HOMEWORK #8
SOLUTION

1) Time-constants associated with capacitors C_1 & C_2, assuming the other capacitor behaves like an open circuit:

$$\tau = C \cdot \langle R \text{ seen by } C \rangle$$

$$\tau_1 = C_1 \cdot \frac{1}{g_{m2}}$$

$$\tau_2 = C_2 \quad r_{o1} // r_{o2}$$

ⓐ $\tau_1 \gg \tau_2 \quad \Rightarrow \quad \boxed{\omega_{p1} = \frac{1}{\tau_1} = \frac{g_{m2}}{C_1}}$

Since this pole cuts off half of our current flowing to the output, there will also be a zero at twice the pole frequency:

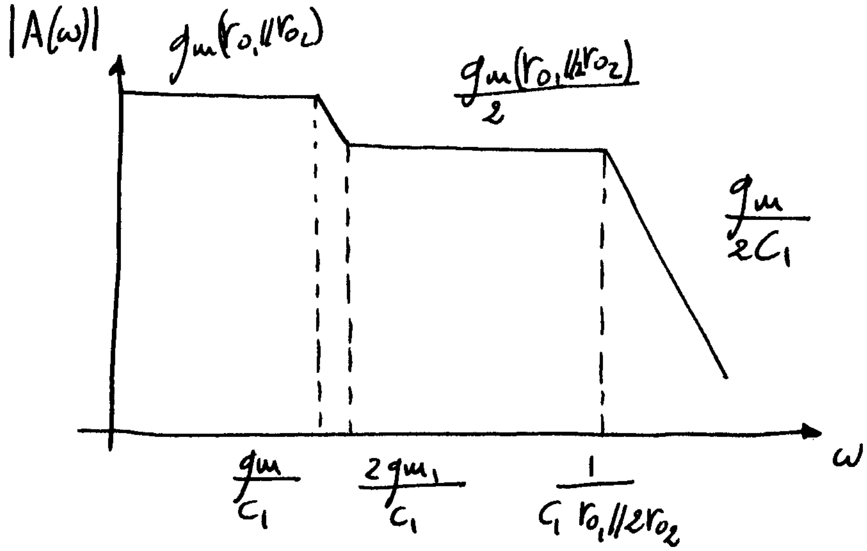
$$i_{out,total} = \underbrace{\frac{i_d}{2}}_{\text{current through M1b}} + \underbrace{\frac{i_d}{2}}_{\text{current through M1a and mirror}} \cdot \frac{1}{1 + s\tau_1}$$

$$= i_d \; \frac{1 + s\tau_1/2}{1 + s\tau_1} \quad \text{where } i_d = g_m V_{id}$$

$$\Rightarrow \boxed{\omega_Z = \frac{2}{\tau_1} = \frac{2 g_{m2}}{C_1}}$$

For $\omega \gg \omega_{p_1}$, the current mirror is shorted out by C_1 and the impedance seen by C_2 becomes $r_{o_2} \parallel 2r_{o_1}$

$$\Rightarrow \boxed{\omega_{p_2} = \frac{1}{C_1(r_{o_1} \parallel 2r_{o_2})}}$$

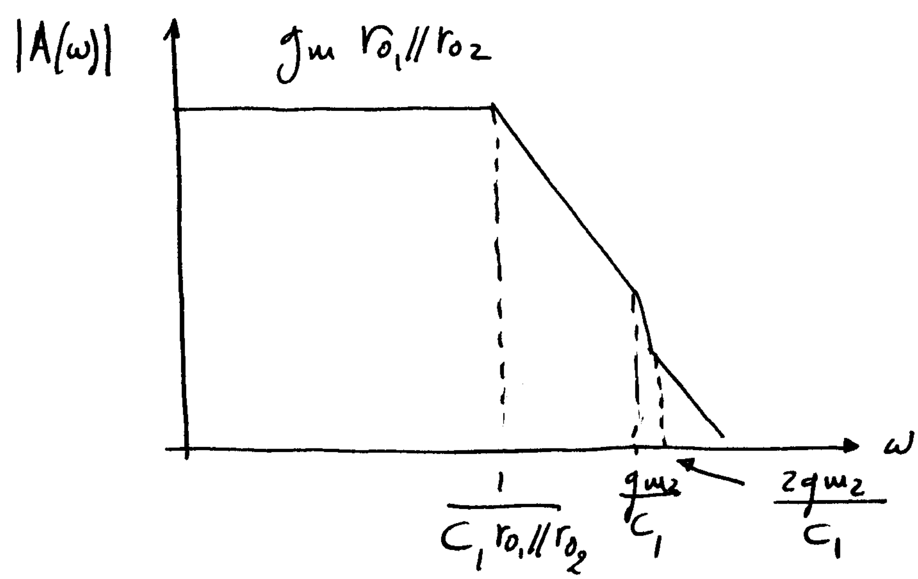


(b) $\tau_2 >> \tau_1 \Rightarrow$ $\boxed{\omega_{p_1} = \frac{1}{\tau_2} = \frac{1}{C_2\, r_{o_1} // r_{o_2}}}$

Even for $\omega >> \omega_{p_1}$, when C_2 looks like a short, the resistance seen by C_1 is still $\frac{1}{g_{m_2}}$

$$\Rightarrow \boxed{\omega_{p_2} = \frac{1}{\tau_1} = \frac{g_{m_2}}{C_1}}$$

$$\boxed{\omega_z = \frac{2}{\tau_1} = \frac{2 g_{m_2}}{C_1}}$$



2) ⓐ $A_{dm} = -\frac{1}{2} \cdot g_{m_1} \left(\frac{1}{g_{m_2}} \,//\, r_{o_1} \,//\, r_{o_2} \right) g_{m_3} \left(R_D \,//\, r_{o_3} \right)$

$$V_{DSat_1} \approx V_{Dsat_3} \approx \sqrt{\frac{2 I_{DS}}{k_n' \; W/L_{eff}}} = \sqrt{\frac{2 \cdot 200\mu A}{332 \frac{\mu A}{V^2} \frac{4}{0.08}}}$$

$$\approx 155 \, mV$$

$$V_{Dsat_2} \approx \sqrt{\frac{2 I_{DS}}{k_p' \; W/L_{eff}}} = \sqrt{\frac{2 \cdot 200 \mu A}{133 \frac{\mu A}{V^2} \frac{4}{0.08}}} = 245 \, mV$$

$$g_{m_1} \approx g_{m_3} \approx \frac{2 I_{DS}}{V_{Dsat_{1,3}}} = 2.6 \, mS$$

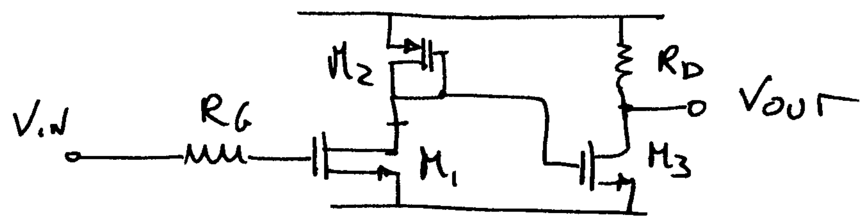
$$g_{m_2} \approx \frac{2 I_{DS}}{V_{Dsat_2}} = 1.6 \, mS$$

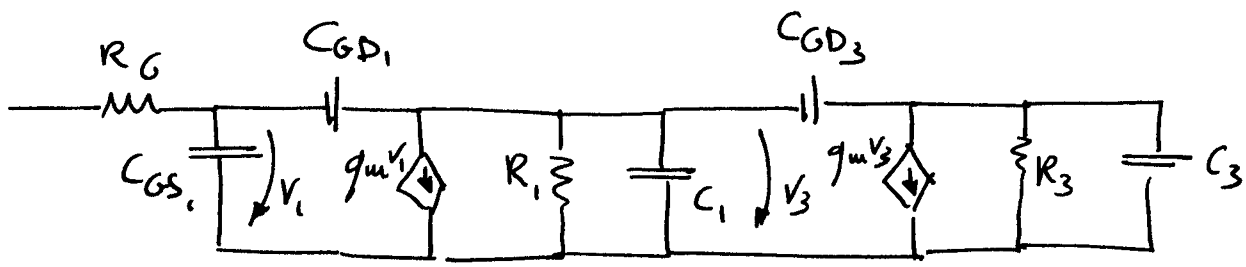
$$r_{o_1} \approx r_{o_3} \approx \frac{1}{\lambda_n I_{DS}} = 25 \, k\Omega$$

$$r_{o_2} \approx \frac{1}{\lambda_p I_{DS}} = 33 \, k\Omega$$

$\Rightarrow$ **$A_{dm} = -6.9$** (SPICE : -6.7)

Differential mode half-circuit:





where $R_1 = \frac{1}{g_{m2}} \,//\, r_{o1} \,//\, r_{o2} \approx 590\,\Omega$

$R_3 = r_{o2} \,//\, R_D = 3.4\,k\Omega$

$C_1 = C_{DB1} + C_{DB2} + C_{GS2} + C_{GS3}$

Resistance seen by C_{GD1}:

$R_{CGD1} = R_G + R_1 + g_{m1} R_1 R_G = 3.1\,k\Omega$

Resistance seen by C_{GD3}:

$R_{CGD3} = R_1 + R_3 + g_{m3} R_1 R_3 = 9.2\,k\Omega$

$$C_{GS} = \frac{2}{3} W(L - 2L_D) C_{ox} + W L_D C_{ox} = 4.77 fF$$

$$C_{GD} = W L_D C_{ox} = 1.33 fF$$

$$V_{DB_1} = V_{D_1} - V_{B_1} = (V_{DD} - V_{GS_2}) - (V_{IN,CM} - V_{GS_1})$$

$$= (1.2V - 300mV - 245mV) - (0.9V - 300mV - 155mV)$$

$$= 210mV$$

$$C_{DB_1} = \frac{C_J \cdot W \cdot 2HDIF}{\left(1 + \frac{V_{DB_1}}{P_J}\right)^{m_j}} + \frac{C_{JSW} \cdot (W + 4HDIF) + C_{JG} \cdot W}{\left(1 + \frac{V_{DB_1}}{P_{JSW}}\right)^{m_{JSW}}}$$

$$= 1.47 fF$$

Similarly: $|V_{DB_2}| = |V_{GS_2}| = 545mV$

$$C_{DB_2} = 1.29mV$$

$$V_{DB_3} = V_{D_3} - V_{B_3} = (V_{DD} - R_D \cdot I_{DS}) - (V_{DD} - V_{GS_2} - V_{GS_3})$$

$$\approx (1.2V - 0.8V) - (1.2V - 0.155V - 0.3V - 0.245V - 0.3V)$$

$$= 0.4V - 0.2V$$

$$= 200mV \approx V_{DB_1}$$

$$\Rightarrow C_{DB_3} \approx C_{DB_1} = 1.47 fF$$

$$\Rightarrow C_1 = C_{DB_1} + C_{DB_2} + C_{GS_2} + C_{GS_3} = 12.5 fF$$

$$C_3 = C_{DB_3} = 1.47 fF$$

$$\tau = R_G C_{GS_1} + R_{C_{GD_1}} C_{GD_1} + R_1 C_1 + R_{C_{GD_3}} C_{GD_3} + R_3 C_3$$

$$= 4.7\,ps + 4.1\,ps + 7.4\,ps + 12\,ps + 5\,ps$$

$$= 33\,ps$$

(Note how the largest contribution is from C_{GD_3})

$f_{3db,\,estimate} = \dfrac{1}{2\pi\tau} = 4.8\,GHz$

(b) $f_{pd} = 7\,GHz$ → much larger than our estimate 3db frequency,

but $\underbrace{\left(\sum_i f_{p_i}^{-1}\right)^{-1}}$ $= 5\,GHz$

→ correct expression for $\dfrac{1}{2\pi\tau}$ (see book)

(c) $f_{3db} = 6.5\,GHz$

(a little lower than f_{pd} due to relatively close non-dominant poles)

```
1 ******  Star-HSPICE -- 2001.4   (20011215) 11:07:34  11/03/2002  solaris
 ******
  homework 8 - problem 2
  ******   pole/zero analysis              tnom=  25.000 temp=  25.000
 ******

      input =  0:vdm                  output = v(vo2b


      input =  0:vdm                  output = v(vo2b

      poles (rad/sec)                 poles ( hertz)
**********************************************************************
    real            imag            real            imag
  -44.1440g        0.              -7.0257g        0.
 -104.7433g       -73.6406g       -16.6704g       -11.7203g
 -104.7433g        73.6406g       -16.6704g        11.7203g
 -153.3417g        0.             -24.4051g         0.
 -199.3073g        0.             -31.7207g         0.
 -216.4428g      -147.4541g       -34.4479g       -23.4680g
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 -380.1907g        0.             -60.5092g         0.
 -469.6599g        0.             -74.7487g         0.
      zeros (rad/sec)                 zeros ( hertz)
**********************************************************************
    real            imag            real            imag
 -104.7433g       -73.6406g       -16.6704g       -11.7203g
 -104.7433g        73.6406g       -16.6704g        11.7203g
 -199.3073g        0.             -31.7207g         0.
 -216.4428g      -147.4541g       -34.4479g       -23.4680g
 -216.4428g       147.4541g       -34.4479g        23.4680g
 -469.6599g        0.             -74.7487g         0.

 ***** constant factor = -1.728e+34      *****
  Opening plot unit= 15
 file=problem2.ac0

 ******
  homework 8 - problem 2
  ******  ac analysis                     tnom=  25.000 temp=  25.000
 ******
   a0=  1.6544E+01
   f3db=  6.4568E+09

          ***** job concluded
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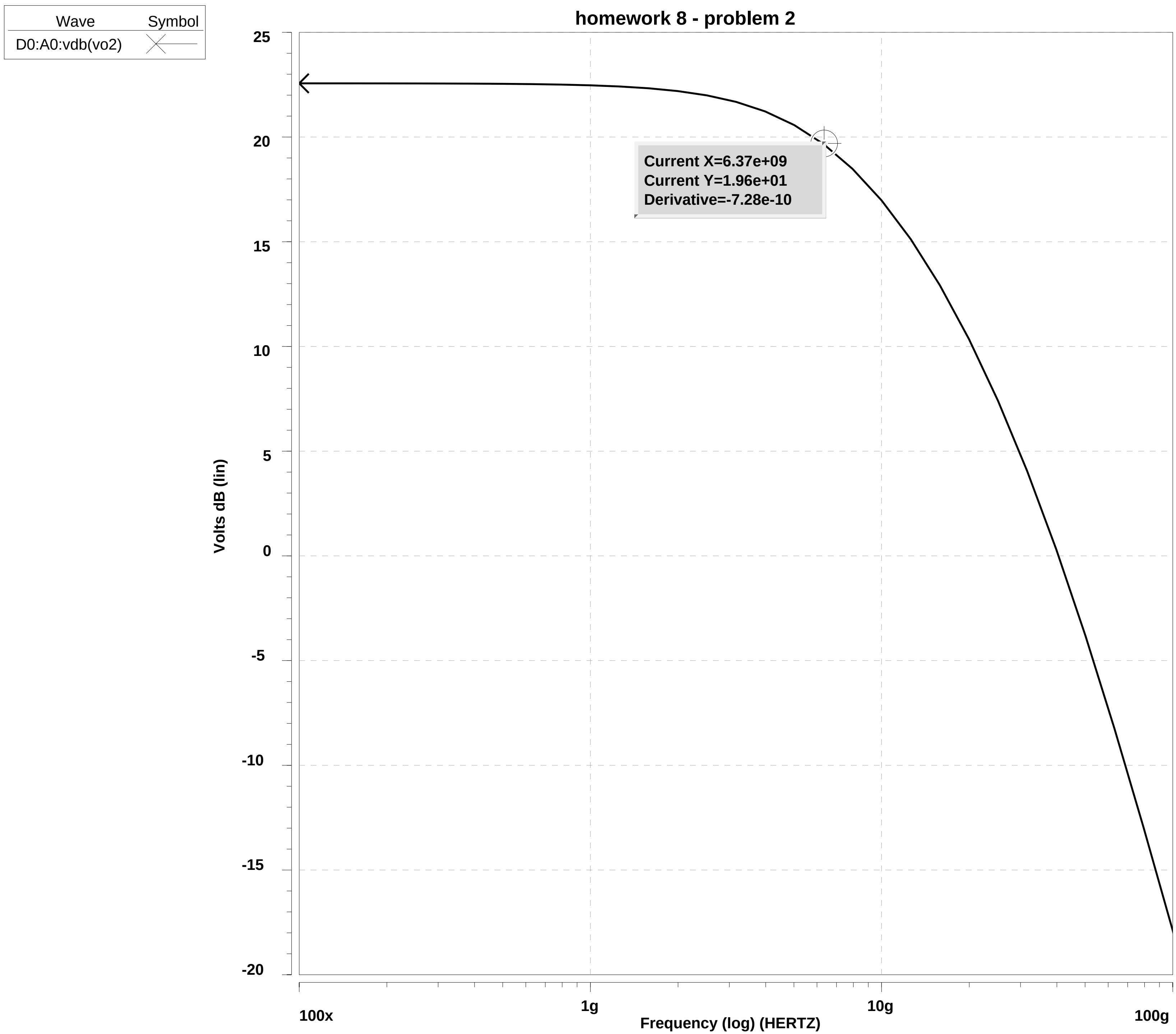
Wave
Symbol
D0:A0:vdb(vo2)
homework 8 - problem 2
25
20
15
10
5
0
-5
-10
-15
-20
Volts dB (lin)
Current X=6.37e+09
Current Y=1.96e+01
Derivative=-7.28e-10
100x
1g
10g
100g
Frequency (log) (HERTZ)