University of California
Berkeley

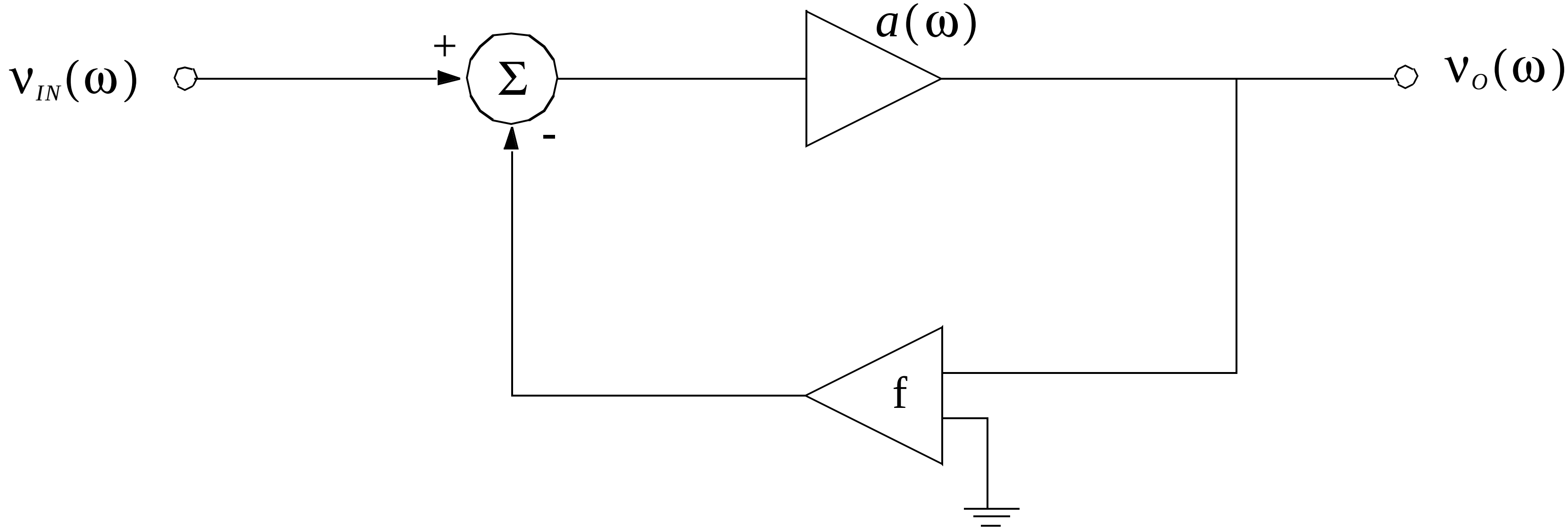
College of Engineering
Department of Electrical Engineering
and Computer Science

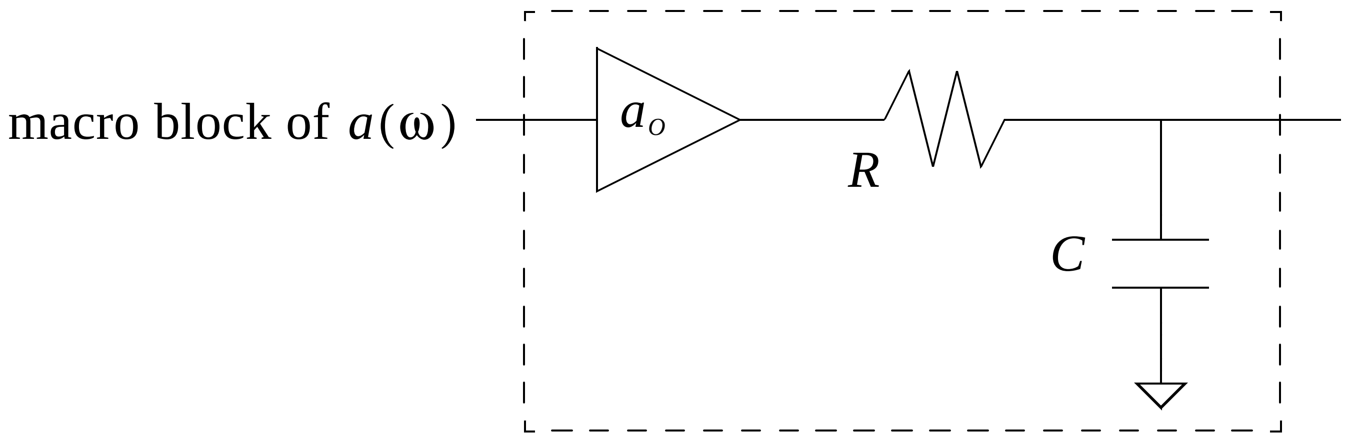
Robert W. Brodersen
EECS140

Analog Circuit Design

Lectures
on
STABILITY

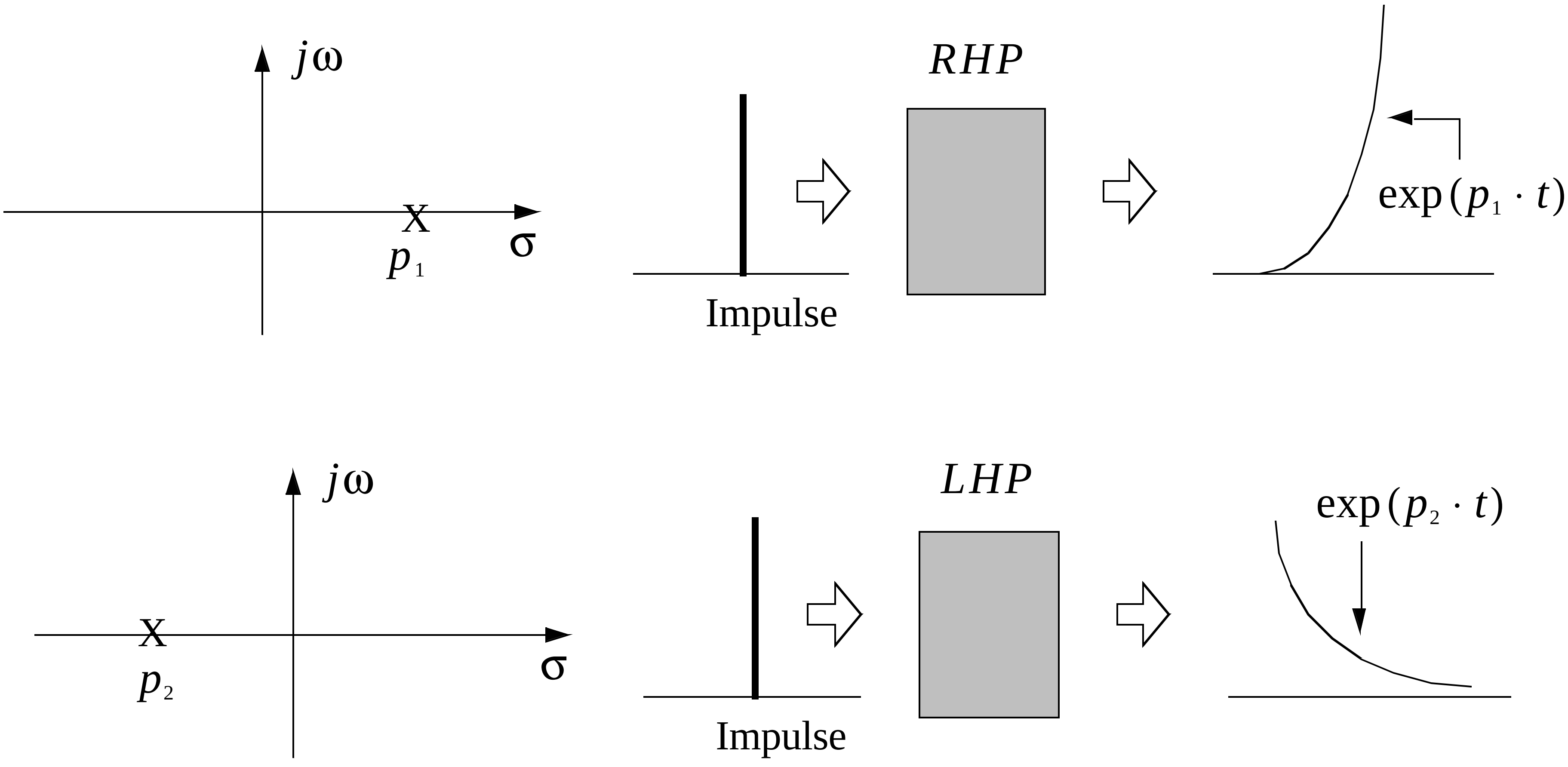
Effect of Feedback on Frequency Response





Effect of Feedback on Frequency Response (Cont.)

SB-2



Effect of Feedback on Frequency Response (Cont.)

Let $a(\omega)$ be a single pole response,

$$a(s) = \frac{a_O}{1 - \frac{s}{p_1}} \Leftrightarrow a(\omega) = \frac{a_O}{1 + j\frac{\omega}{\omega_{p1}}}$$

$$p_1 = -\omega_{p1}$$

$$\frac{\nu_{OUT}(s)}{\nu_{in}(s)} = A(s) = \frac{a(s)}{1 + a(s) \cdot f} = \frac{1}{f}\left(\frac{T(s)}{1 + T(s)}\right)$$

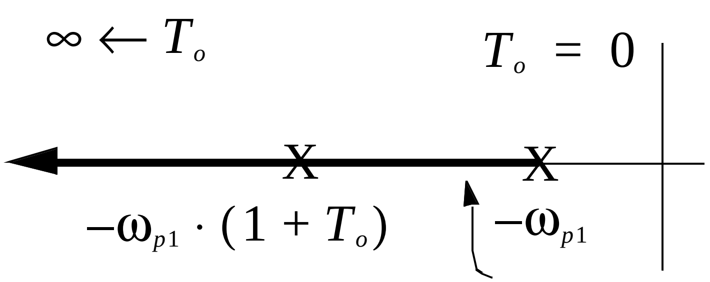
$$A(s) = \frac{\frac{a_O}{1 - \frac{s}{p_1}}}{1 + \frac{a_O}{1 - \frac{s}{p_1}} \cdot f} = \frac{a_O}{1 + a_O \cdot f}\left(\frac{1}{1 - \frac{s}{p_1 \cdot (1 + a_O \cdot f)}}\right)$$

Effect of Feedback on Frequency Response (Cont.)

Let $T_O = a_O \cdot f$

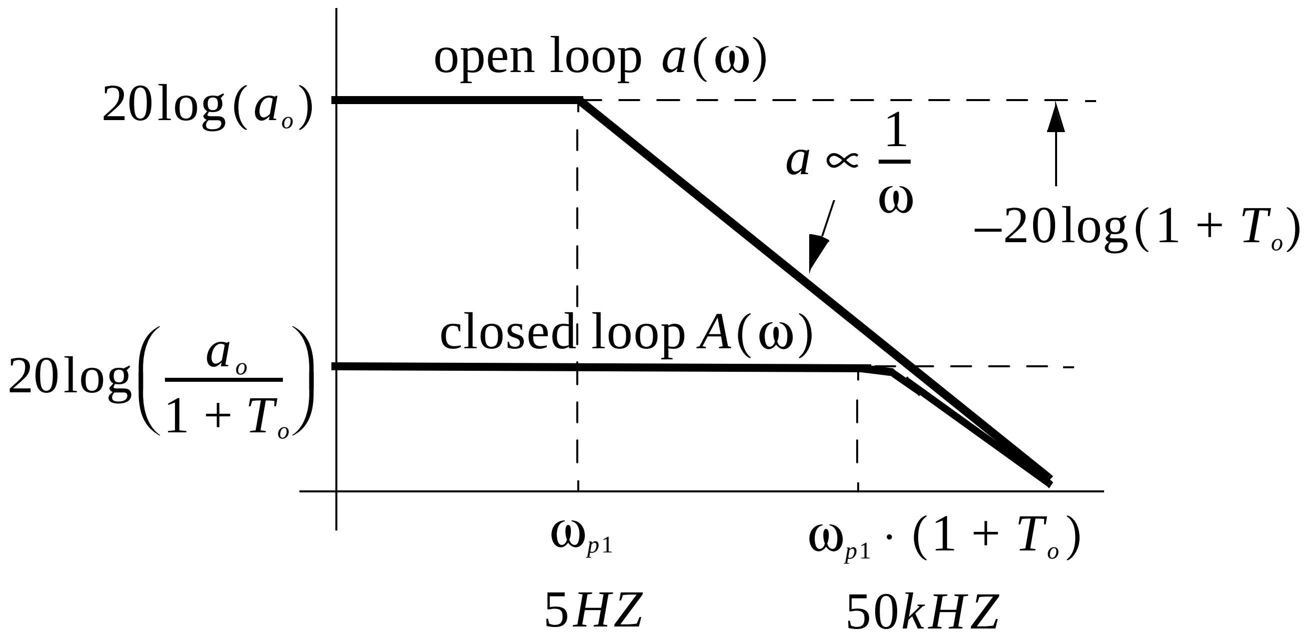
$$A(s) = \frac{a_O}{1 + T_O} \cdot \left(\frac{1}{1 - \frac{s}{p_1 \cdot (1 + T_O)}} \right)$$

Pole is at $p_1 \cdot (1 + T_O) \Rightarrow -\omega_{p1} \cdot (1 + T_O)$



Root Locus - motion of poles as loop Gain is increased.

Effect of Feedback on Frequency Response (Cont.)

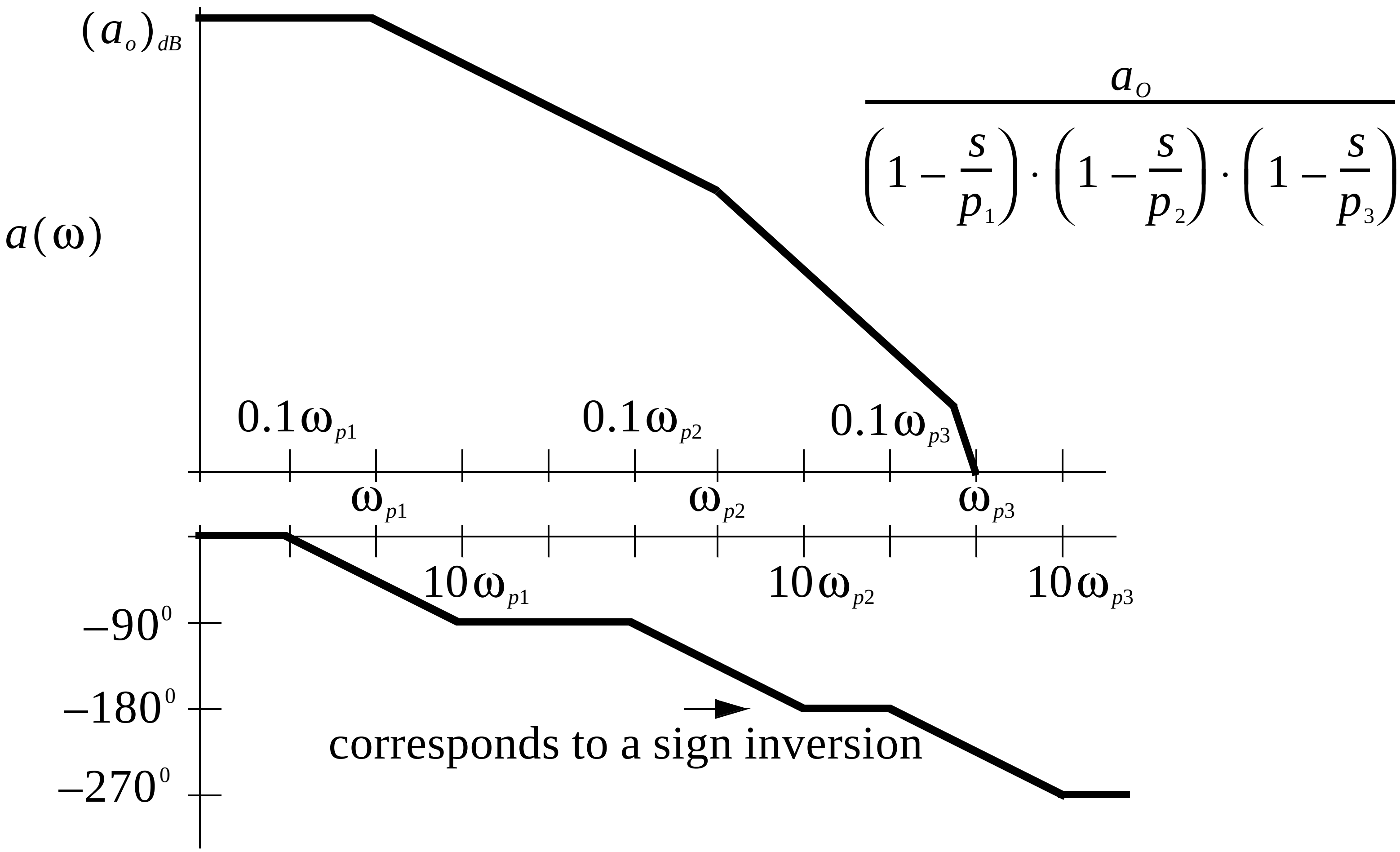


$T_o = 10^4$

Gain reduction by negative feedback reduces Gain by $\left(\frac{1}{1 + T_o}\right)$ and increases bandwidth by $(1 + T_o)$

Effect of Feedback on Frequency Response (Cont.)

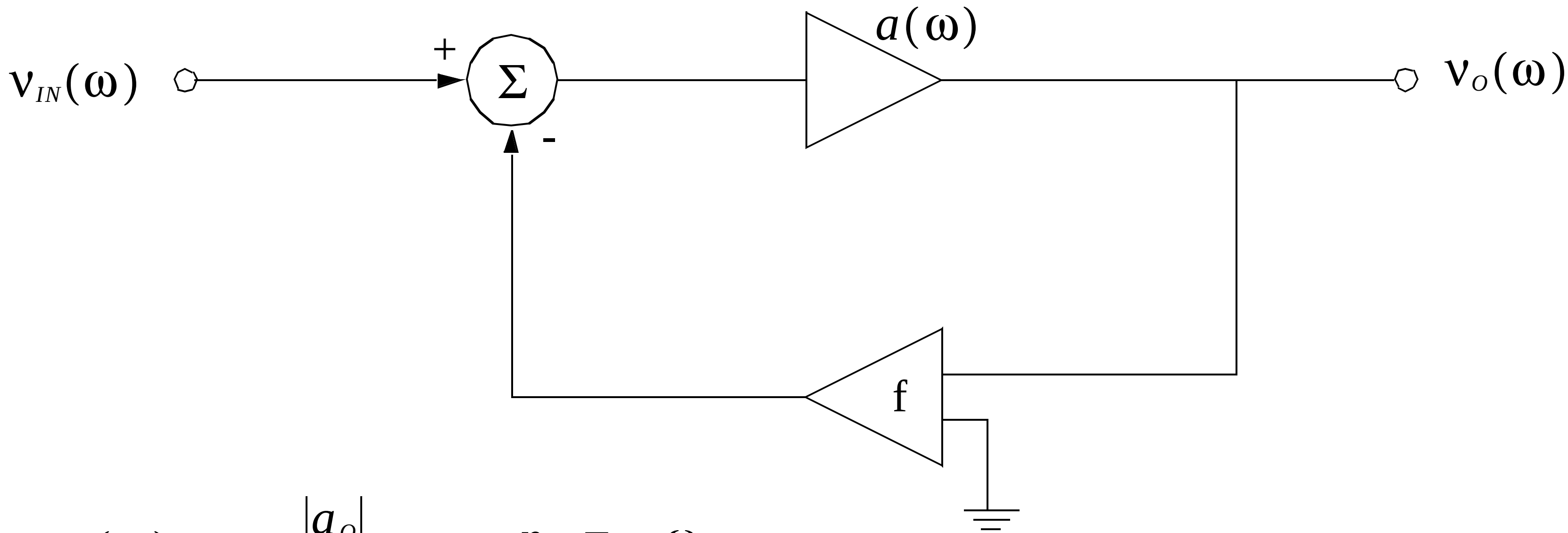
Why not let $T_O \rightarrow \infty$? Problems if we have more than one pole.



At the frequency ($10\omega_{p2}$) the pahse shift is 180^0 or negative feedback at DC is now positive feedback.

Effect of Feedback on Frequency Response (Cont.)

Lets look at the motion of a single pole with positive feedback :



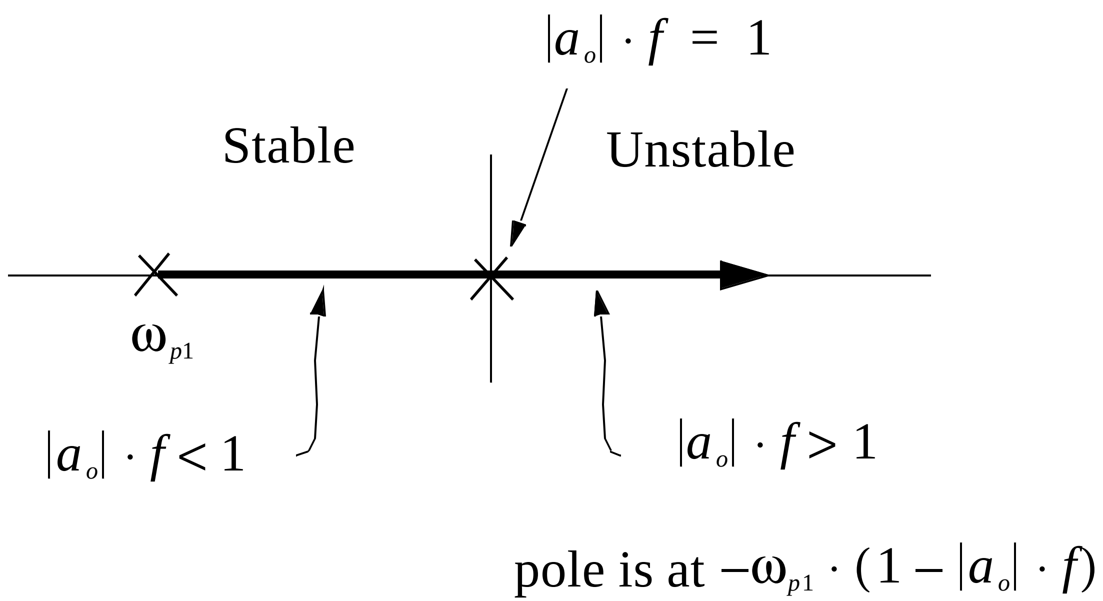
$$a(\omega) = -\frac{|a_O|}{1 - \frac{s}{p_1}} \qquad p_1 = -\omega_{p1}$$

$$A(s) = \frac{a_O}{1 + T_O} \cdot \left[\frac{s}{p_1 \cdot (1 - |a_O| \cdot f)}\right]$$

Effect of Feedback on Frequency Response (Cont.)

Since,

$$T_o = -|a_o| \cdot f$$



If $T < -1$ or $(1+T) < 0$ the circuit is unstable

Effect of Feedback on Frequency Response (Cont.)

The condition for stability of a multipole response is the Nyquist Criteria.

$$A(s) = \frac{a(s)}{1 + a(s) \cdot f} = \frac{a(s)}{1 + T(s)}$$

Simple Version :

If $|Tj\omega| > 1$ at the frequency where the phase of $T(j\omega) = -180^0$,

then the circuit is unstable.

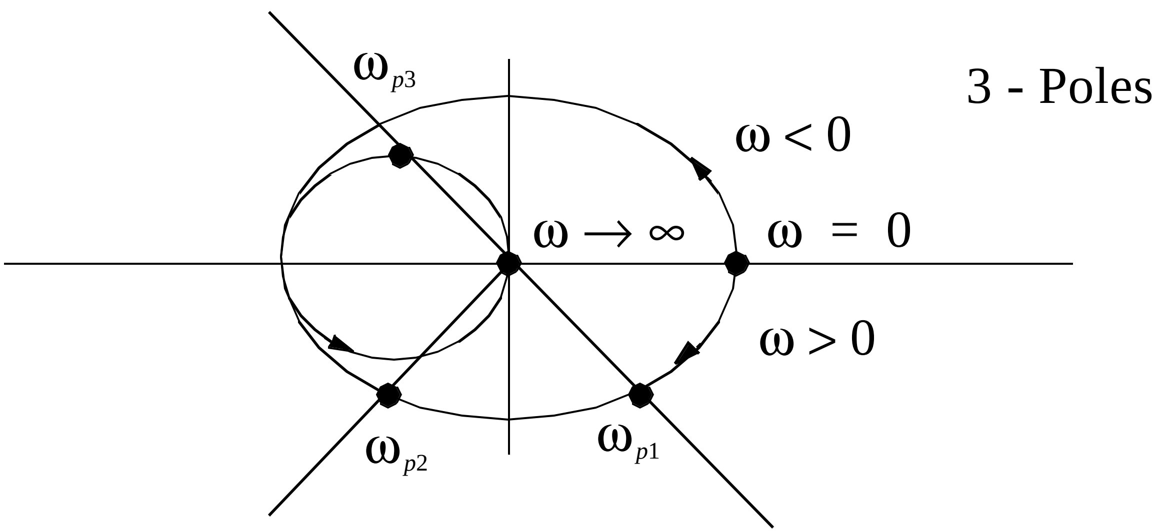
$$T(j\omega) = T(s)|_{s = j\omega}$$

$$\theta_{T(j\omega)} = arc\tan\left[\frac{Im\{T(j\omega)\}}{Re\{T(j\omega)\}}\right]$$

Effect of Feedback on Frequency Response (Cont.)

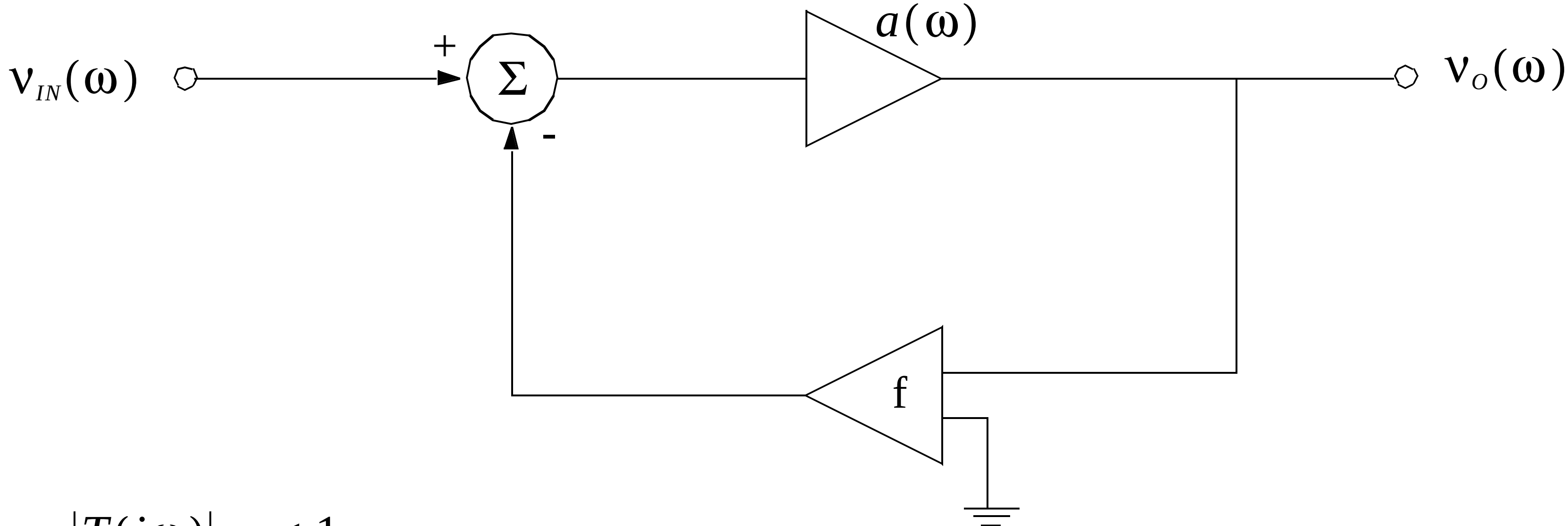
Complex Nyquist Criteria :

Plot $T(j\omega)$ on complex plane. As ω increases count number of times -1 is circled - even number means unstable (I Think).

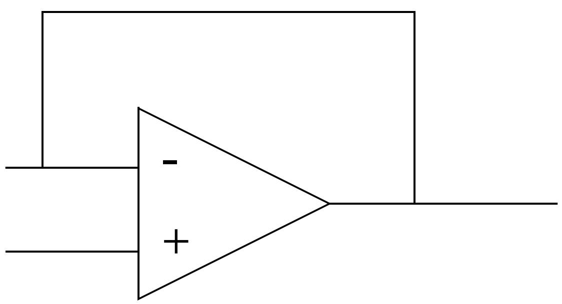


Effect of Feedback on Frequency Response (Cont.)

SB-11

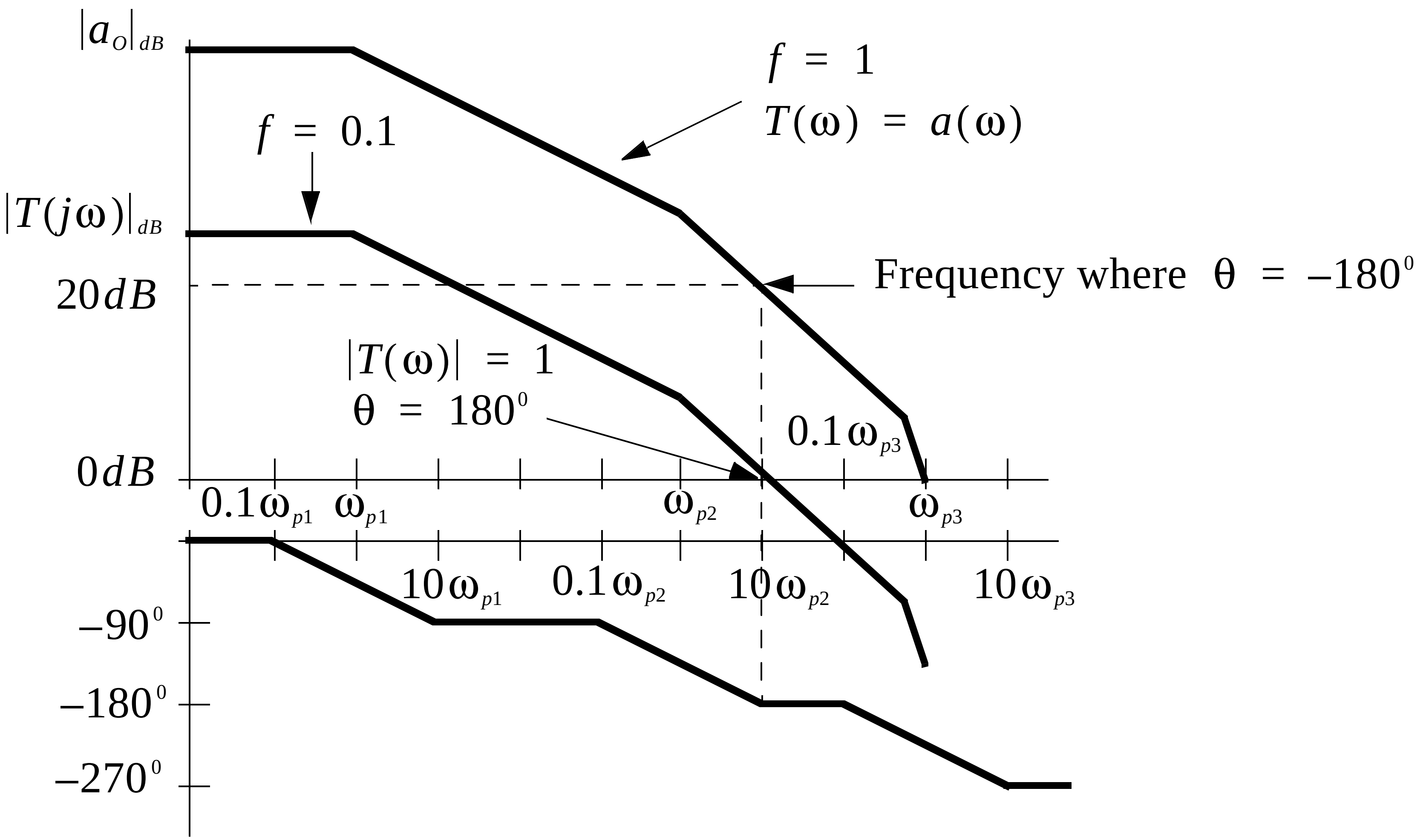


$$\left|T(j\omega)\right|_{180^\circ} < 1$$



Worst Case Stability Condition

Effect of Feedback on Frequency Response (Cont.)



Effect of Feedback on Frequency Response (Cont.)

PHASE MARGIN : Difference between the actual phase shift and -180^0

when $|T(\omega)| = 1$

i.e. $\theta_m \equiv \text{Phase Margin} = \theta[T(\omega)] - (-180^0)$

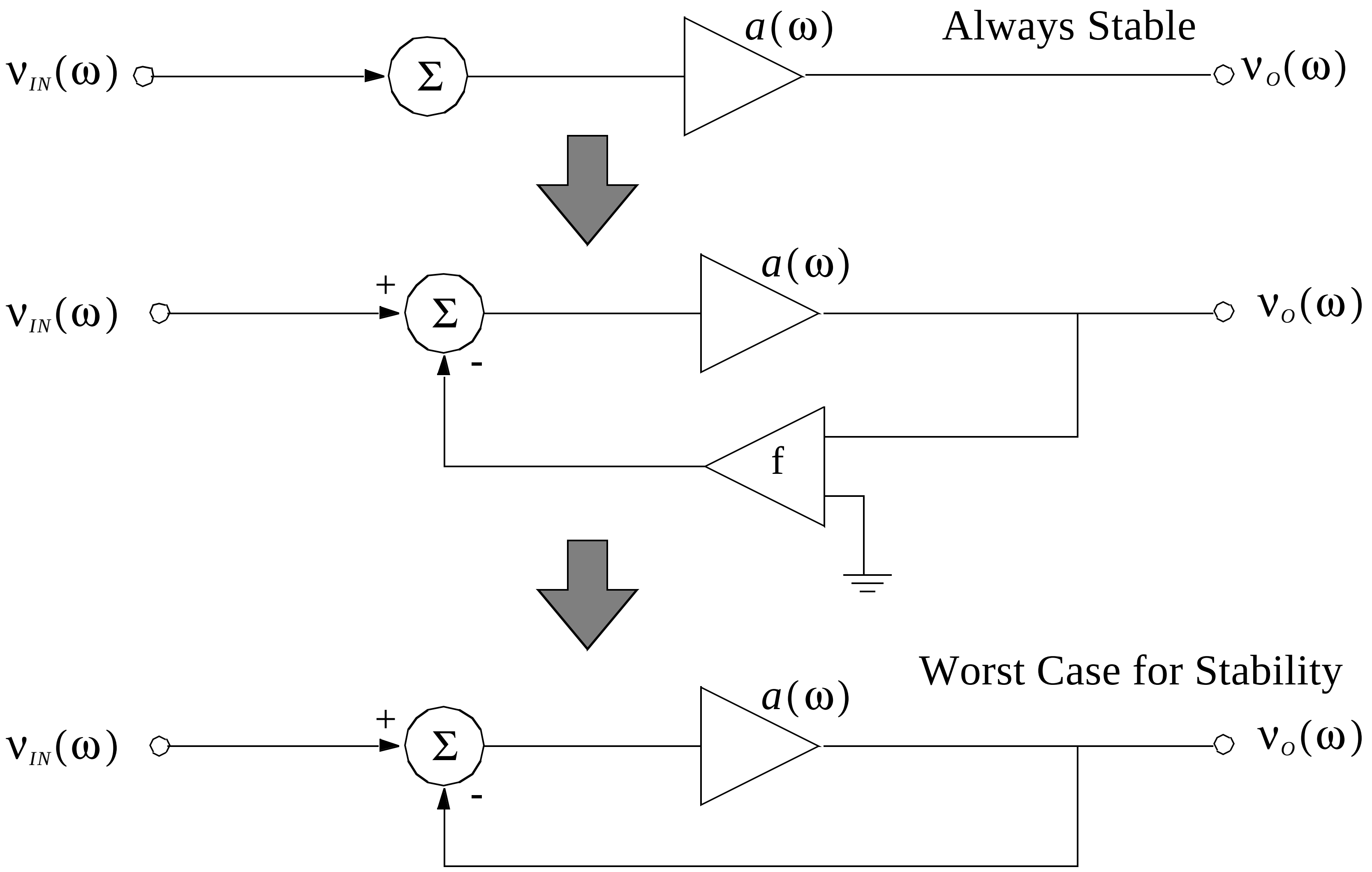
if $\theta_m > 0$ then the amplifier is stable - typically $45^0 - 60^0$

$$A = \frac{1}{f}$$ more gain more stable

$$R_{OUT} = \frac{r_o}{1+T}$$ higher R_{OUT} with more gain

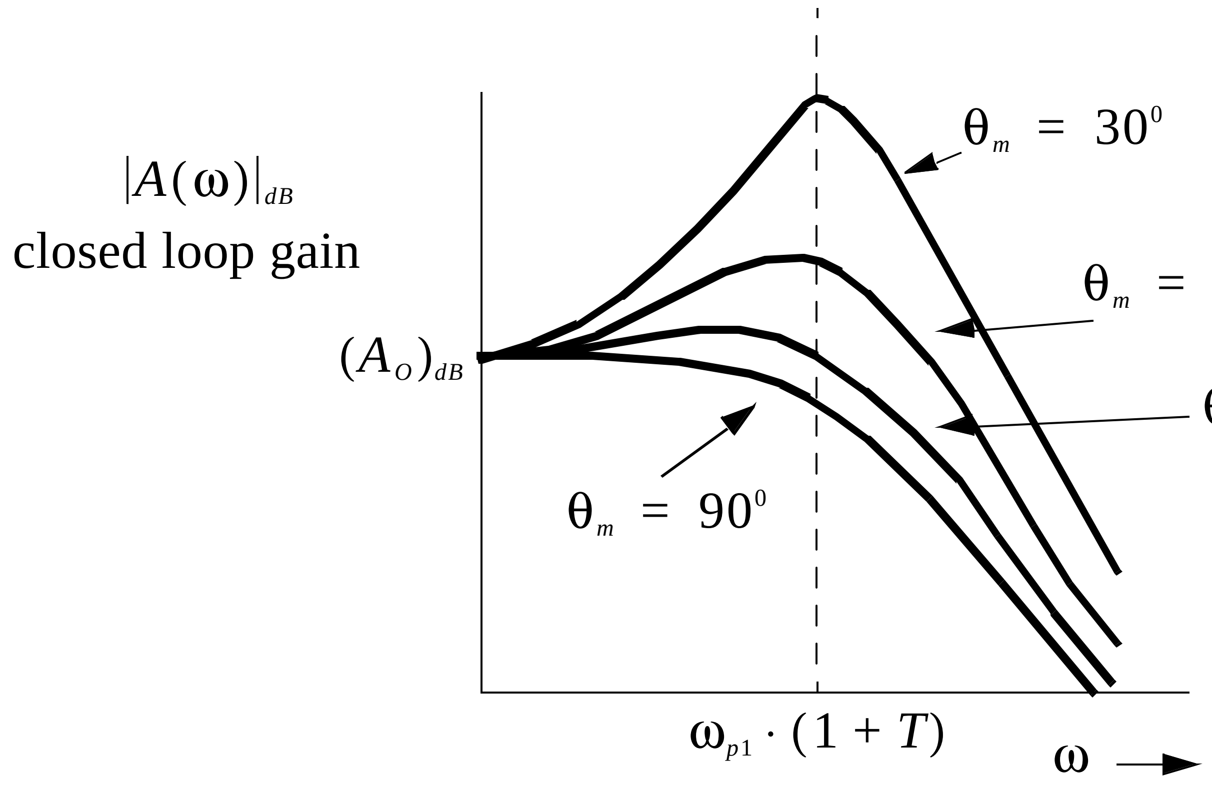
Effect of Feedback on Frequency Response (Cont.)

SB-14



Effect of Feedback on Frequency Response (Cont.)

As θ_m approaches 0 the amplifier is becoming unstable.



Effect of Feedback on Frequency Response (Cont.)

$$A(\omega) = \frac{a(\omega)}{1 + a(\omega) \cdot f}$$

$$a(s) = \frac{N(s)}{D(s)}$$

$$A(s) = \frac{\dfrac{N(s)}{D(s)}}{1 + \dfrac{N(s)}{D(s)} \cdot f} = \frac{N(s)}{D(s) + N(s) \cdot f}$$

zeros of a(s)

poles of a(s)

if the feedback factor is frequency dependent, then,

$$f(s) = \frac{N_f(s)}{D_f(s)}$$

$$A(s) = \frac{N(s)D(s)}{D(s)D_f(s) + N(s)N_f(s)}$$

Compensation

Compensation is the method in which an amplifier is modified so that it is stable.

One way is to decrease f (less feedback).

If ω_{180} is the frequency where,

$$\theta(a(\omega_{180^0})) = -180^0$$

then if,

$$f < \left|\frac{1}{a(\omega_{180^2})}\right|$$

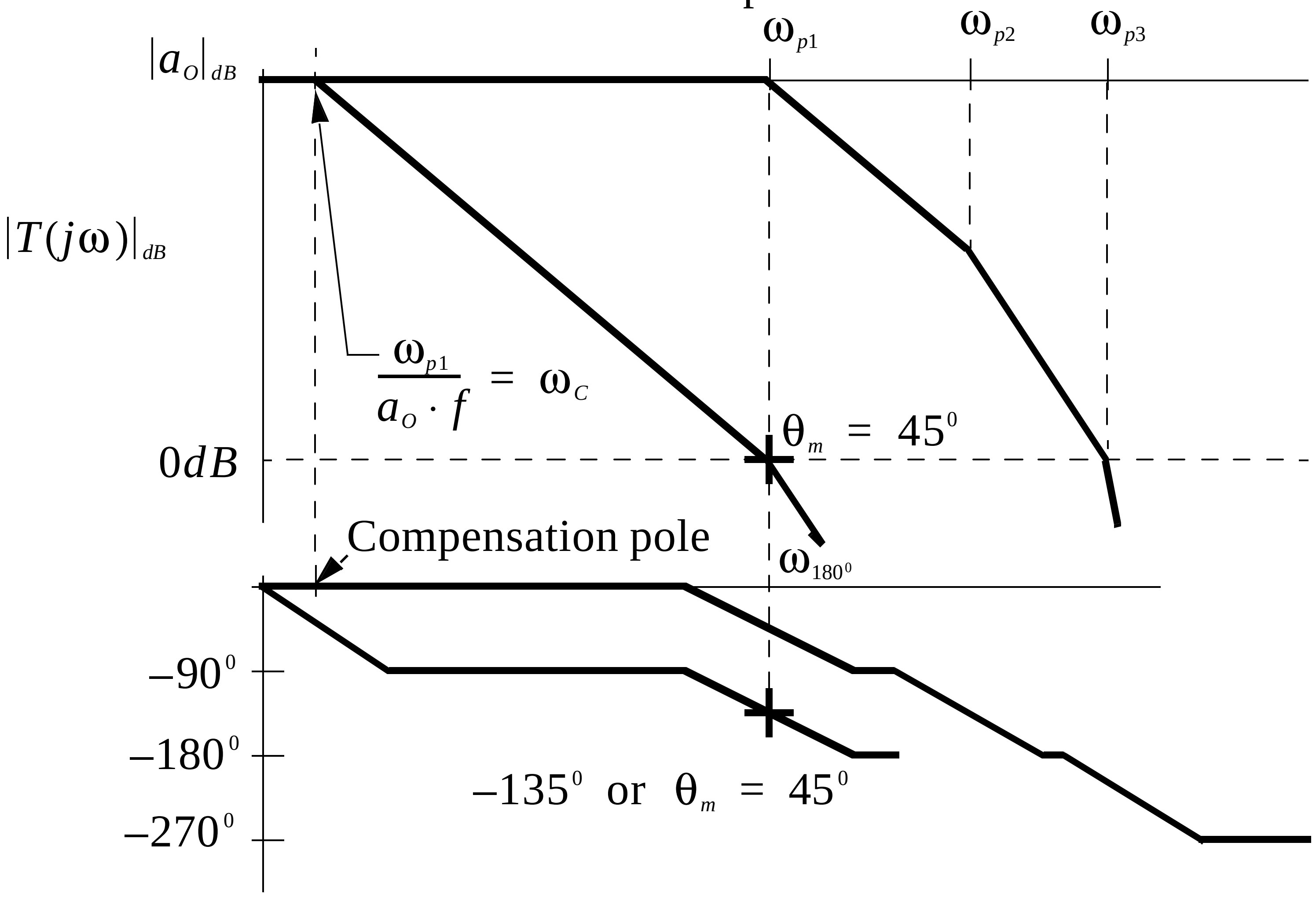
then,

$$|T(\omega_{180^0})| = f \cdot |a(\omega_{180^0})|$$

and stability is ensured.

Narrowbanding for Compensation

This entails the addition of a dominant pole



Narrowbanding for Compensation (Cont.)

For,

$$\theta_m = 45°$$

add a compensation pole, ω_C at the fequency,

$$\frac{\omega_{P1}}{|a_o f|} = \omega_C$$

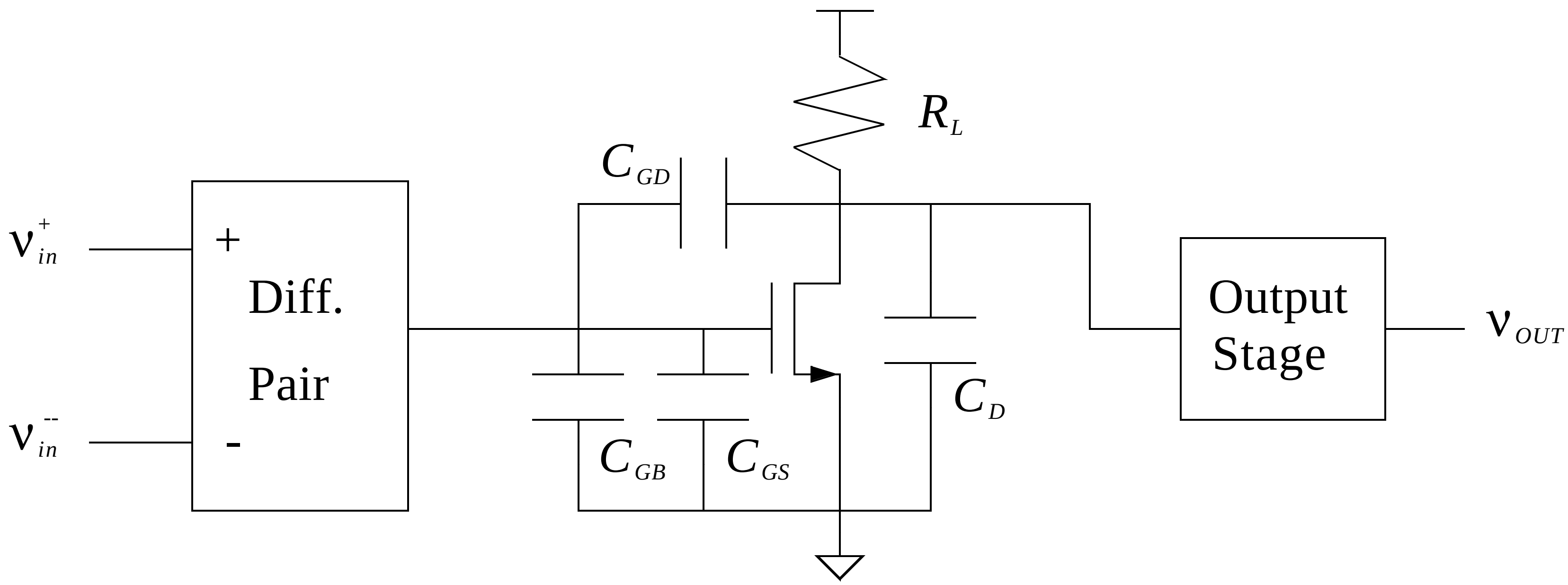
$$\omega_P = 1MHz$$
$$|a_o f| = 10^4$$
$$\omega_C = 100Hz$$

$-90°$ of phase shift from the new compensation pole.

$-45°$ from the second pole.

Pole Splitting

It is better to use an existing pole rather than add another.



$$\frac{a_1}{1 + j\dfrac{\omega}{\omega_{p1}}} \qquad \frac{g_m R_L}{\left(1 + j\dfrac{\omega}{\omega_{p2}}\right)\cdot\left(1 + j\dfrac{\omega}{\omega_{p3}}\right)} \qquad \frac{a_2}{1 + j\dfrac{\omega}{\omega_{p4}}}$$

Pole Splitting (Cont.)

Lets say ω_{p1} and ω_{p4} are given with,

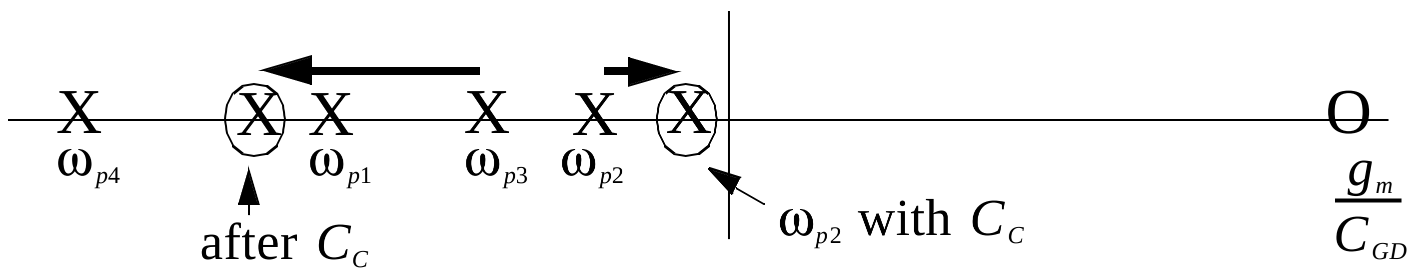
$$\omega_{p4} \gg \omega_{p1}$$

$$C_{GD} \ll C_{GS}, C_D$$

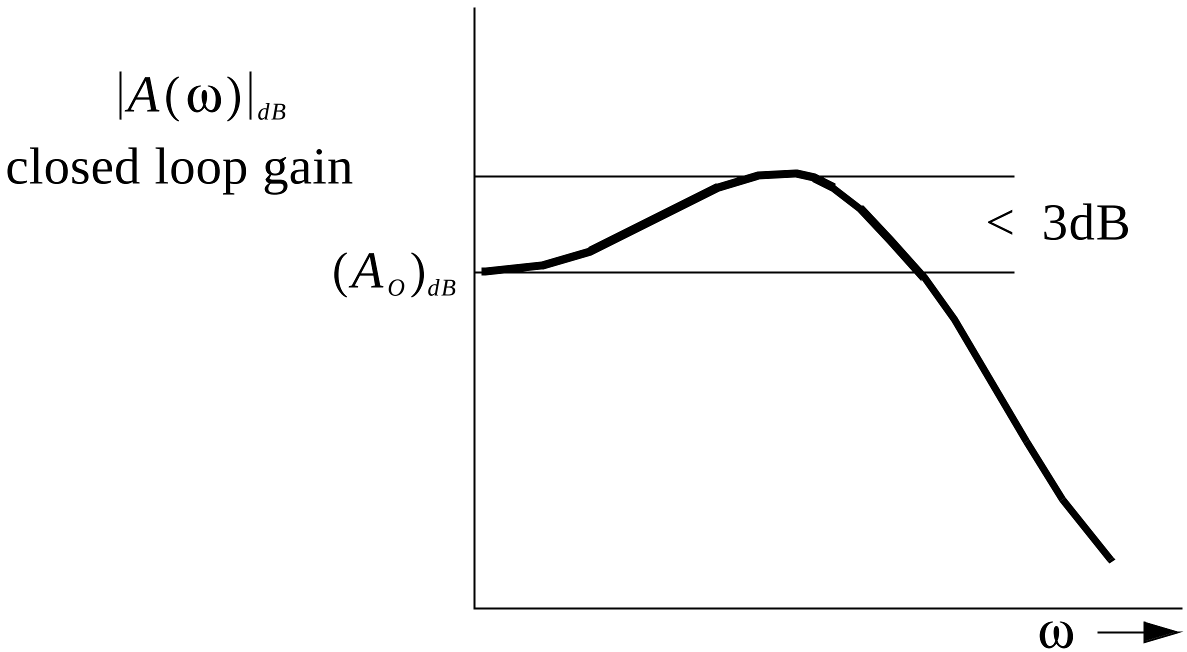
then,

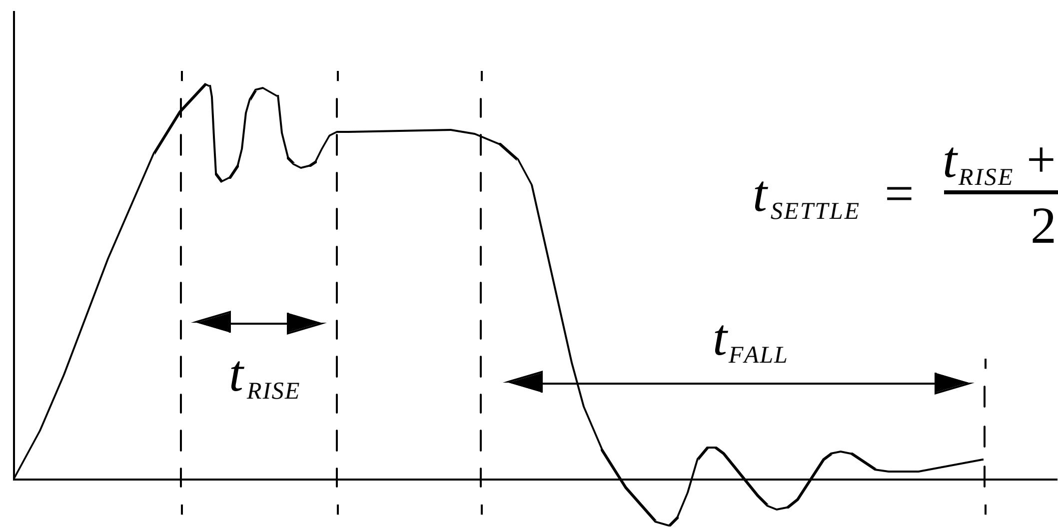
$$\omega_{p2} = \frac{1}{R_{DIFF} C_{GS}} \qquad \omega_{p3} = \frac{1}{R_L C_D}$$

$$\omega_Z = \frac{g_m}{C_{GD}}$$



SB-22





$$t_{SETTLE} = \frac{t_{RISE} + t_{FALL}}{2}$$

Pole Splitting (Cont.)

If we add a compensation capacitor, C_C in parallel with C_{GD} :

$$\omega_{p2} = \frac{1}{R_{DIFF} \cdot (1 + g_m \cdot R_L) \cdot C_C}$$

$$\omega_{p3} = \frac{g_m}{C_{GS} + C_D}$$

Lets put numbers in :

$$R_{DIFF} = 10 Meg\Omega$$

$$R_L = 5 Meg\Omega$$

$$C_{GS} = 0.1 pF$$

$$C_D = 0.1 pF$$

$$g_m = 10^{-3} Mhos$$

$$\omega_{p1} = 10 \cdot 10^6 \frac{rad}{\sec}$$

$$\omega_{p4} = 100 \cdot 10^6 \frac{rad}{\sec}$$

$$a_1 = 10^3$$

$$a_2 = 1$$

Pole Splitting (Cont.)

Before compensation, and with,

$$C_{GD} = 0$$

$$\omega_{p2} = \frac{1}{10^7 \cdot 10^{-13}} = 10^6 \frac{rad}{\sec}$$

$$\omega_{p3} = \frac{1}{5 \cdot 10^6 \cdot 10^{-13}} = 2 \cdot 10^6 \frac{rad}{\sec}$$

$$a(\omega) = \left(\frac{10^3}{1 + j\frac{\omega}{4 \cdot 10^6}}\right) \cdot \left(\frac{10^{-3} \cdot 5 \cdot 10^6}{\left(1 + j\frac{\omega}{10^6}\right) \cdot \left(1 + j\frac{\omega}{2 \cdot 10^6}\right)}\right) \cdot \left(\frac{1}{1 + j\frac{\omega}{10^8}}\right)$$

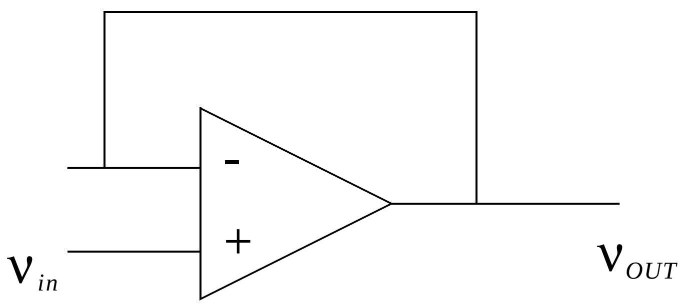
Pole Splitting (Cont.)

Compensate this amplifier for the worst case,

$$f = 1$$

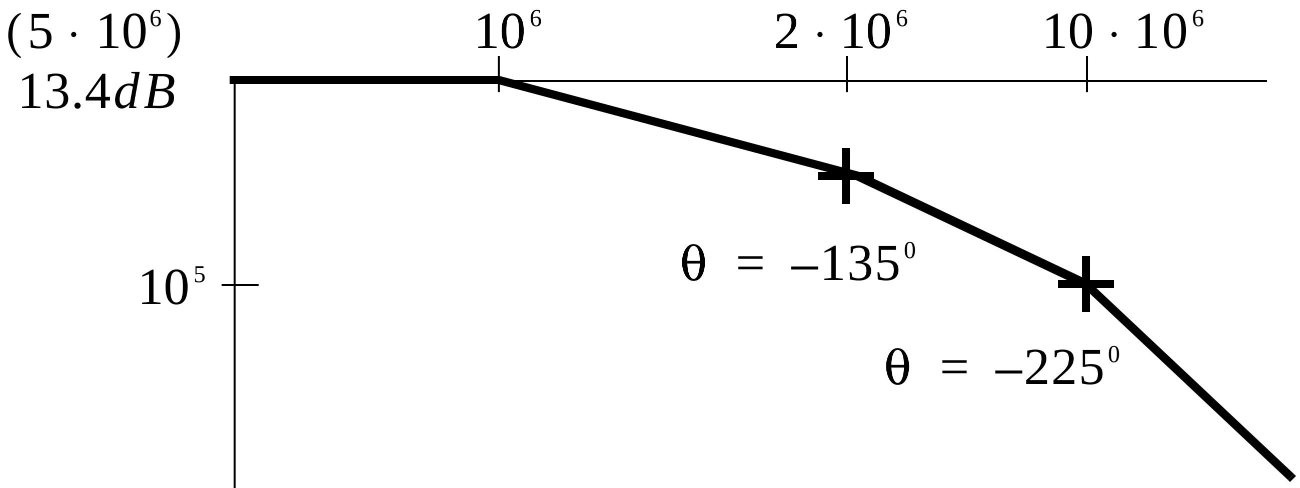
with,

$$\theta_m = 45^0$$



Worst Case Stability Condition

Pole Splitting (Cont.)



Somewhere between 2MHz and 10MHz,

$$\theta = -180^0$$

but the loop gain,

$$T \gg 1 \qquad T \sim 10^5$$

So how to compensate it?

Add C_c so that the gain at the first non-dominant pole (w_{p1}). since w_{p3} will move to a higer frequency and w_{p2} will move lower

Pole Splitting (Cont.)

$$\omega_{P2} = \frac{\omega_{P1}}{5 \times 10^6} = \frac{10^7}{5 \times 10^6} = 2 \cdot \frac{rad}{\sec}$$

Formula for ω_{P2} & ω_{P3} with C_C :

$$C_C » C_{GS}, C_D$$

$$\omega_{P2} = \frac{1}{R_{DIFF} \cdot (1 + g_m \cdot R_L) \cdot C_C}$$

$$\omega_{P3} = \frac{g_m}{C_{GS} + C_D}$$

$$\omega_Z = \frac{g_m}{C_C}$$

Pole Splitting (Cont.)

$$C_C = 10pF$$

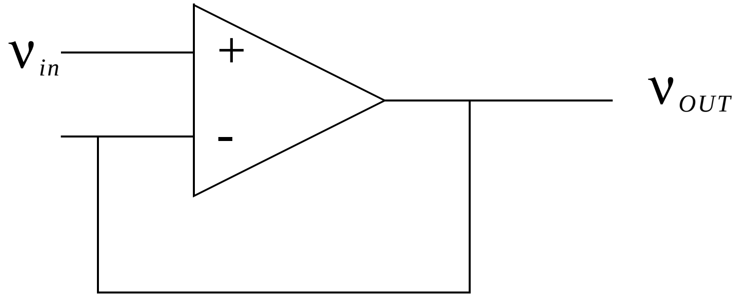
$$\omega_{P2} = \frac{1}{10^7 \times 5 \times 10^3 \times C_C}$$

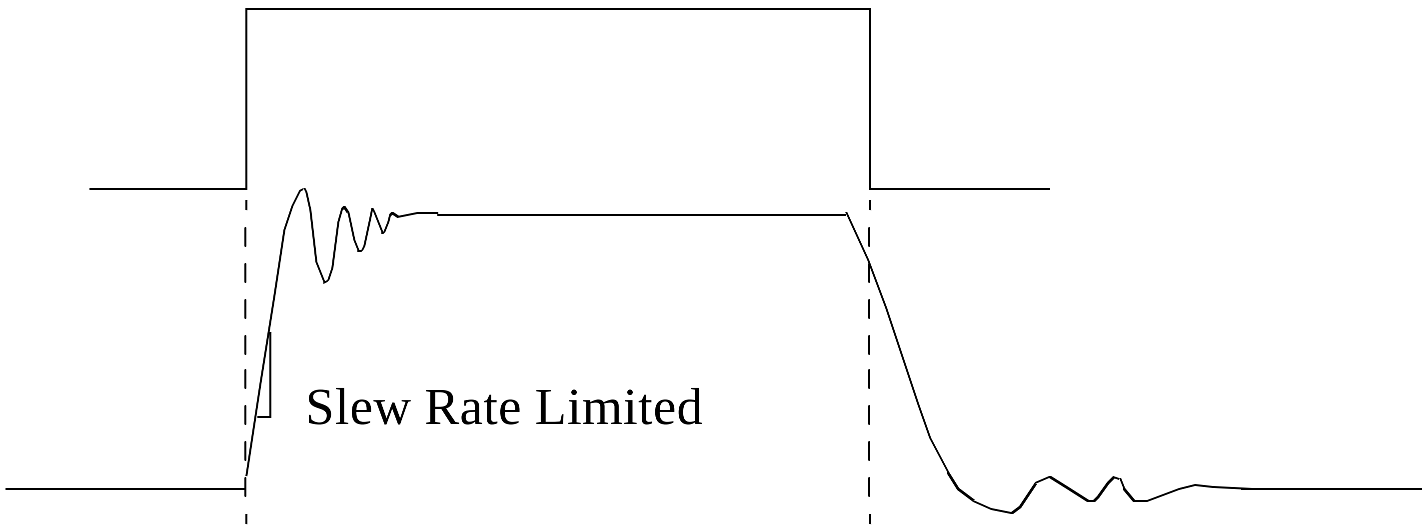
$$\omega_{P3} = \frac{10^{-3}}{0.2 \times 10^{-12}} = 5 \times 10^9 \cdot rad/\sec = 5000 Mrad/\sec$$

$$\omega_Z = \frac{10^{-3}}{10^{-11}} = 10^8 \cdot \frac{rad}{\sec}$$

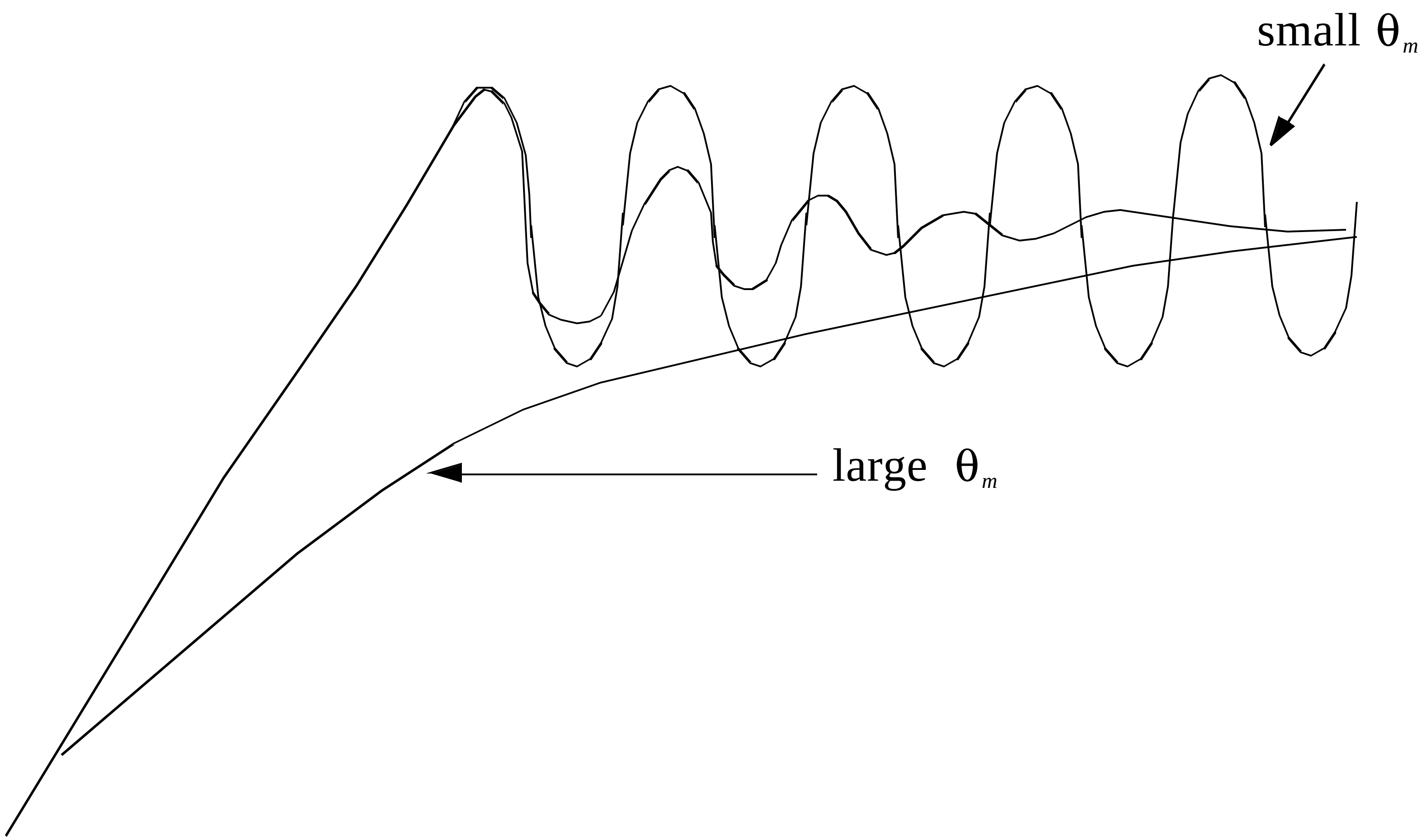
So by adding a 10pF capacitor this circuit is made stable.

SB-29

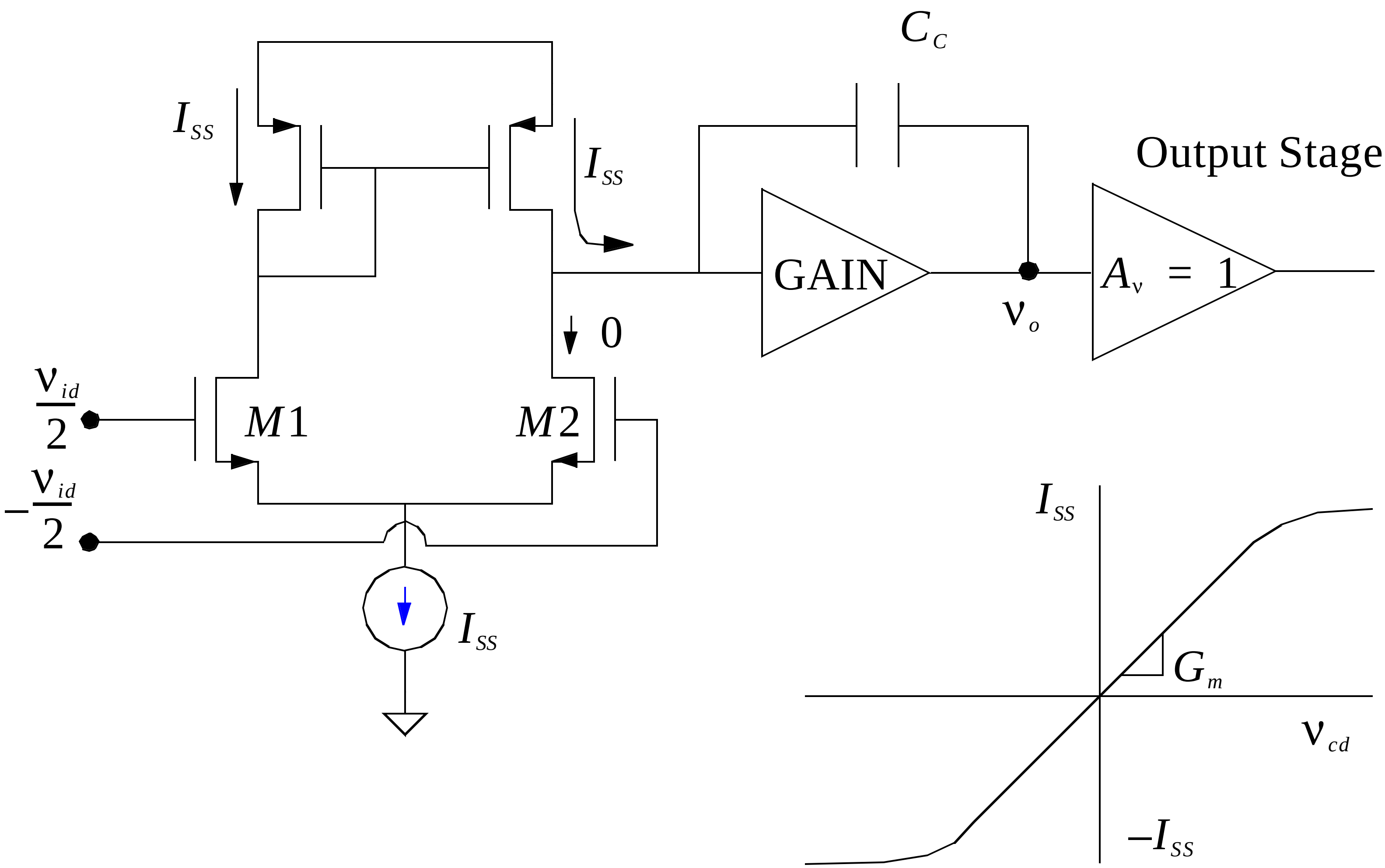




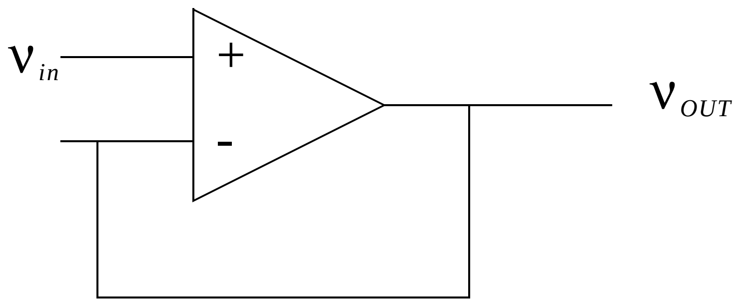
SB-30

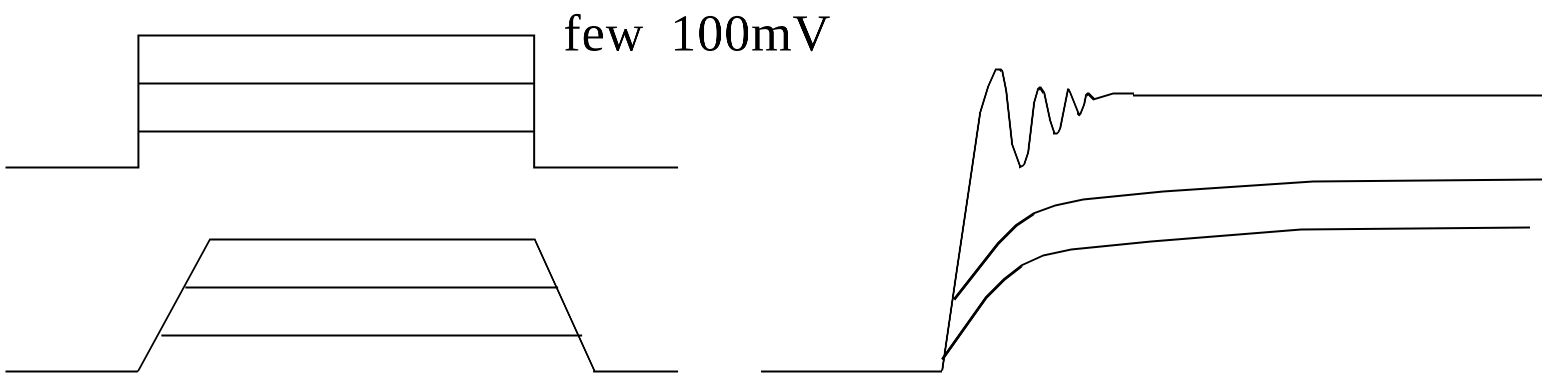


Slew Rate & Compensation Miller Op Amp



Slew Rate & Compensation Miller Op Amp (Cont.)





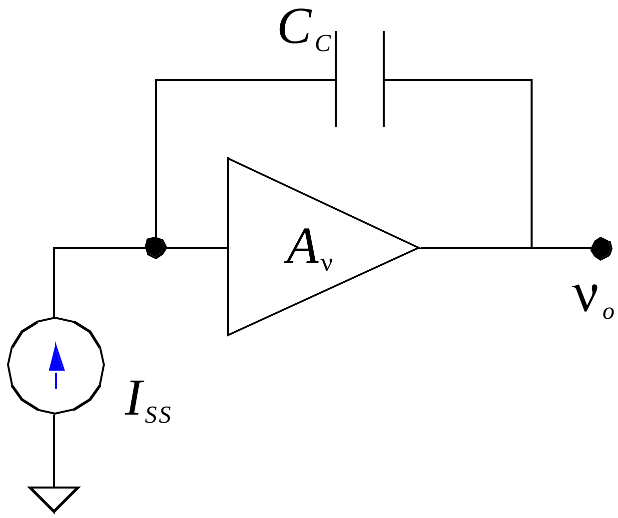
Slew Rate Volts/μsec 10 low

20 – 50 med

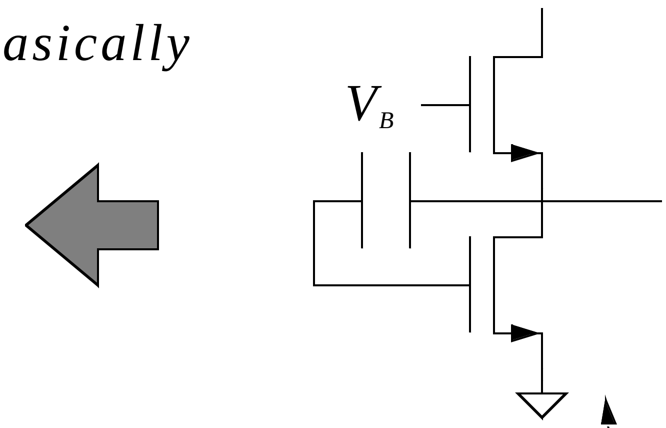
100 high

Slew Rate & Compensation Miller Op Amp (Cont.)

Circuit situation with large v_{id}



basically

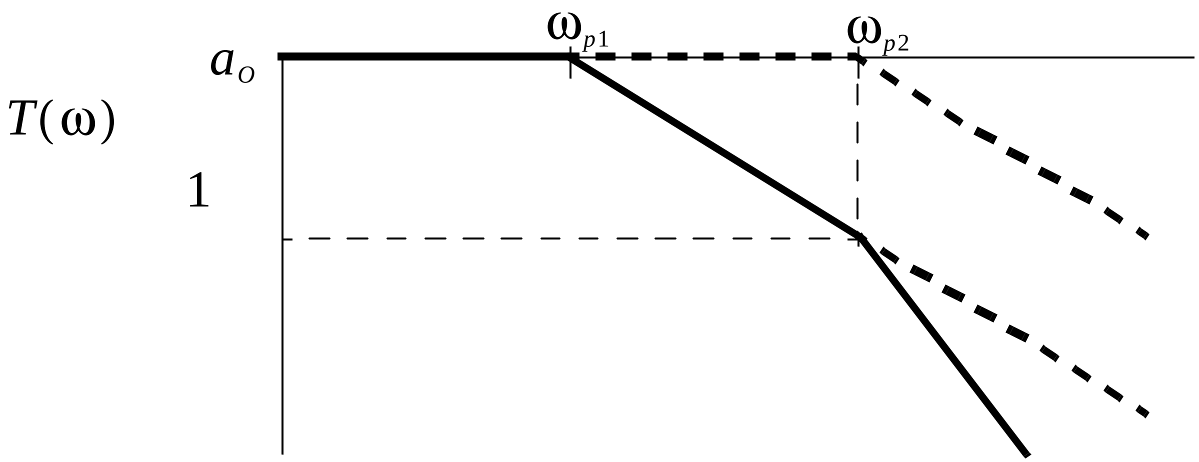


Slewrate

$$I_{SS} = -C_C \cdot \frac{dv_o}{dt} \quad \text{or} \quad \frac{dv_o}{dt} = -\frac{I_{SS}}{C_C} = \text{slew rate}$$

$$v_o = \frac{1}{C} \cdot \int I_{SS} \cdot dt = \frac{I_{SS}}{C} \cdot t \qquad \leftarrow \text{linear with time}$$

Slew Rate & Compensation Miller Op Amp (Cont.)



$$\theta_m = 45°$$

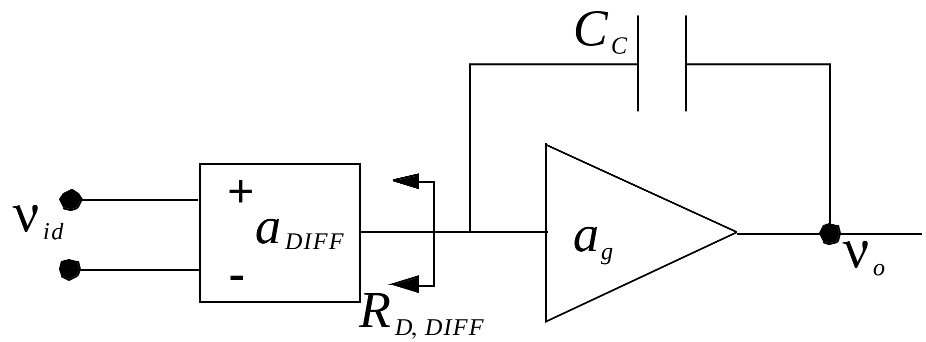
$$f = 1$$

$$a_o = a_{DIFF} \cdot a_g$$

Location of compensation pole :

$$\omega_C = \frac{\omega_{P2}}{a_o}$$

Slew Rate & Compensation Miller Op Amp (Cont.)



$$a_{DIFF} = g_m \cdot R_{D,\,DIFF}$$

$$\omega_C = \frac{1}{R_{D,\,diff} \cdot a_g \cdot C_C} = \frac{\omega_{P2}}{a_o} = \frac{\omega_{P2}}{a_{DIFF} \cdot a_g}$$

$$\frac{\omega_{P2}}{a_{DIFF}} = \frac{1}{\cancel{R_{D,\,DIFF}} \cdot C_C} = \frac{\omega_{P2}}{g_m \cdot \cancel{R_{D,\,DIFF}}}$$

$$C_C = \frac{g_m}{\omega_{P2}}$$ The size of the compensation depends only on g_m & ω_{P2}

Slew Rate & Compensation Miller Op Amp (Cont.)

$$g_m = \frac{2 \cdot I_{DS}}{V_{DSAT}}$$

$$V_{DSAT} = \frac{2 \cdot I_{DS}}{g_m}$$

$$g_{m1}\Big|_{I_{DS} = \frac{I_{SS}}{2}}$$

$$\frac{d\nu_o}{dt} = \frac{I_{SS}}{C_C} = \frac{I_{SS}}{g_{m1}} \cdot \omega_{P2} = \frac{\frac{I_{SS}}{2} \cdot 2}{g_{m1}} \cdot \omega_{P2}$$

$$\boxed{\text{Slewrate} = \frac{d\nu_o}{dt} = V_{DSAT1} \cdot \omega_{P2}}$$

$$V_{DSAT} = 0.1V$$

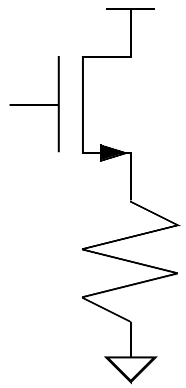
$$\omega_{P2} = 10MHz \cdot 2\pi \qquad SR = 6.3V/\mu\text{sec}$$

Slew Rate & Compensation Miller Op Amp (Cont.)

How to increase slew rate :

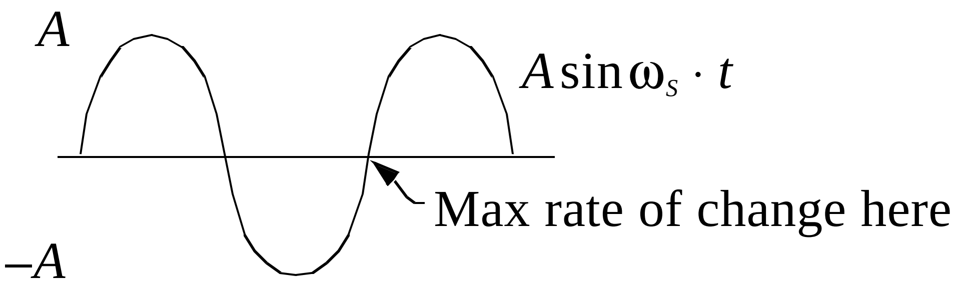
Increase $V_{DSAT1} \Rightarrow$ More current, smaller $\dfrac{W}{L}$

Increase ω_{P2}



$$\frac{g_m}{1 + g_m R_S}$$

Slew rate limits max change :



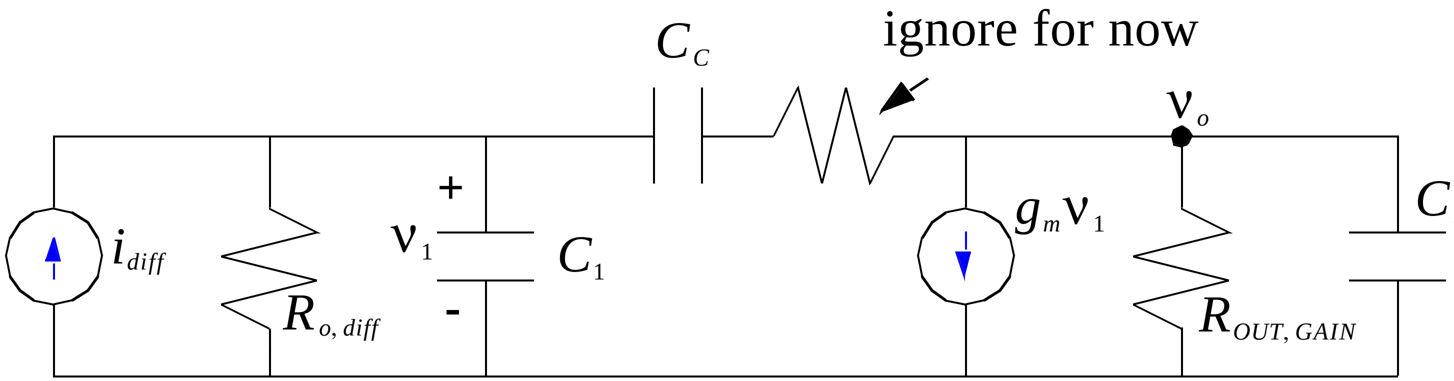
$A = 2V \qquad \omega_S = 10^6 \cdot 2\pi$

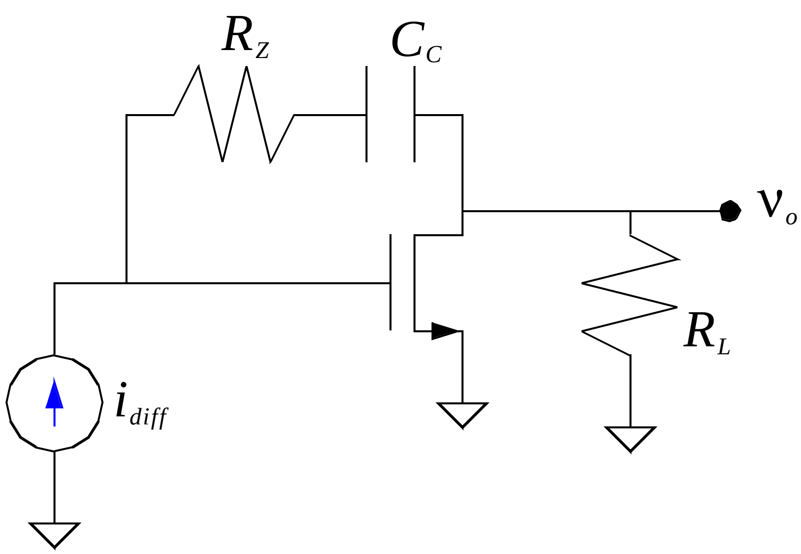
$SR = 13V/\mu\text{sec}$

$$\frac{dV_{SIG}}{dt} = \omega_S \cdot A \cos \omega_S \cdot t$$

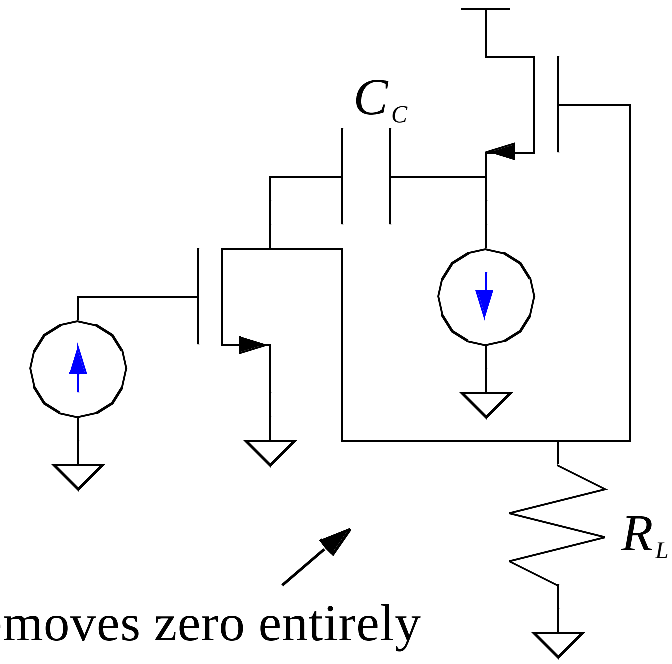
Max Value $\omega_S \cdot A$

MOS Miller Amp - Right Half Plane Zero





also



MOS Miller Amp - Right Half Plane Zero (Cont.)

$$\omega_Z = \frac{1}{C_C \cdot \left(\frac{1}{g_m} - R_Z\right)}$$