HOMEWORK # 9
SOLUTION

General assumption: $RR >> 1$, $T >> 1$

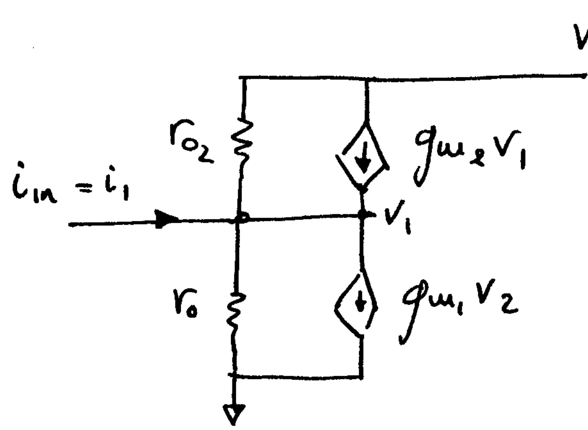
1) (a) M_2 is the forward amplifier
M_1 is the feedback element

(b) shunt - shunt feedback

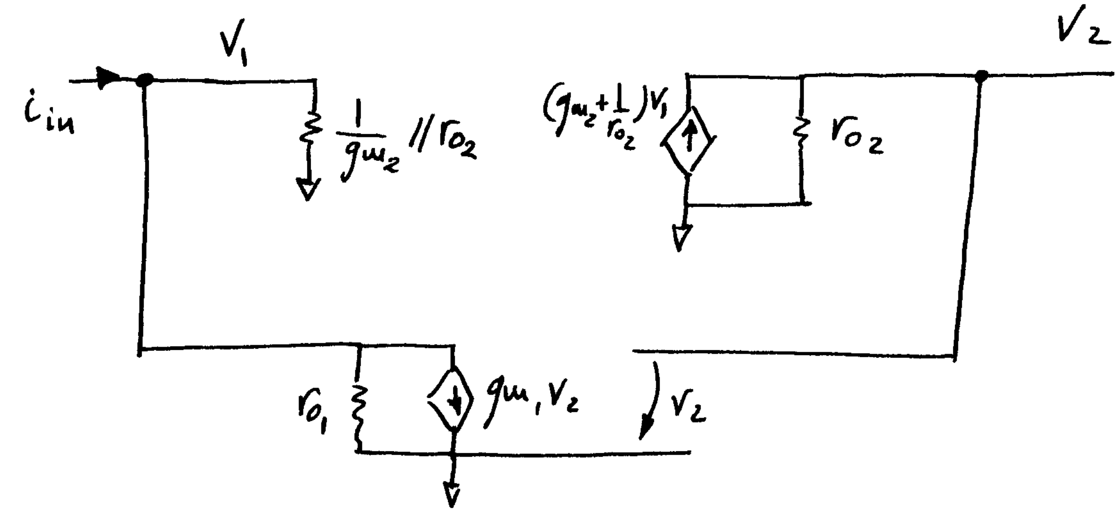
shunt at the input : current summing
(we are feeding in current and feeding back current through $M1$; current summing happens at the drain of M_1, gate of M_2)

shunt at the output : we sample voltage

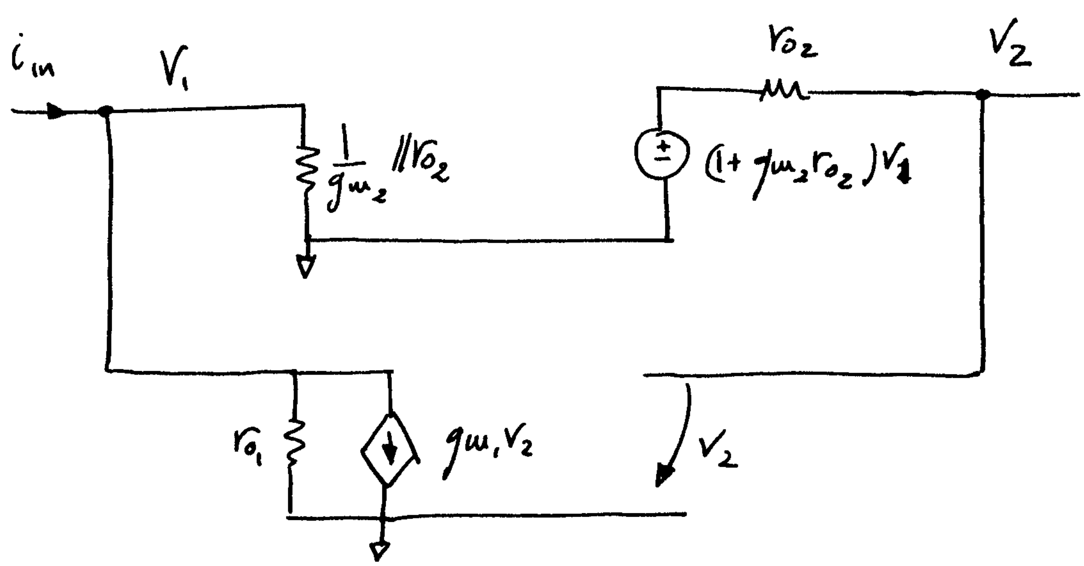
small signal model :



Small signal model showing forward and feedback amplifier



$$\parallel$$



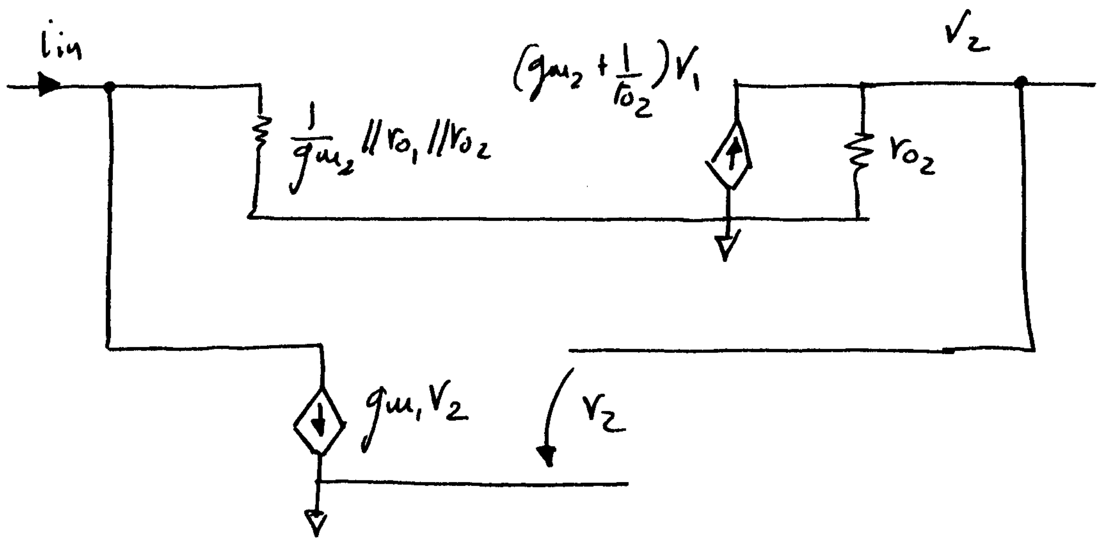
Since this is shunt-shunt feedback, the input and output resistances for the feedback and the forward amplifier have to be computed with the <u>the other terminal shorted</u> (in small signal)

e.g. $\frac{1}{g_{m_2}} // r_{o_2} = \left.\frac{i_1}{v_1}\right|_{v_2=0}$ (only considering M_2)

$r_{o_2} = \left.\frac{i_2}{v_2}\right|_{v_1=0}$ (only considering M_1)

These parameters are called the "Y-parameters", which you should use for shunt-shunt feedback to consider the loading of the feedback network <u>when the terminal resistances of the forward amplifier depend on the other terminal</u>

Now we take the loading of the feedback network into account by moving all non-ideal elements to the forward amplifier:



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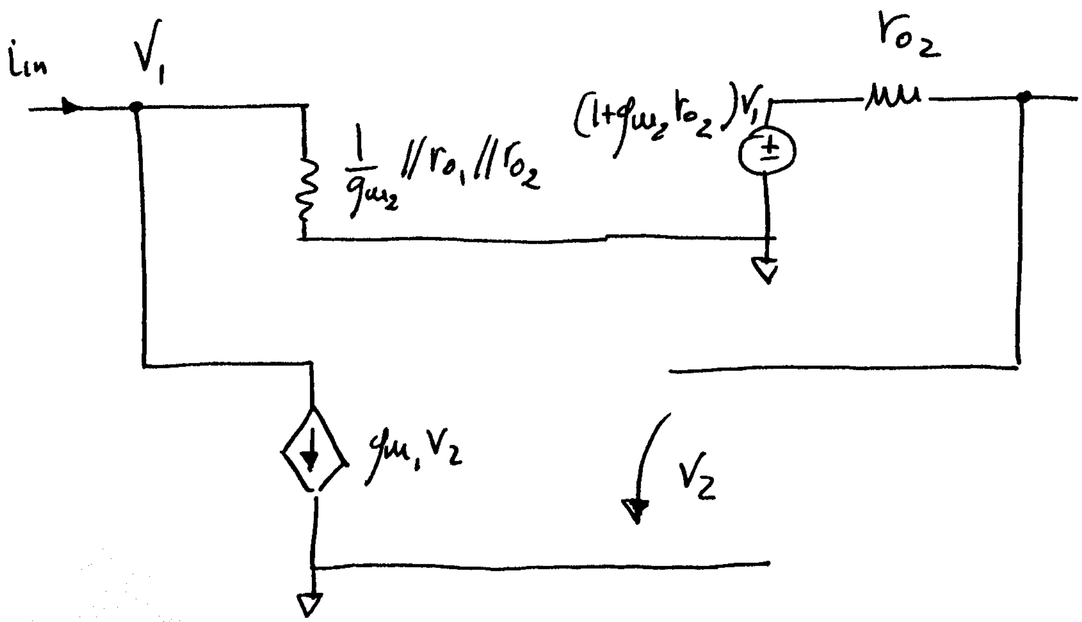


$$A_R = \left(\frac{1}{g_{m2}} \| r_{o2} \| r_{o1}\right)(1 + g_{m2} r_{o2}) \approx r_{o2}$$

$$f_G = -g_{m1}$$

$$T \approx r_{o2}\, g_{m1}$$

(d) After feedback the overall transresistance becomes:

$$\frac{A_R}{1+T} \approx \frac{1}{g_{m1}}$$

$$Z_{in} = \frac{1/g_{m2} \| r_{o1} \| r_{o2}}{1+T} \approx \frac{1}{g_{m1}\, g_{m2}\, r_{o2}}$$

$$Z_{out} = \frac{r_{o2}}{1+T} \approx \frac{1}{g_{m1}}$$

2) (a) Forward amplifier : G_m

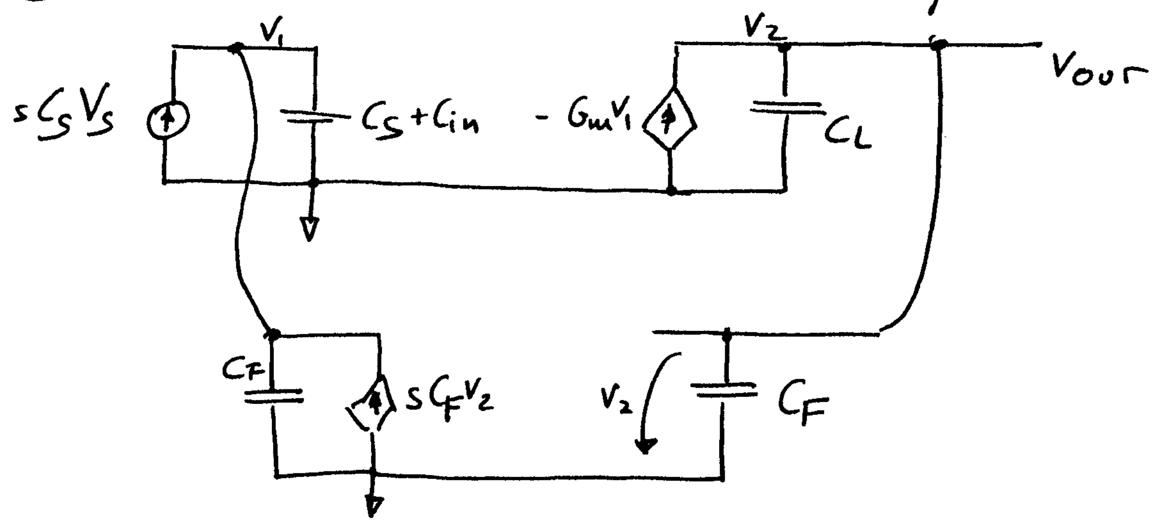
Feedback : through C_F

(b) shunt-shunt feedback

shunt at the input : current summing at the negative input node of "G_m"

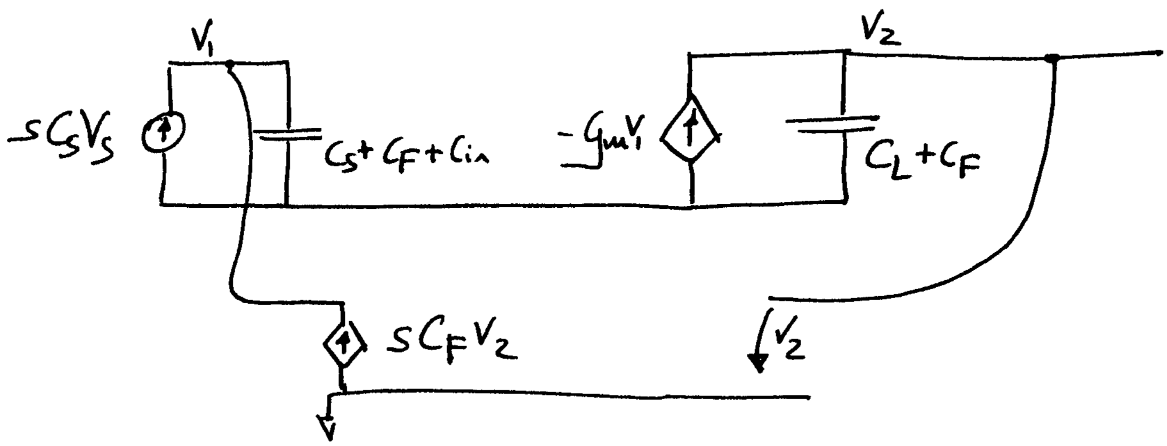
shunt at the output : voltage sensing

(c) we can redraw the circuit as follows:



(strictly speaking, we neglected the feedforward through the feedback capacitor; this will give a "sC_FV_1" term that has to be added to "$-g_mV_1$")

Moving all the loading of the feedback network to the main amplifier, we get:



$$A_R = \frac{1}{s(C_S+C_F+C_{in})}(-g_m)\frac{1}{s(C_L+C_F)}$$

$$\beta_G = sC_F$$

$$\boxed{T = \frac{C_F}{C_F+C_S+C_{in}}\,\frac{g_m}{s(C_L+C_F)}}$$

(d) After feedback, the transresistance gain becomes:

$$\frac{A_R}{1+T} \approx \frac{A_R}{T} = \frac{1}{sC_F} = \frac{V_{out}}{i_{in}}$$

$$\Rightarrow \boxed{\frac{V_{out}}{V_{in}} = \frac{C_S}{C_F}} \qquad (\text{since } i_{in} = sC_S V_{in})$$

$$Z_{out} = \frac{1}{s(C_L+C_F)}\,\frac{1}{1+T} \approx \frac{1}{s(C_L+C_F)}\,\frac{1}{T}$$

$$\Rightarrow \boxed{Z_{out} = \frac{1}{G_m}\,\frac{C_F+C_S+C_{in}}{C_F}}$$

$\tilde{Z}_{in}$ for equivalent circuit (norton equivalent)

$$\tilde{Z}_{in} = \frac{1}{s(C_S+C_F+C_{in})}\,\frac{1}{1+T}$$

Real Z_{in}:

$$Z_{in} = \frac{1}{sC_S} + \left[\tilde{Z}_{in}^{-1} - sC_S\right]^{-1}$$

$$\approx \frac{1}{sC_S} \qquad \text{for } f \ll \text{GBW of } T$$

3) (a) Forward amplifier : M_1

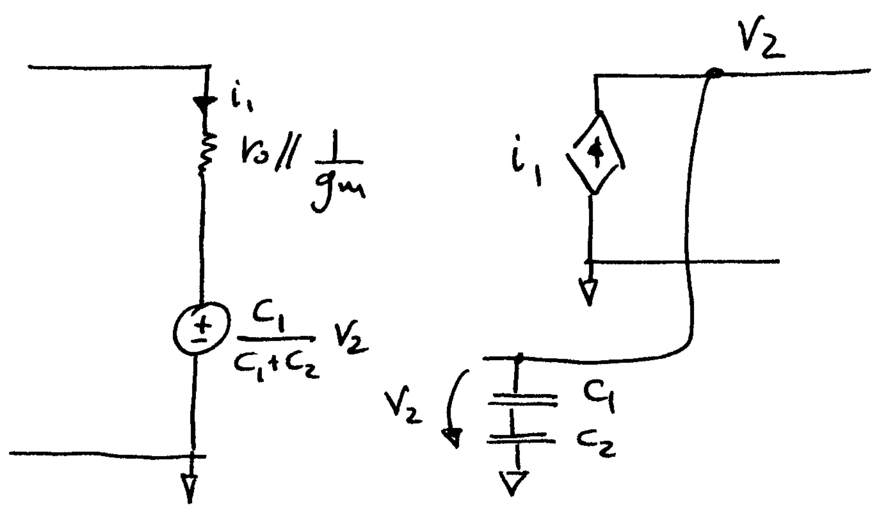
Feedback : through C_1 & C_2 (capacitive divider)

(b) $\boxed{\text{series - shunt}}$

series at the input : voltage difference between V_{IN} and $V_{out} \cdot \frac{C_1}{C_1+C_2}$ drives M_1

shunt at the output : voltage sensing

(c)



$$f = \frac{C_1}{C_1+C_2} \qquad A = \frac{1}{(r_o // \frac{1}{g_m})\, s\, C_1 // C_2}$$

where $C_1 // C_2 = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1}$

$$T = Af = \frac{1}{sC_2 \left(r_o // \frac{1}{g_m}\right)}$$

$$r_o \parallel \frac{1}{g_m} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$$

($V_2 = 0$, since the output is shunt)

$$\left.\frac{i_2}{v_2}\right|_{i_1=0} = \infty$$ when only considering the forward amplifier (M_1)

$$\left.\frac{i_2}{V_2}\right|_{i_1=0} = s\, C_1 \parallel C_2$$ when considering the feedback network

($i_1 = 0$, since the input is series)

(d) Voltage gain with feedback:

$$\frac{A}{1+Af} \approx 1 + \frac{C_2}{C_1}$$

$$Z_{in} = \underline{(r_o \parallel 1/g_m)}\,(1+Af) = \frac{1}{sC_2}$$

$$Z_{out} = \frac{1}{s\, C_1 \parallel C_2}\,\frac{1}{1+Af} \approx \frac{1}{g_m}\left(1 + \frac{C_2}{C_1}\right)$$

Return Ratio Approach

(1) break at $g_{m1} \longrightarrow \boxed{RR = g_{m1} r_{o1} (1 + g_{m2} r_{o2})}$

$A_\infty = 0$ (closed-loop gain for $g_m \to \infty$)

$d = r_{o1}(1 + g_{m2} r_{o2})$ (closed-loop gain for $g_m \to 0$)

$$A = \frac{A_\infty RR}{1 + RR} + \frac{d}{1 + RR} = \frac{r_{o1}(1 + g_{m2} r_{o2})}{1 + g_{m1} r_{o1}(1 + g_{m2} r_{o2})} \approx \frac{1}{g_{m1}}$$

$$Z_{in} = Z_{in}\Big|_{g_m = 0} \cdot \frac{1 + RR\big|_{\text{input shorted}}}{1 + RR\big|_{\text{input open}}}$$

$$= r_{o1} \cdot \frac{1}{1 + RR} \approx \frac{1}{g_{m1} g_{m2} r_{o2}}$$

$$Z_{out} = Z_{out}\Big|_{g_m = 0} \cdot \frac{1 + RR\big|_{\text{output shorted}}}{1 + RR\big|_{\text{output open}}}$$

$$= \frac{r_{o1}(1 + g_{m2} r_{o2}) + r_{o2}}{1 + RR} \approx \frac{1}{g_{m1}}$$

(2) Break at the input of "Gm"

$$\Rightarrow \boxed{RR = \frac{G_m}{s\left(C_L + C_F \| (C_S + C_{in})\right)} \cdot \frac{C_F}{C_F + C_S + C_{in}}}$$

$$A_\infty = -\frac{C_S}{C_F} \quad \text{(assuming } RR \gg 1\text{)}$$

neglect d (will give a zero in the closed-loop transfer function, but not in the RR)

$$A \approx \frac{A_\infty RR}{1 + RR} \approx -\frac{C_S}{C_F} \quad \text{(assuming } RR \gg 1\text{)}$$

$$Z_{in} = Z_{in}\Big|_{G_m = 0} \frac{1 + RR\big|_{\text{input shorted}}}{1 + RR\big|_{\text{output open}}}$$

$$= \left[s C_S \| \left(C_{in} + C_F \| C_L\right)\right]^{-1} \cdot \frac{1 + \frac{G_m}{s(C_L + C_F \| (C_S + C_{in}))} \cdot \frac{C_F}{C_S + C_F + C_{in}}}{1 + \frac{G_m}{s(C_L + C_F \| C_{in})} \cdot \frac{C_F}{C_F + C_{in}}}$$

$$\approx \frac{1}{s C_S} \quad \text{(assuming } RR \gg 1\text{)}$$

$$Z_{out} = Z_{out}\Big|_{G_m = 0} \frac{1 + RR\big|_{\text{output shorted}}}{1 + RR\big|_{\text{output open}}}$$

$$= \frac{1}{s\left(C_L + C_F \| (C_S + C_{in})\right)} \frac{1}{1 + RR} = \frac{1}{G_m} \frac{C_F + C_S + C_{in}}{C_F}$$

(3) Break at the gate of M_1:

$$RR = g_m \left[r_o \,//\, \frac{1}{s\, C_1 // C_2} \right] \frac{C_1}{C_1 + C_2}$$

$$\boxed{RR = g_m \frac{r_o}{1 + s r_o\, C_1//C_2} \frac{C_1}{C_1 + C_2}}$$

$$A_\infty = 1 + \frac{C_2}{C_1} \qquad \text{neglect } \delta$$

$$A = \frac{A_\infty\, RR}{1 + RR} \approx A_\infty = 1 + \frac{C_2}{C_1}$$

$$Z_{in} = Z_{in}^{o} \frac{1 + RR\,(\text{input shorted})}{1 + RR\,(\text{input open})}$$

$$Z_{in}^{o} = \frac{\frac{1}{pC_1} + \frac{1}{pC_2} + r_o}{1 + g_m r_o} = \frac{1 + s r_o\, C_1//C_2}{1 + g_m r_o} \frac{1}{s\, C_1//C_2}$$

$$\Rightarrow Z_{in} = Z_{in}^{o} \cdot (1 + RR) \approx \frac{1}{sC_2}$$

$$Z_{out}^{o} = r_o \,//\, \frac{1}{s\, C_1//C_2} = \frac{r_o}{1 + s r_o\, C_1//C_2}$$

$$Z_{out} = Z_{out}^{o} \frac{1 + RR\,(\text{output shorted})}{1 + RR\,(\text{output open})} = \frac{Z_{out}^{o}}{1 + RR}$$

$$\approx \frac{1}{g_m}\left(1 + \frac{C_2}{C_1}\right)$$