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A BASIC PROPERTY OF MOS TRANSISTORS AND ITS CIRCUIT IMPLICATIONS

E. Vittoz[1,2], C.Enz[1,2] and F.Krummenacher[2]

[1] CSEM, Neuchâtel, Switzerland

[2] EPFL , Lausanne, Switzerland

evittoz@csem.ch

INTRODUCTION

- Goals of transistor modeling:
  - simulation by quantitative calculation on computer
  - highlighting properties to facilitate
    - understanding circuits
    - synthesis of robust circuits

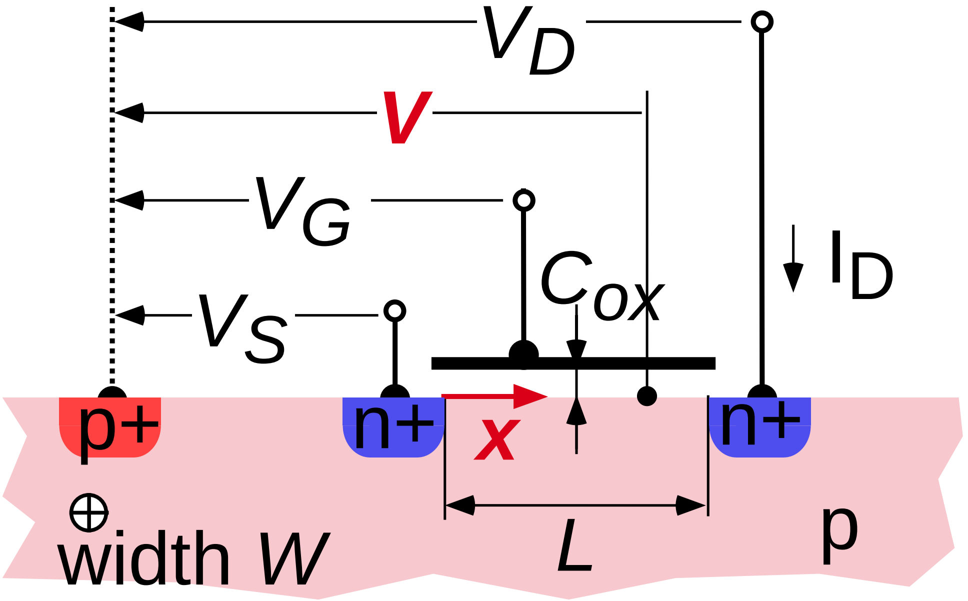
- Best models: combine both goals by hierarchical structure

  example: EKV model.

- EKV approach will be used to explicit a basic property.

DEFINITIONS

for EKV model



- Substrate referred-voltages

  V_S, V_D, V_G

- Local "channel voltage" ***V***

  splitting of quasi-Fermi levels due non-0 V_S and/or V_D

  V = V_S at source

  V = V_D at drain

  n-channel: holes at equilibrium

  thus ***V*** = electron quasi-Fermi level + constant.

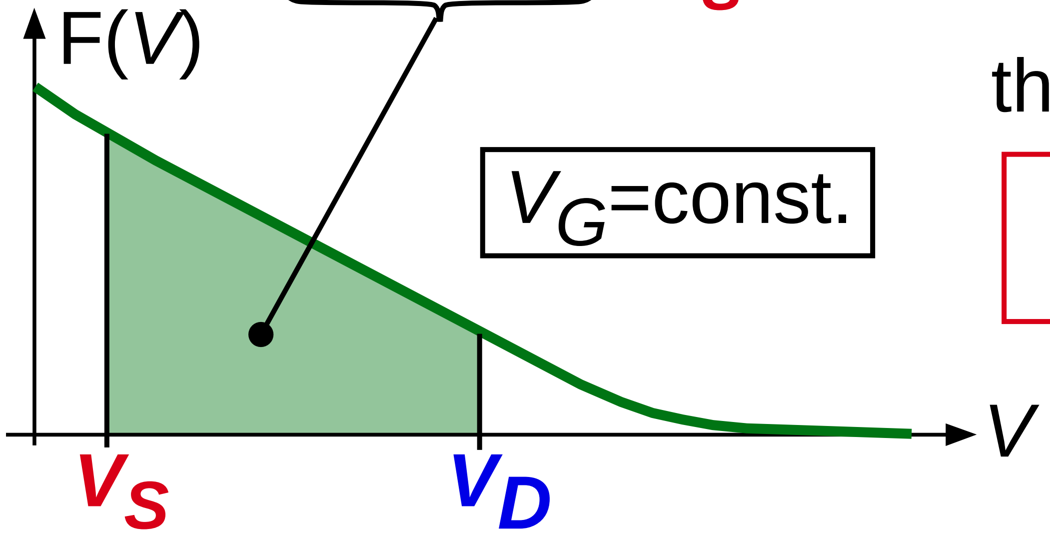
BASIC PROPERTY (1)

- For a long and wide channel:

$$I_D = \mu W(-Q_i)\,\frac{dV}{dx} = \frac{F(V,V_G)\,dV}{G(x,V_G)\,dx}$$

- Condition: separable in V and x

$$\text{then: } \underbrace{I_D\int_0^L G\,dx}_{} = \int_{V_S}^{V_D} F\,dV \equiv \int_{V_S}^{\infty} F\,dV - \int_{V_D}^{\infty} F\,dV$$



thus:

$$\boxed{I_D = I(V_S,V_G) - I(V_D,V_G)}$$

BASIC PROPERTY (2)

$$\boxed{I_D = \mathrm{I}(V_S,V_G) - \mathrm{I}(V_D,V_G)}$$

- The drain current is the superposition of independent and symmetrical effects of source and drain voltages.

- Definitions:
    - **Forward** current $I_F = \mathrm{I}(V_S,V_G)$, independent of V_D
    - **Reverse** current $I_R = \mathrm{I}(V_D,V_G)$, independent of V_S

$$\boxed{\text{then } I_D = I_F - I_R}$$

DOMAIN OF VALIDITY (1)

- Condition: $$\boxed{\mu W(-Q_i) = \frac{F(V,V_G)}{G(x,V_G)}}$$

with: $$-Q_i = \underbrace{C_{ox}(V_G-V_{FB}-\Psi_s)}_{\text{total charge}} - \underbrace{\sqrt{2qN_b\varepsilon_{si}\Psi_s}}_{\text{depletion charge } Q_b}$$

- Mobile charge Q_i depends on surface potential Ψ_s, and Ψ_s=f(V), thus Q_i should not be a (direct) function of x to be part of F. Therefore:

- V_G-V_{FB}
- C_{ox}
- N_b

- must be independent of position x along the channel : homogeneous channel.

- but may depend on Ψ_s or V.

(*e.g.*: $C_{ox}(\Psi_s)$: polydepletion)

DOMAIN OF VALIDITY (2)

- Condition: $$\boxed{\mu W(-Q_i) = \frac{F(V,V_G)}{G(x,V_G)}}$$

- W is independent of V; thus:
  - part of G, may depend on x: ⇒ any shape of channel.

- Mobility μ depends on vertical field thus on Ψ_S , thus
  - included in F, provided velocity $v \ll v_{sat}$

    (otherwise depends on I_D itself)

- Furthermore, the effective value of L along which $G(x,V_G)$ is integrated must be independent of I_D, V_S and V_D.

EFFECT OF NARROW CHANNEL

- Increased importance of side effects.
- Equivalent to parallel connection of several transistors with different characteristics.
  - if each transistor *i* fulfils

$$I_{Di} = \mathrm{I}_i(\boldsymbol{V_S}, V_G) - \mathrm{I}_i(\boldsymbol{V_D}, V_G)$$

  - then the sum I_D of I_{Di} fulfils it as well.
- The property is not degraded.

DOMAIN OF VALIDITY (SUMMARY)

The basic property is available

- For long and homogeneous channel
- Independently of channel shape
- Even if the channel is very narrow
- Even for large gate voltages reducing the mobility.

CAUSES OF DEGRADATION (1)

- Non homogeneous channel: Q_i direct function of x.

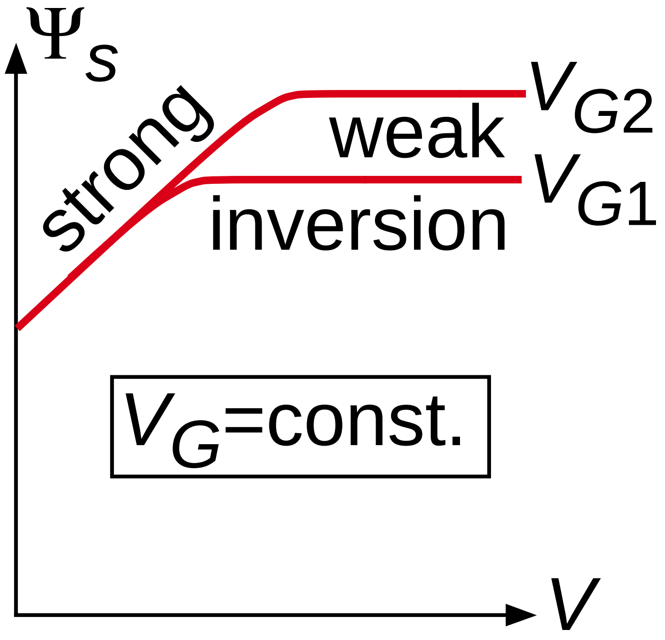
$$Q_i = -C_{ox}(V_G - V_{FB} - \Psi_s) + \sqrt{2q\,N_b\,\varepsilon_{si}\Psi_s}$$

There may be variations with position x in the channel of:

- substrate doping N_b, which can be
  - intentional (e.g.: LDD)
  - artifact of process (gradient or piling-up)
    (always present at very ends of channel)
- flat-band voltage V_{FB} , caused by
  - variation of N_b
  - variation of charge in oxide
- effective C_{ox}, always present at very ends of channel.

SPECIAL CASE OF WEAK INVERSION

- Weak inversion characterized by $Q_i \ll Q_b$, therefore:
  - Q_i has negligible effect on potential and field
- Can be expressed as $-Q_i = G_q(\Psi_s)e^{-V/U_T}$
  - with Ψ_s independent of V, thus:
  - G_q can be any function of x and is included in G, therefore:
- The property is valid even if the channel is not homogeneous.



- Mobility μ independent of V, thus part of G,
  F is reduced to $F = e^{-V/U_T}$: independent of V_G.

CAUSES OF DEGRADATION (2)

- Channel long ⇒ non-long ⇒ short
  - property progressively degraded by...
  - several independent mechanisms:

a. Voltage effects:

- channel length modulation
  - I_F or I_R becomes function of both V_D and V_S
  - effect proportional to $1/L$
- barrier lowering and 2-D effects: further degradation.

b. Current effects:

- if I_D is increased by reducing L, then

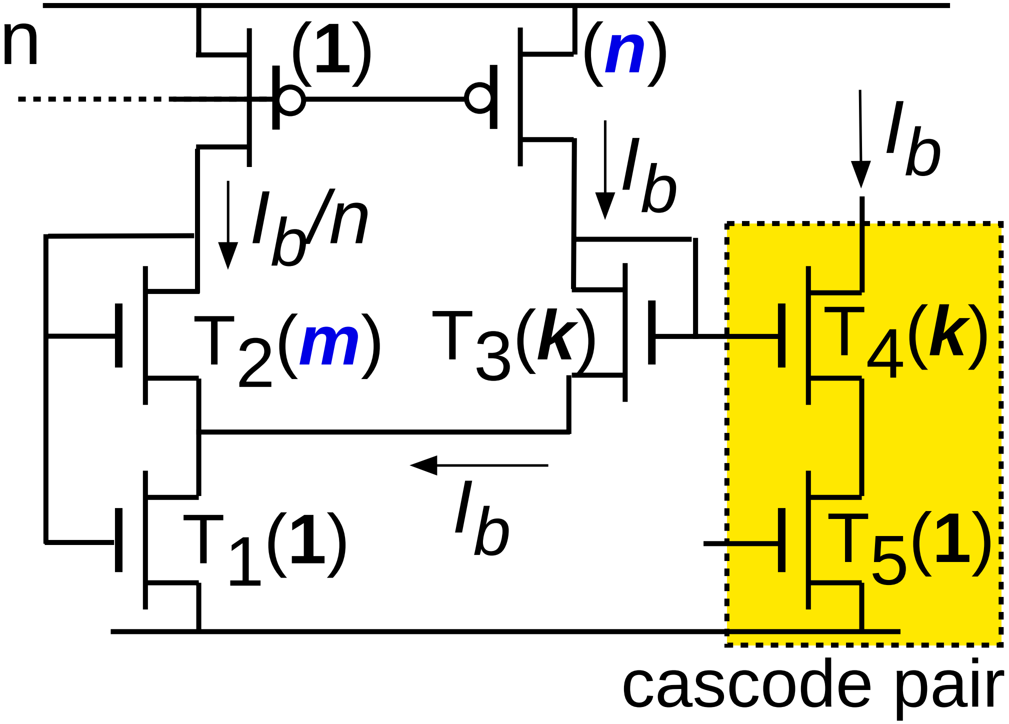
  ⇒ carrier velocity increases towards saturation

  ⇒ mobility reduced, thus function of I_D

c. Non-homogeneous channel (except in weak inversion):

- importance of end-effects proportional to $1/\boldsymbol{L}$.

CIRCUIT EXAMPLE: LOW-VOLTAGE CASCODE



- Goal: V_{D5} min. for saturation
- Means: control $\frac{I_{R5}}{I_{F5}} \ll 1$:
  - ***m*** and ***n*** » 1, hence:
  - $I_{D5} \cong I_{D1}$ with $V_{D5}=V_{D1}$
  - thus $I_{R5}/I_{F5} \cong I_{R1}/I_{F1}$

$$\left.\begin{array}{l} \bullet\ I_{F2} = I_b/n = m\, I_{R1} \\ \bullet\ I_{F1} \cong I_b \end{array}\right\} \Rightarrow \frac{I_{F1}}{I_{R1}} \cong \boxed{mn \cong \frac{I_{F5}}{I_{R5}}}$$

- Large enough to ensure saturation
- Independent of I_b.

CONCEPT OF PSEUDO-RESISTOR

- We have shown that: $$I_D = \frac{1}{\int_0^L G\mathrm{d}x}\left[\int_{V_S}^{\infty} F\mathrm{d}V - \int_{V_D}^{\infty} F\mathrm{d}V\right]$$
- Definitions:
  - pseudo-voltage: $$\boxed{V^* = -K_0 \int_V^{\infty} F(V,V_G)\mathrm{d}V}$$
  - pseudo-resistor: $$\boxed{R^* = K_0 \int_0^L G\,(x,V_G)\mathrm{d}x}$$

(where K_0: any positive constant)

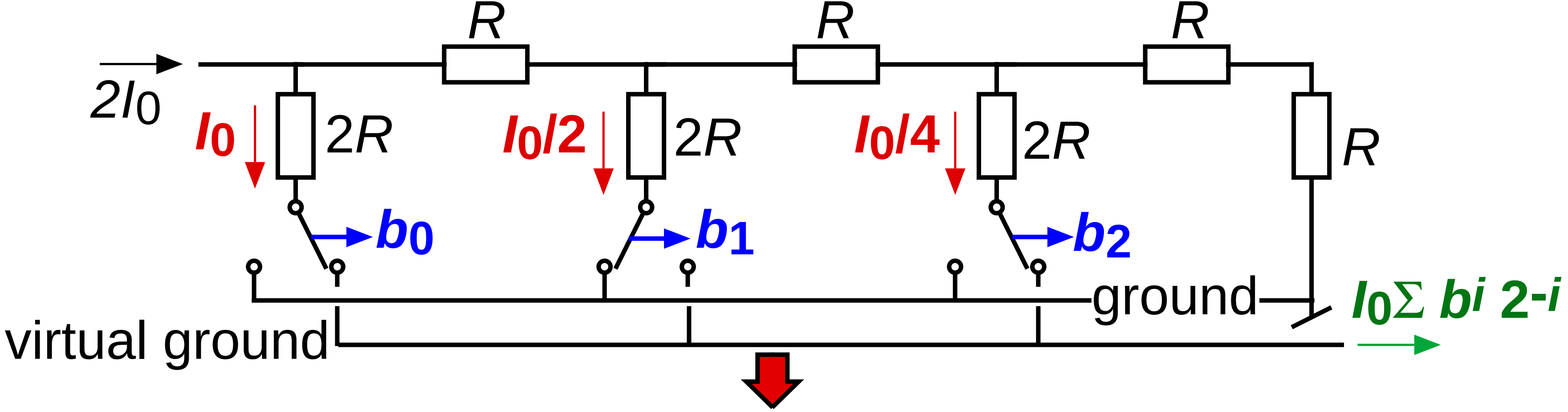
- Results in pseudo Ohm's Law: $$\boxed{I_D = (V_D^* - V_S^*)/R^*}$$

LINEAR CURRENT-MODE CIRCUITS

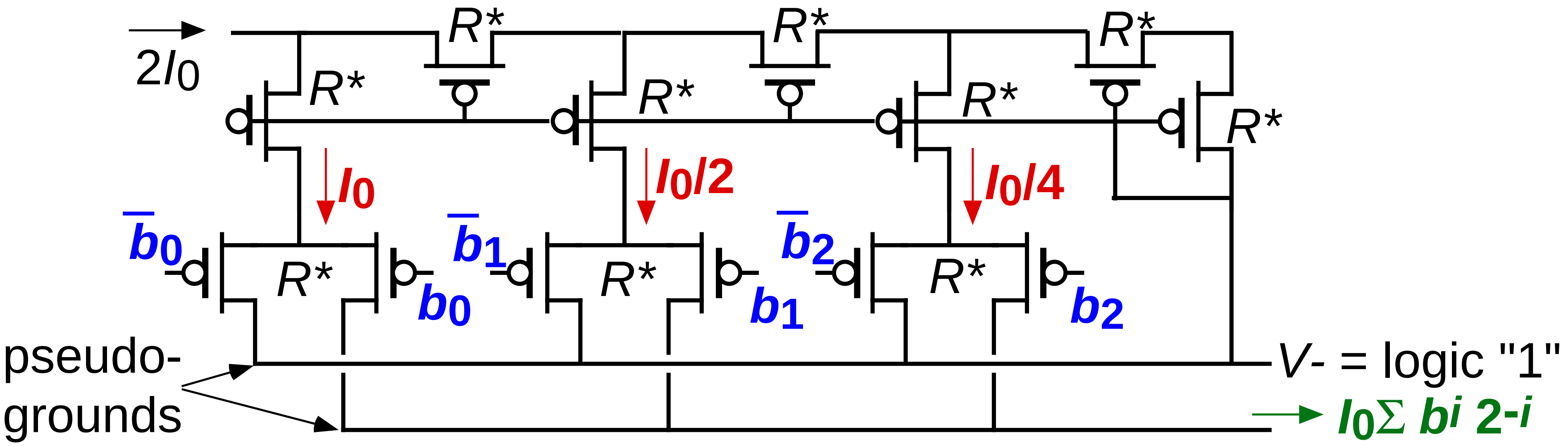
- Implications of pseudo Ohm's law $I_D = (V_D^* - V_S^*)/R^*$
  - Any network interconnecting transistors with same F(V,V_G) and same V_G is linear with respect to currents.
  - Any circuit of linear resistors can be implemented by transistors only, provided only currents are considered.
  - A resistor connected to ground (V=0) in the resistive prototype corresponds to a saturated transistor that provides a pseudo-ground (V^*=0).
- In weak inversion:
  - F indep. of V_G, but V_G included in function G, hence:
  - Different V_G possible for each transistor
  - Each R^* can be separately adjusted by its V_G.

EXAMPLE OF APPLICATION: *R*-2*R* D/A CONVERTER

- Standard resistive circuit (example for 3-bit): [6,7]



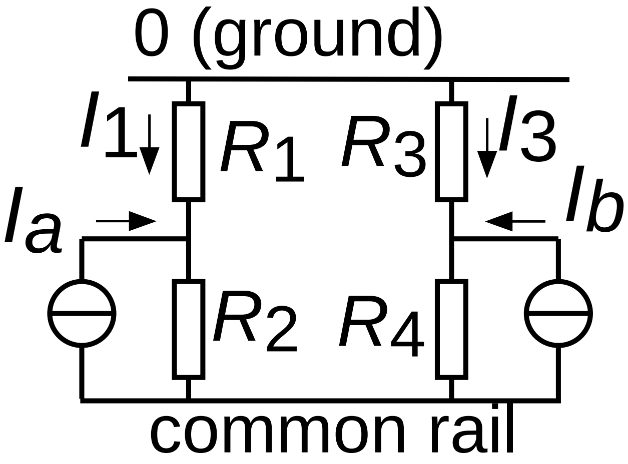
- Pseudo-resistive implementation (by p-channel transistors):



- For best precision:
  - strong inversion
  - non-satutated devices (avoid pseudo-grounds)

APPLICATION OF PSEUDO-RESISTORS

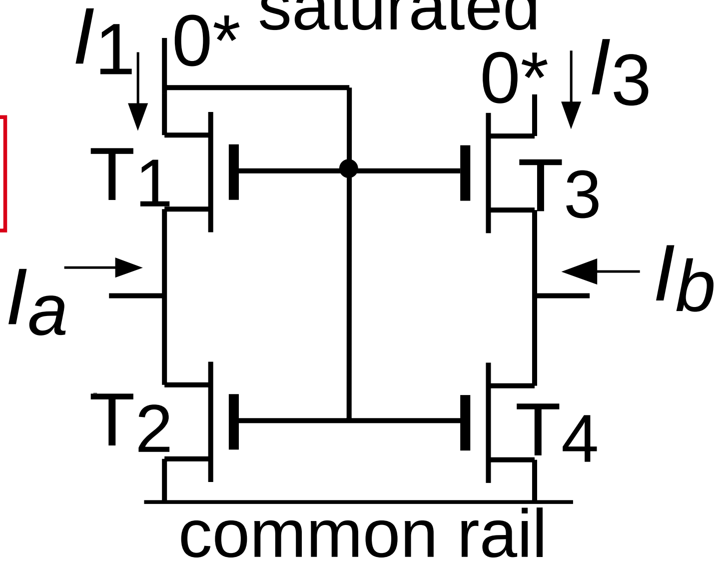
(example)



$$(R_3+R_4)I_3+R_4I_b=(R_1+R_2)I_1+R_2I_a$$

$$\boxed{R_i \sim L_i\,/\,W_i}$$

⇔

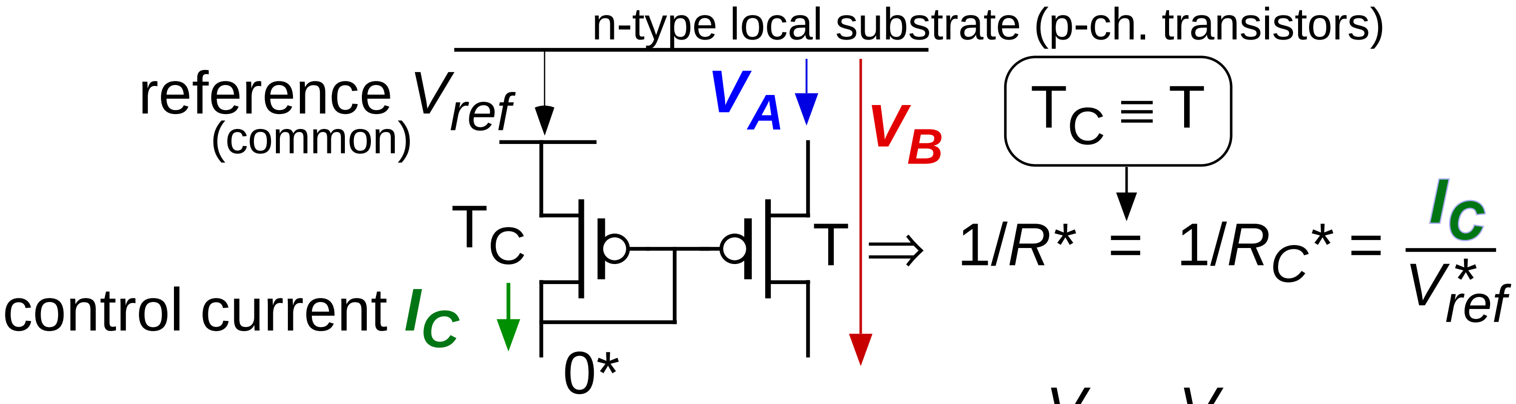


thus: $$\boxed{I_3 = k_1I_1+ k_aI_a -k_bI_b}$$

- Output I_3 is the weighted sum of I_1, I_a and **-**I_b.
- I_a is low-voltage input (T_2 not saturated)
- For R_4=0 (T_4 in s/c) and T_1=T_2=T_3 ⇒ $k_1 = 2$
  - for N unit trans. in series at input ⇒ $k_1 = N$
    - for M units in parallel at output ⇒ $k_1 = M.N$

PSEUDO-RESISTORS IN WEAK INVERSION

- If I_F and $I_R \ll I_S = 2n\beta U_T^2$ → weak inversion
- Then: $I(V_G, V) = I_S \exp\frac{V_G - V_{T0}}{nU_T} \exp\frac{-V}{U_T}$ **separable** in V_G and V
- Therefore:
  - pseudo-voltage $V^* = -V_0 \exp\frac{-V}{U_T}$ **independent** of V_G
  - pseudo-conductance $1/R^* = \frac{I_S}{V_0} \exp\frac{V_G - V_{T0}}{nU_T}$ **controllable** by V_G
- Low voltage swing (in wide current range), thus compatible with **low V_B**
- Control of R^* by a current:



n-type local substrate (p-ch. transistors)

reference V_{ref}
(common)

V_A

V_B

$T_C \equiv T$

T_C

T

control current I_C

0*

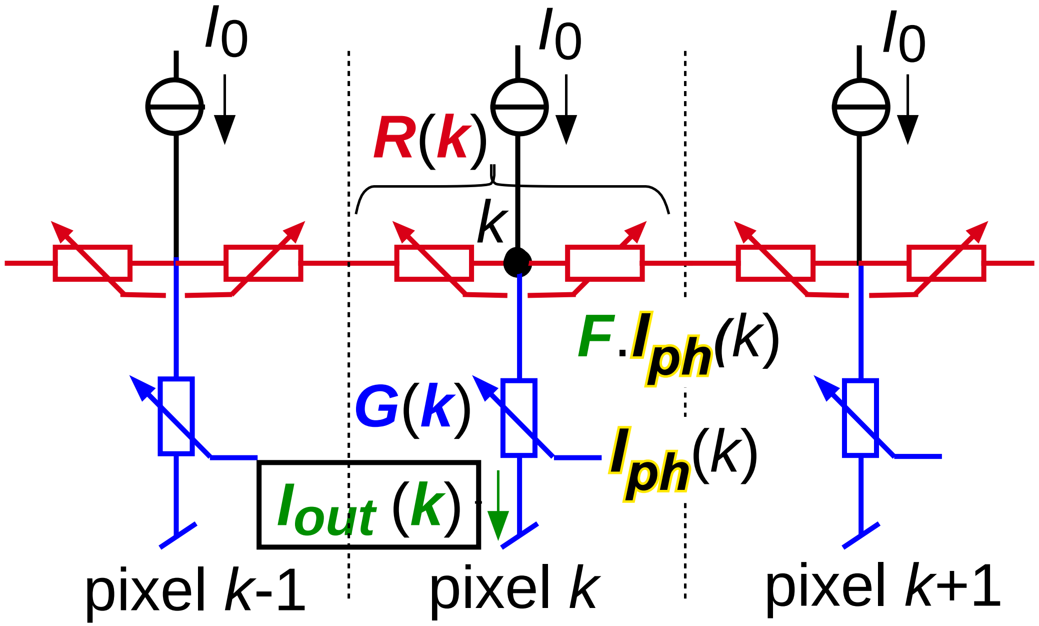
$$\Rightarrow 1/R^* = 1/R_C^* = \frac{I_C}{V^*_{ref}}$$

- weak inversion ensured if $I_C \exp\frac{V_{ref} - V_{A(B)}}{U_T} \ll I_S$

EXAMPLE OF APPLICATION: LOCALLY ADAPTIVE RETINA

[97-1]

- Schematic for 1-D array:

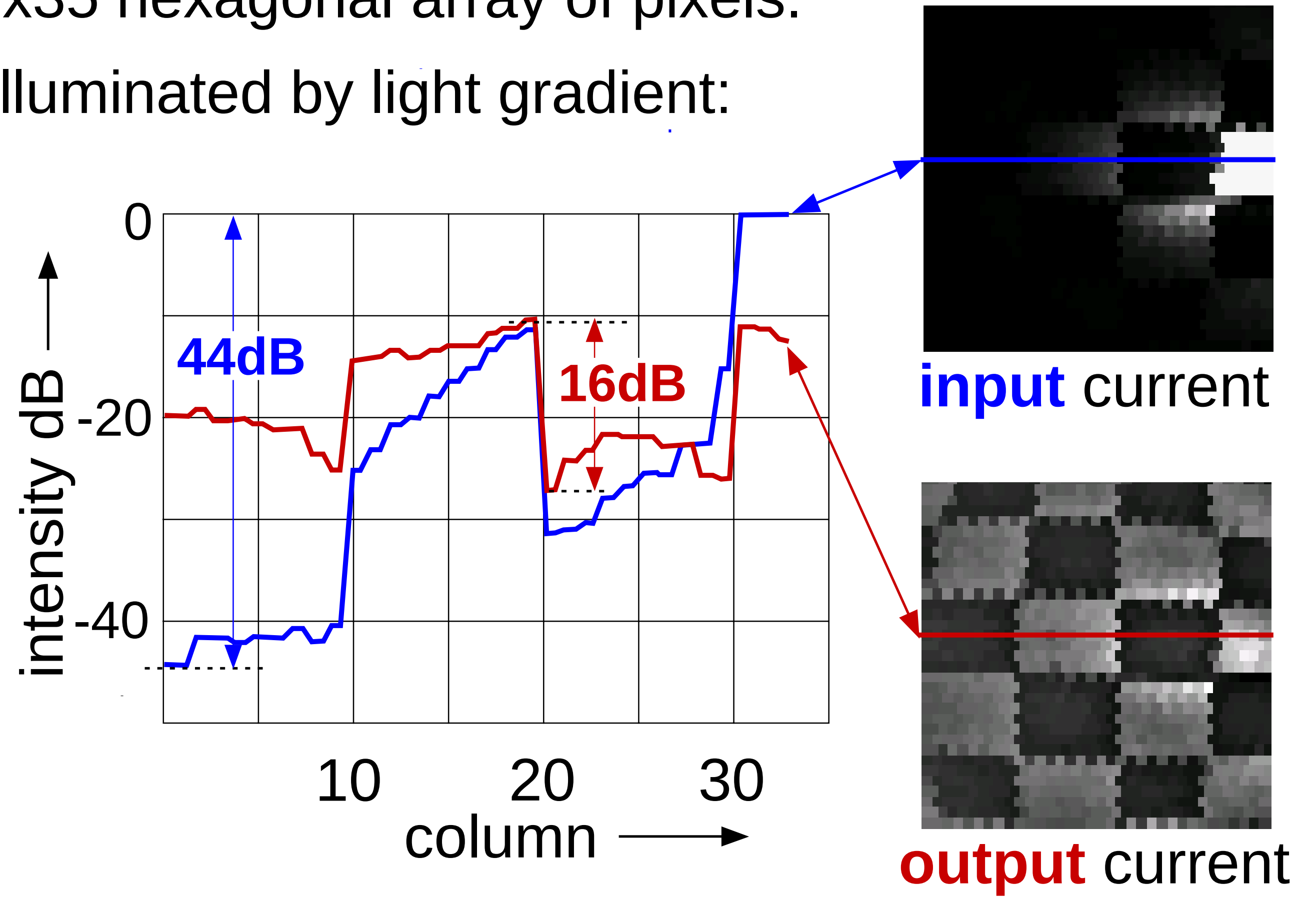


- Real implementation: 2-D array
  with $R,G \rightarrow R^*,G^*$
  (pseudo-resistors)

- $G(k)$ proportional to $I_{ph}(k)$ (local photocurrent)
  - $1/R(k)$ proportional to $F.I_{ph}(k)$
- If $R(k) = 0$ then $\Sigma I_{out}(k) = \Sigma I_0$: global normalisation
- If $R(k) > 0$, each I_0 distributed across area $A \sim L^2 = 1/(RG) \sim F$
  - **local normalisation** in adjustable area A.

MEASUREMENTS ON
LOCALLY ADAPTIVE RETINA

- Experimental 35x35 hexagonal array of pixels.
- Checker-board illuminated by light gradient:

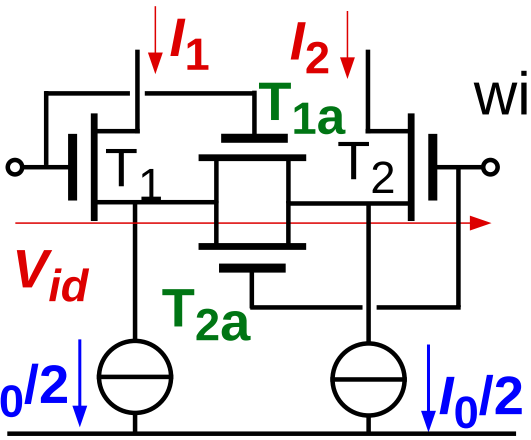


- Dynamic range reduced to that of the object **contrast** (independently of light distribution).

LINEARIZATION OF DIFFERENTIAL PAIR BY TRANSISTORS

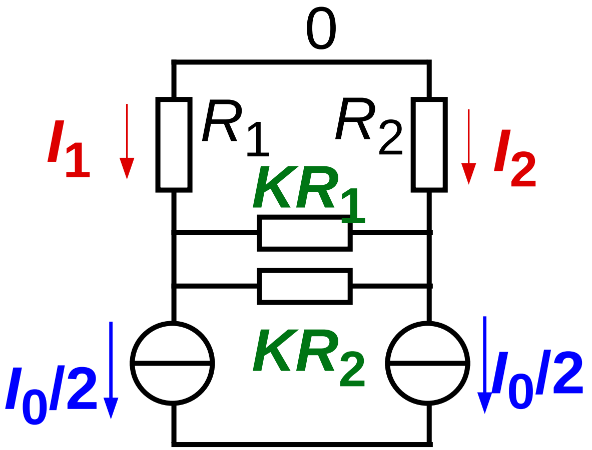
LVT-18

[13]



with $\frac{\beta_1}{\beta_{1a}} = \frac{\beta_2}{\beta_{2a}} = K$

Equivalent circuit in **weak inv.**
(concept of pseudo-resistors):

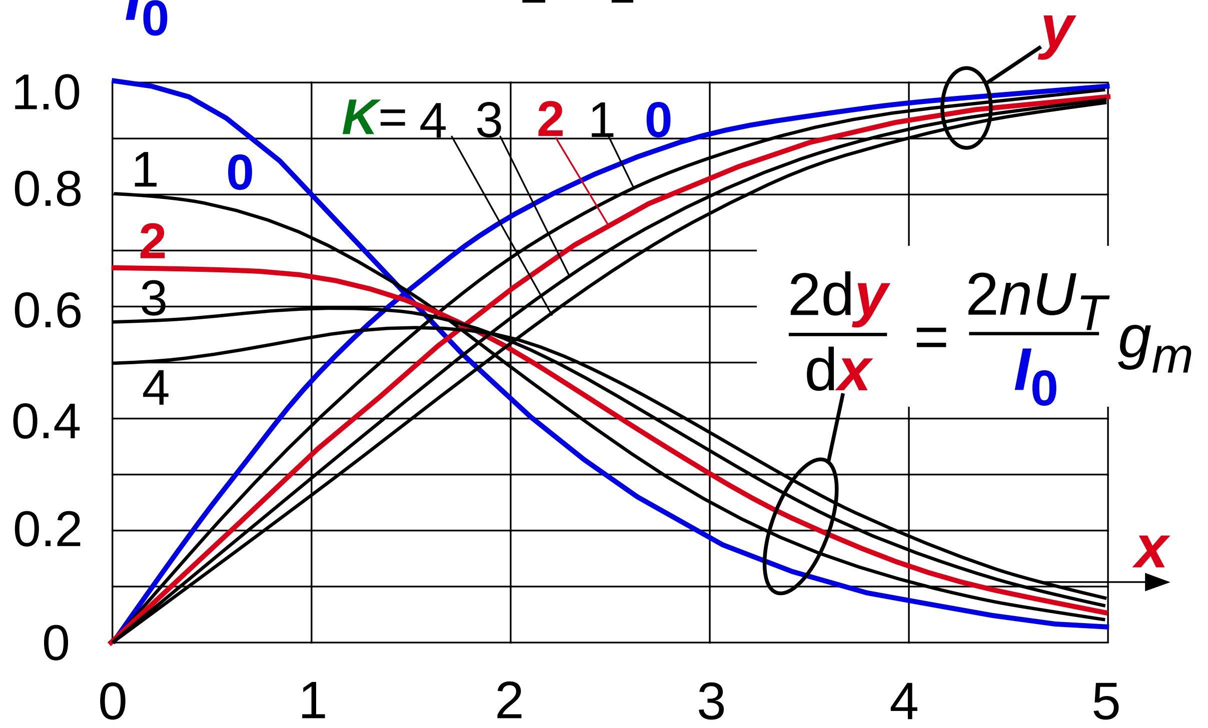


definitions: $x = \frac{V_{id}}{nU_T}$ and $y = \frac{I_1 - I_2}{I_0}$ then: $R_2/R_1 = e^x$

- Results in:

$$y = \frac{e^x - 1}{e^x + 1 + K\frac{e^x}{e^x + 1}}$$

- Also applicable in strong inversion



$$\frac{2dy}{dx} = \frac{2nU_T}{I_0} g_m$$

FURTHER APPLICATIONS OF PSEUDO-RESISTORS

- Linear attenuators (electrical control in weak inversion)
  - R-2R network for D/A conversion.
- Spatial information processing:
  - n^{th} oder moment computation
  - diffusion networks (isotropic or not)
  - 2-D emulation of physical media
  - path finding.
- In weak inversion: exploitation of current distribution in voltage- (or current-) dependent linear networks:
  - local normalisation in vision processing
  - generation of nonlinear functions
  - ...

CONCLUSION

- Basic MOS property for long and homogeneous channels:

$$I_D = \mathrm{I}(V_S,V_G) - \mathrm{I}(V_D,V_G) = I_F - I_R$$

  - superposition of independent and symmetrical effects of S and D voltages.
  - forward and reverse components.
- Property progressively degraded when channel shortened.
- Underlies the concept of pseudoresistor:
  - linear current mode circuits
  - transistor implementation of arrays of resistors.
  - simpler analysis of transistor circuits.

POST-SCRIPTUM: INGREDIENTS OF DC CORE OF EKV MODEL

- Voltages referred to (local) substrate
- Quasi-Fermi level of channel carriers $\boldsymbol{V}$
- Gradual channel and charge sheet approximations
- Mobile ch. density $Q_i = -C_{ox}[V_G - V_{TB}(\psi_s)]$
- $\psi_s = \psi_0 + V$ in strong inversion (independent of V_G)
- Pinch-off value of V : $V_P(V_G) = \psi_P(V_G) - \psi_0$
- Linearization of $Q_i(\psi_s) \Rightarrow n \cong$ constant
- General expression of $Q_i(V_P - V)$
- $I_D \approx \int_{V_S}^{V_D} Q_i \, dV = \mathrm{I}(V_G, V_S) - \mathrm{I}(V_G, V_D) = I_F - I_R$
- Specific current $I_S = 2n\beta U_T^2$
- Inversion coeff. IC = the larger of I_F/I_S or I_R/I_S