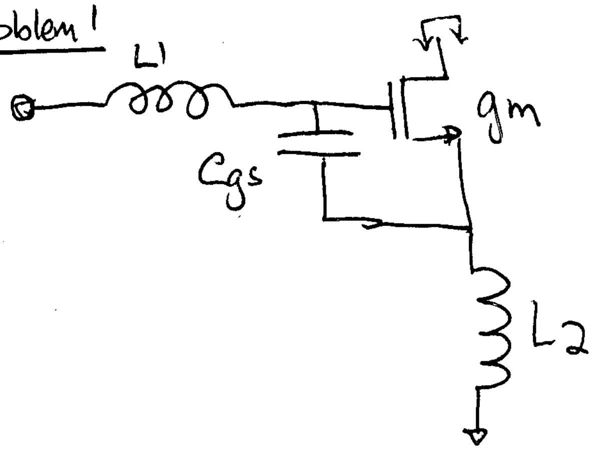
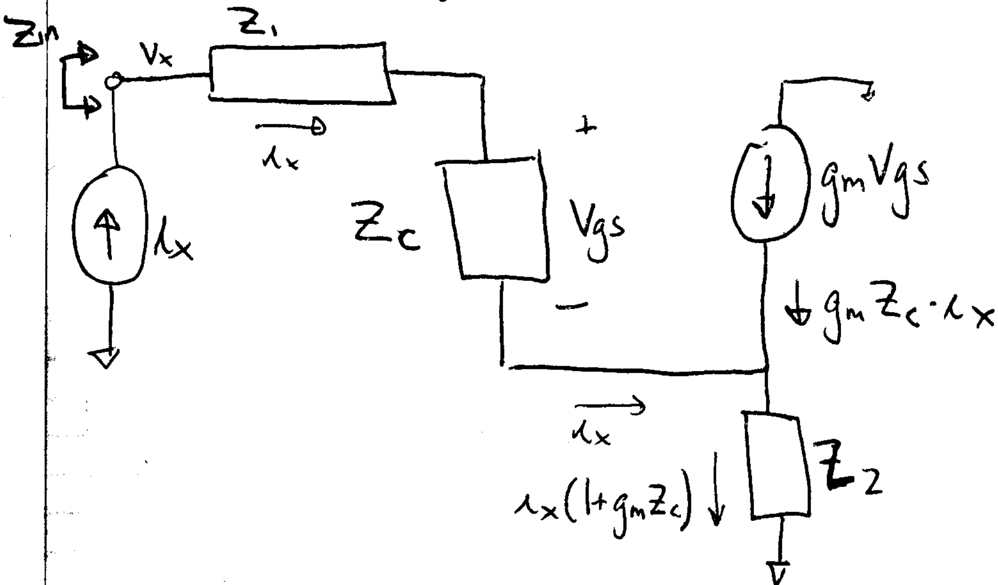
Problem 1



Small signal equivalent:



(a) KVL: $V_x = Z_1 \cdot i_x + Z_c \cdot i_x + (1 + g_m Z_c) Z_2 \cdot i_x$

$$Z_{IN} = \frac{V_x}{i_x} = Z_1 + Z_2 + Z_c + g_m Z_c \cdot Z_2$$

Let $\left.\begin{array}{l} Z_1 = j\omega_0 L_1 \\ Z_2 = j\omega_0 L_2 \\ Z_c = \frac{-j}{\omega_0 C_{gs}} \end{array}\right\} \Rightarrow Z_{IN} = j\left[\omega_0(L_1 + L_2) - \frac{1}{\omega_0 C_{gs}}\right] + \frac{g_m L_2}{C_{gs}}$

$$= j \cdot X_{IN} + R_{IN}$$

At ω_0, $Z_{IN} \overset{!}{=} 50\Omega$

$$\Rightarrow R_{IN} = \frac{g_m L_2}{C_{gs}} \overset{!}{=} 50\Omega$$

$$\Rightarrow L_2 = \frac{50\Omega \cdot C_{gs}}{g_m} = \frac{50 \cdot 80e\text{-}15}{10e\text{-}3} = 0.4\,nH$$

~~$X_{IN} \overset{!}{=} 0$~~ -

$$X_{IN} \overset{!}{=} 0 \Rightarrow \omega_0(L_1 + L_2) - \frac{1}{\omega_0 C_{gs}} = 0$$

$$\Rightarrow L_1 = \frac{1}{\omega_0^2 \cdot C_{gs}} - L_2 = \frac{1}{(2\pi \cdot 5e9)^2 \cdot 80e\text{-}15} - 0.4\,nH$$

$$\Rightarrow L_1 = 12.2\,nH$$

$$\boxed{\begin{array}{l} L_1 = 12.2\,nH \\ L_2 = 0.4\,nH \end{array}}$$

(b) At the operating frequency, $Z_{IN} = 50\Omega$

$$G_m = \frac{Z_c}{Z_{IN}} \cdot g_m = \frac{1}{50\Omega} \cdot \left(\frac{-j}{\omega_0 C_{gs}}\right) \cdot g_m$$

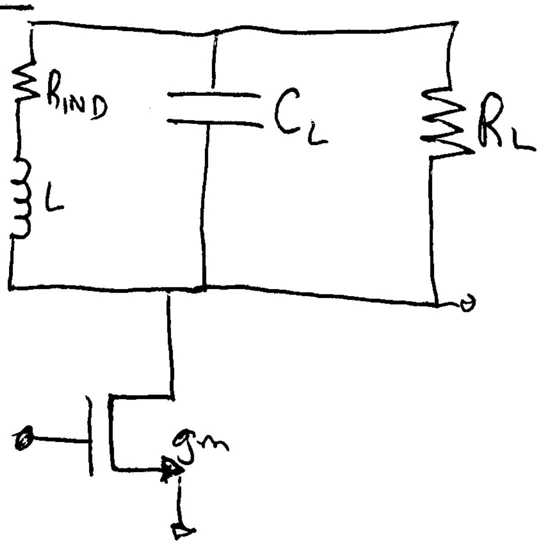
$$= \frac{1}{50\Omega} \cdot \left(\frac{-j}{(31.4\,Grad/s)(80\,fF)}\right) 10\,mS = -j\,79.6\,mS$$

$$\boxed{G_m = -j \cdot 79\,mS}$$

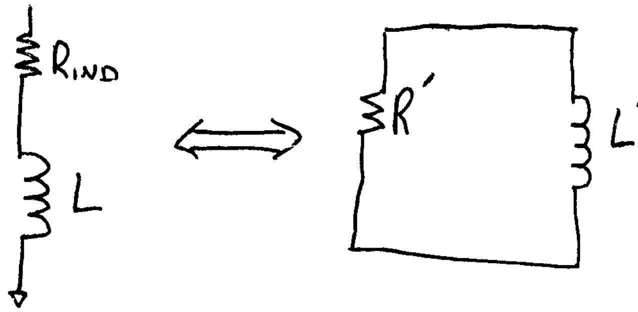
i.e. 79mS w/ a 90° phase lag

Problem 2

(a)



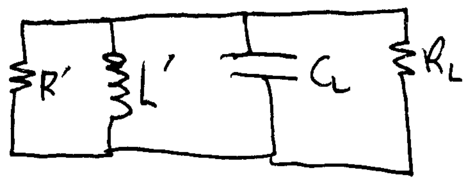
we know that



where $L' = L \cdot \left(1 + \frac{1}{Q_{IND}^2}\right)$

$R' = R_{IND} \cdot \left(1 + Q_{IND}^2\right)$

Therefore, we can represent the tank as:



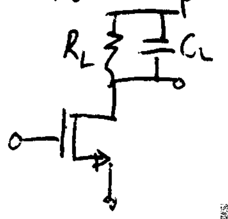
and obviously to ~~can~~ cancel out C_L:

$$\omega_0 L' = \frac{1}{\omega_0 C_L}$$

$$\Rightarrow L' = \frac{1}{\omega_0^2 C_L} \quad \& \quad L' = L\left(1 + \frac{1}{Q_{IND}^2}\right)$$

$$\Rightarrow \boxed{L = \frac{1}{\omega_0^2 C_L} \cdot \frac{1}{1 + \frac{1}{Q_{IND}^2}}}$$

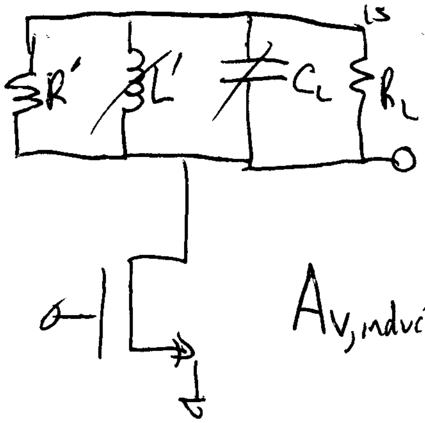
without inductor present:



$$A_{V,\text{no inductor}} = g_m \cdot (R_L // Z_{cap})$$

$$= \frac{g_m \cdot R_L \cdot \frac{1}{j\omega_0 C_L}}{R_L + \frac{1}{j\omega_0 C_L}} = \boxed{\frac{g_m R_L}{1 + j\omega_0 R_L C_L}}$$

with an inductor present: ~~Volt~~ NOTE that the inductor cancels out the capacitor, so all we are left with is R_L and R'



$$\Rightarrow \quad A_{V,\text{inductor}} = g_m (R_L // R')$$

$$R' = R_{IND} \cdot (1 + Q_{IND}^2) \qquad R_{IND} = \frac{\omega_0 L}{Q_{IND}}$$

$$\underbrace{}_{\text{substitute}} \quad R' = \omega_0 L \left(\frac{1}{Q_{IND}} + Q_{IND} \right)$$

Combine with $L = \frac{1}{\omega_0 C_L} \cdot \frac{1}{1 + \frac{1}{Q_{IND}^2}}$

$$\Rightarrow R' = \frac{\omega_0}{\omega_0^2 C_L} \cdot \frac{\left(\frac{1}{Q_{IND}} + Q_{IND}\right)}{1 + \frac{1}{Q_{IND}^2}} = \frac{1}{\omega_0 C_L} \cdot Q_{IND} = R'$$

So, given value of R', $A_V = g_m \cdot (R_L // R')$

$$\boxed{\Rightarrow A_V = \frac{g_m \cdot R_L \cdot \frac{Q_{IND}}{\omega_0 C_L}}{R_L + \frac{Q_{IND}}{\omega_0 C_L}} = \frac{g_m R_L}{1 + \frac{\omega_0 R_L C_L}{Q_{IND}}}}$$

NOTE: How the gain expression looks similar to part a, Problem 2, part c

Solve for L: $L = \frac{1}{\omega_o^2 C_L} \cdot \frac{1}{1+\frac{1}{Q_{IND}^2}} = \frac{1}{(31.4\text{Grad})^2 \cdot 1\text{pF}} \cdot \frac{1}{1+\frac{1}{100}}$ *

$$\boxed{L = 1.003 \text{ nH}}$$

$A_{v,\text{no inductor}} = \frac{g_m R_L}{1 + j\omega_o R_L C_L} \xrightarrow{\text{magnitude}} |A| = \frac{g_m R_L}{\sqrt{1+\omega_o^2 R_L^2 C_L^2}}$ *

$$\boxed{|A_{v,\text{no inductor}}| = \frac{10\text{mS} \cdot 1\text{k}\Omega}{\sqrt{1+[31.4\text{Grad} \cdot 1\text{k} \cdot 1\text{p}]^2}} = 0.318}$$

$$\boxed{|A_{v,\text{with inductor}}| = \frac{g_m R_L}{1+\frac{\omega_o R_L C_L}{Q_{IND}}} = \frac{10\text{mS} \cdot 1\text{k}\Omega}{1+\frac{31.4\text{Grad} \cdot 1\text{k} \cdot 1\text{p}}{10}} = 2.41}$$