle$	SER (float)	target SER	SER (bit true)
A	**d.C.**	12 Bit	12 Bit	$3{,}0\cdot10^{-3}$	$< 10^{-2}$	$3{,}4\cdot10^{-3}$
	LMS	6 Bit	11 Bit	$< 2{,}0\cdot10^{-4}$	$< 10^{-2}$	$6{,}8\cdot10^{-3}$
	QR-RLS	6 Bit	12 Bit	$< 2{,}0\cdot10^{-4}$	$< 10^{-2}$	$6{,}6\cdot10^{-3}$
B	**QR-MRLS**	9 Bit	20 Bit	$5{,}3\cdot10^{-2}$	$5{,}3\cdot10^{-2}$	$4{,}8\cdot10^{-2}$

A ... phase noise + stationary fading channel, **B** ... AWGN + time-variant fading channel

- ➢ **differential coding imposes the highest precision requirements on the FFT output**
- ➢ **square values require high wordlengths**

SER symbol error rate
d.C. differential coding
LMS least mean squares
(M)RLS (modified) recursive least squares
FFT fast Fourier transform
AWGN additive white gaussian noise

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estimated complexity for FDE

	through-put [MHz]	clock [MHz]	NAND gate equivalents
d.C.	27,55	82,64	3 000
LMS	9,18	82,64	7 000
QR RLS r_{11}^{-H} cell	2,17	82,64	5 000
QR RLS $c*_1$ cell	4,35	82,64	6 000
QR MRLS (delay relaxation) - r_{11}^{-H} cell	1,10	68,17	9 000
QR MRLS (delay relaxation) - $c*_1$ cell	4,79	67,11	11 000
QR MRLS (no delay relaxation)	0,923	51,71	20 000

- **differential coding has the lowest complexity**
- **high wordlengths reduce the throughput**
- **divisions and square-roots should be implemented as lookup-tables**

d.C. *differential coding*
LMS *least mean squares*
(M)RLS *(modified) recursive least squares*
FDE *frequency domain equalization*

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- Phase noise introduces severe ICI.
- Linear adaptive FDE in combination with perfect guard intervals provides more flexibility than coherent channel estimation and is less sensitive with respect to ICI than differential coding.
- A novel QRD based architecture for the MRLS algorithm has been developed.
- Neither the RLS, LMS or MRLS algorithms, nor differential coding, can mitigate the requirements on the purity of oscillators in OFDM-systems significantly.
- From a complexity and performance point of view, QR RLS FDE or differential coding are the methods of choice for FRA subject to moderate oscillator phase noise.
- Differential coding has the lowest complexity but imposes the highest precision requirements on the FFT output.
- Adaptive FDEs are applicable to OFDMA-systems and non-orthogonal filter banks with better spectral properties than DFT filter banks, as well.

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