University of California
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College of Engineering
Department of Electrical Engineering
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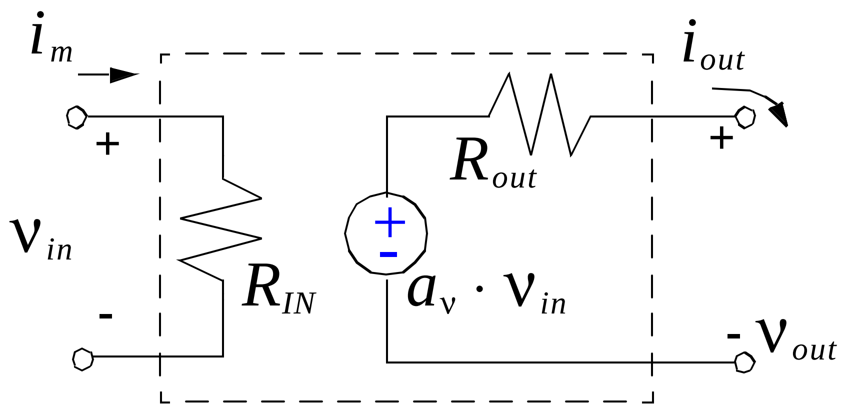
EECS140

Analog Circuit Design

Lectures
on
SINGLE TRANSISTOR CIRCUITS

Two Port Analysis

S-1



Small Signal Analysis

R_{IN} , R_{OUT} , a_υ

Set independent sources to zero

$$R_{in} = \left.\frac{v_{in}}{i_m}\right|_{i_{OUT} = 0}$$

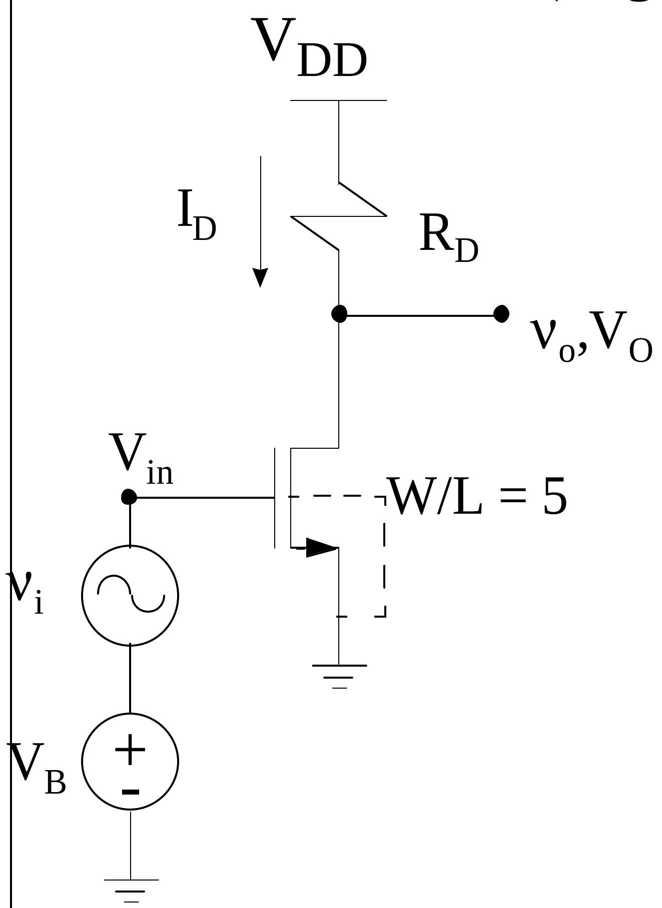
$$a_\upsilon = \left.\frac{v_{out}}{v_{in}}\right|_{i_{out} = 0}$$

v_{in} at input

$$R_{out} = \left.\frac{v_{out}}{(-i_{out})}\right|_{\upsilon_{in} = 0}$$

Common Source (Inverter, Gain Stage)

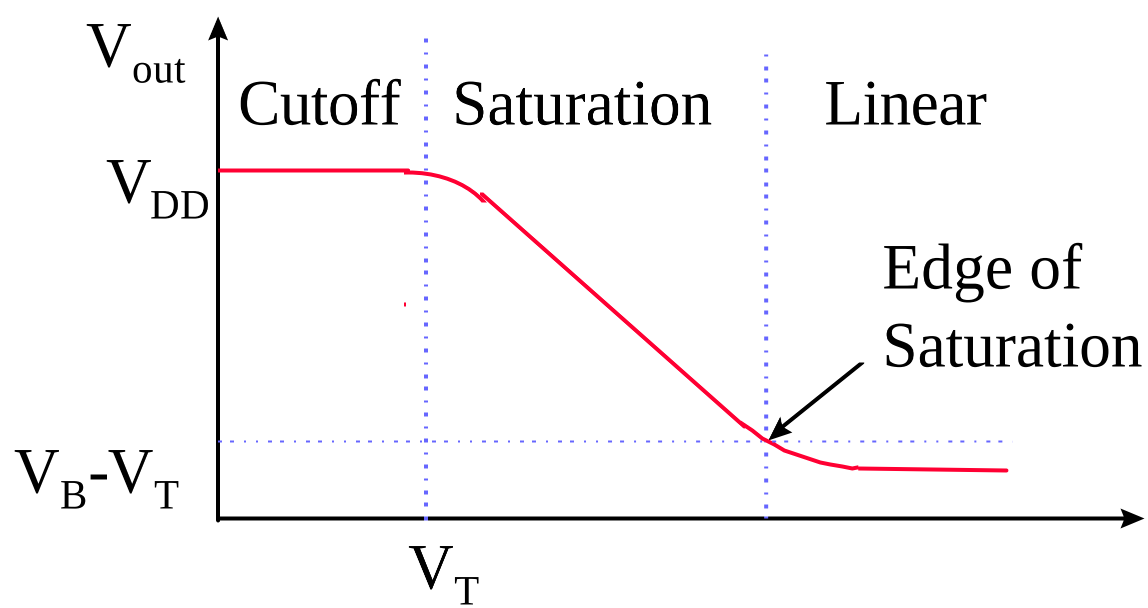
(High Gain, Moderate R_{out})



$V_T = 1V$ $\quad$ $V_{DD} = 5V$

$k' = 60\mu A/V^2$ $\quad$ $R_D = 1k\Omega$

$\lambda = 0.01$ $\quad$ $\gamma = 0.1$



Common Source (Cont.)

DC Analysis :

$$V_O = V_{DD} - I_D R_D$$

$$V_O = V_{DD} - \frac{k'}{2} \cdot \frac{W}{L} \cdot (\underbrace{V_{GS}}_{V_B} - V_T)^2 \cdot R_D$$

$$V_O \geq V_{DSAT} = V_{GS} - V_T = V_B - V_T$$

$$V_{O,EOS} = V_{DD} - \frac{k'}{2} \cdot \frac{W}{L} \cdot (V_B - V_T)^2 \cdot R_D$$

$$V_B = V_T + V_{DD} - \frac{k'}{2} \cdot \frac{W}{L} \cdot (V_B - V_T)^2 \cdot R_D$$

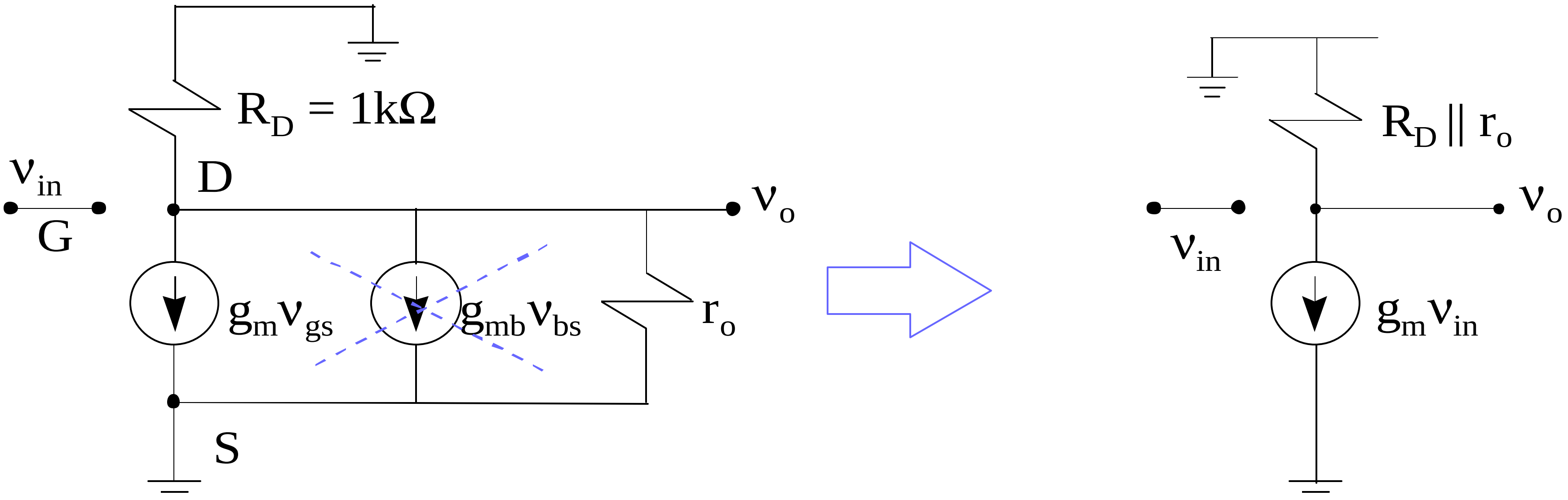
$$V_B^{(A)} = \left[\frac{V_{DD} - V_B^{(B)} + V_T}{\frac{k'}{2} \cdot \frac{W}{L} \cdot R_D} \right]^{\frac{1}{2}} + V_T$$

$V_B^{(A)}$	$V_B^{(B)}$
3.00	4.47
4.47	3.19
3.19	4.33
:	:
	4.12

Common Source (Cont.)

S-4

Small Signal Analysis :



Lets say $V_{OUT} = 2.5V$,then $I_D = 2.5mA$

$$\nu_o = -g_m \cdot (R_D \| r_o) \cdot \nu_{in} = -\left(2k' \cdot \frac{W}{L} \cdot I_{DS}\right)^{\frac{1}{2}} \cdot \underbrace{(R_D \| r_o)}_{1k\Omega} \cdot \nu_{in}$$

$$r_o = \frac{1}{\lambda \cdot I_D} = \frac{1}{0.01 \cdot 2.5 \cdot 10^{-3}} = 40k\Omega$$

$$\frac{\nu_o}{\nu_{in}} = -g_m \cdot (1k\Omega \| 40k\Omega) \approx -g_m \cdot (1k\Omega) = 1.2$$

Common Source (Cont.)

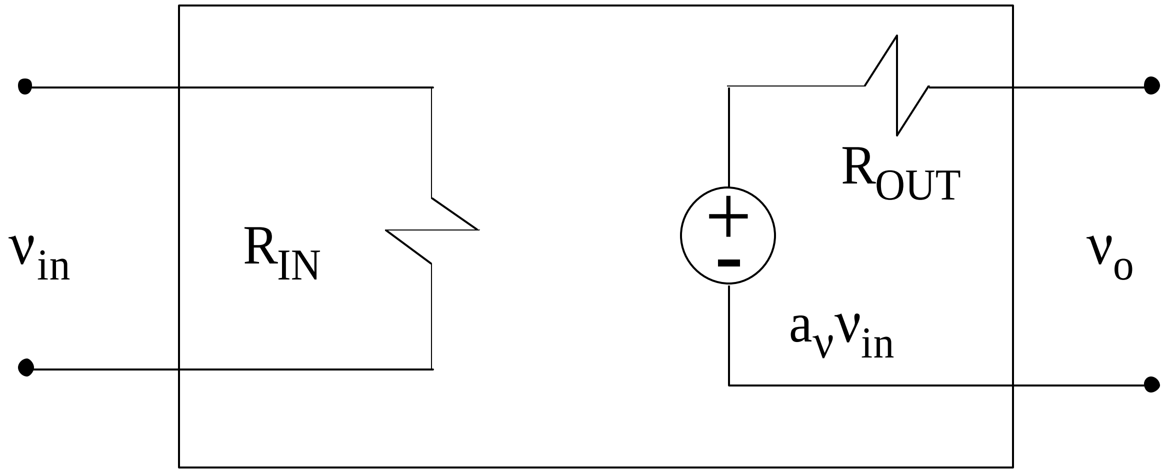
S-5

If we bias the output at $V_{DD}/2$, then the equation for gain is,

$$I_{DS} = \frac{V_{DD}}{2 \cdot R_D}$$

$$a_v = \frac{v_o}{v_{in}} = -g_m \cdot R_D = -\left(2k' \cdot \frac{W}{L} \cdot \frac{V_{DD}}{2}\right)^{\frac{1}{2}} \cdot R_D^{\frac{1}{2}}$$

$$R_{IN} = \infty \qquad R_{OUT} = R_D \parallel r_o$$



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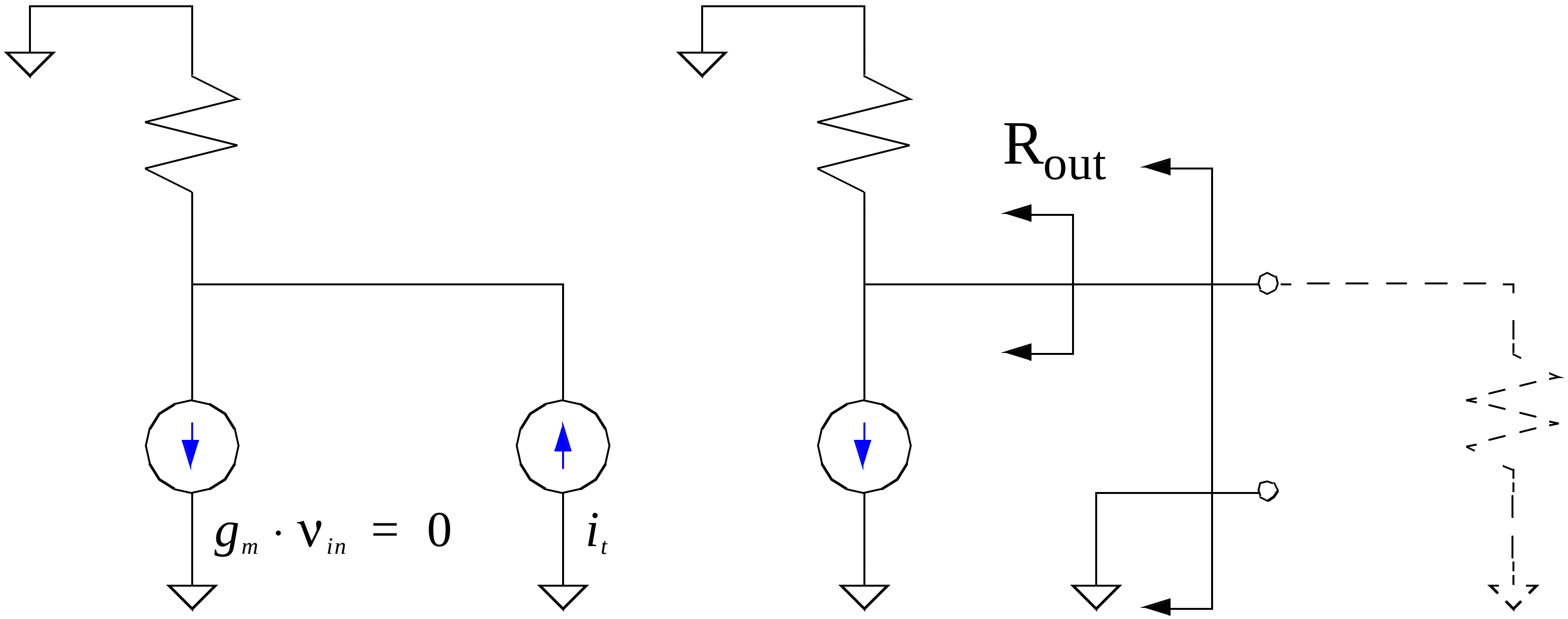
Rout Calculation :



$$R_{out} = \left.\frac{v_t}{i_t}\right|_{v_{in} = 0}$$

$$v_t = (R_D \| r_o) \cdot i_t$$

$$R_{out} = \frac{v_t}{i_t} = R_D \| r_o$$

Circuit Transconductance GM :

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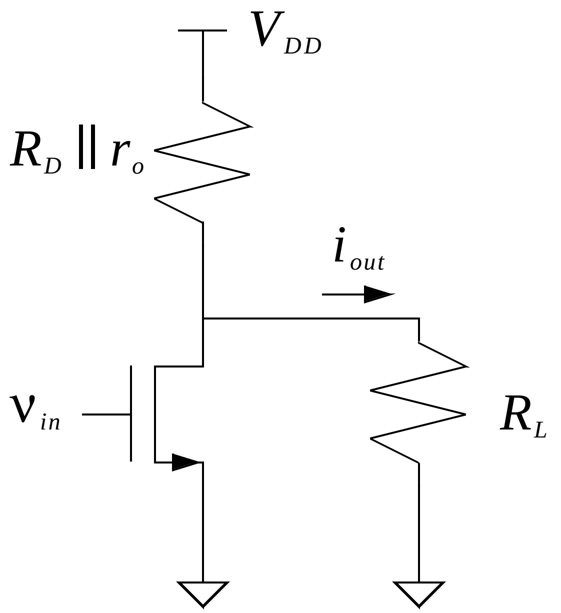
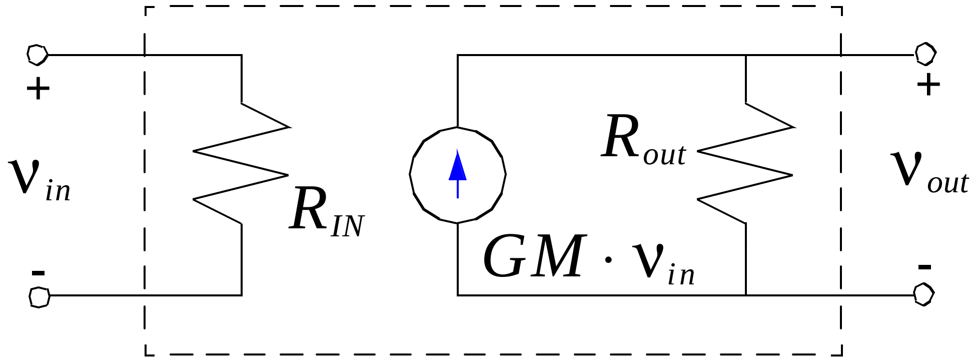


$$GM = \left. \frac{i_{out}}{v_{in}} \right|_{R_L = 0}$$



$$v_{out} = GM \cdot v_{in} \cdot R_{out}$$

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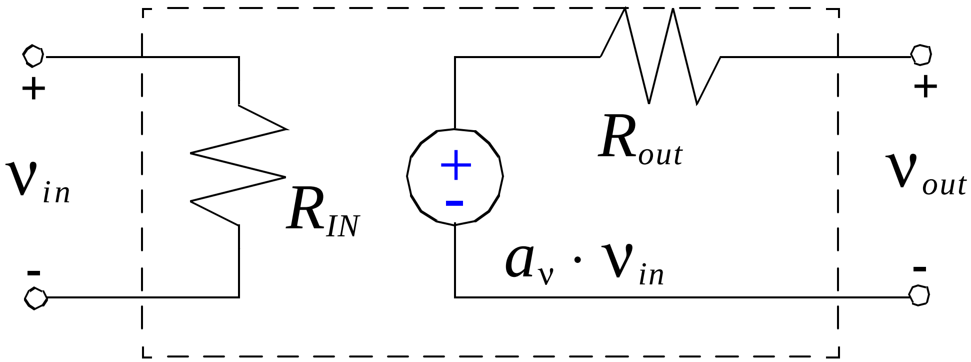
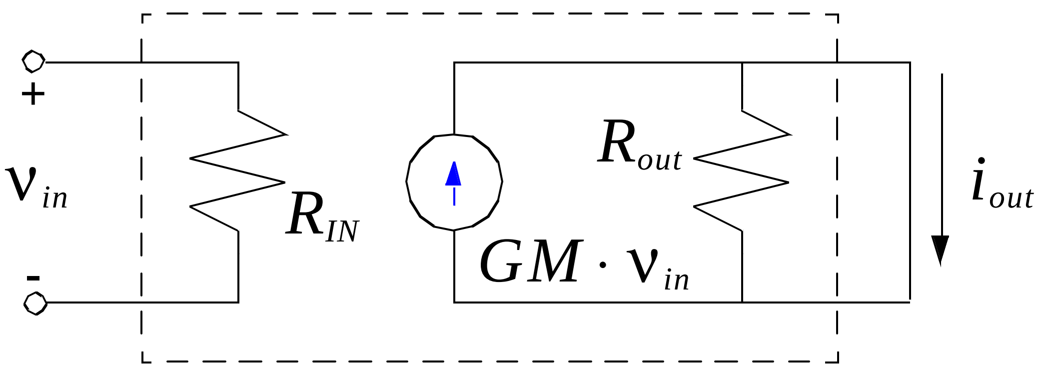


$$v_{out} = a_v \cdot v_{in}$$

$$v_{out} = GM \cdot v_{in} \cdot R_{out}$$

$$\frac{v_{out}}{v_{in}} = a_v = GM \cdot R_{out}$$



for $R_L = 0$,

$$i_{out} = GM \cdot v_{in}$$

$$GM = \frac{i_{out}}{v_{in}}$$

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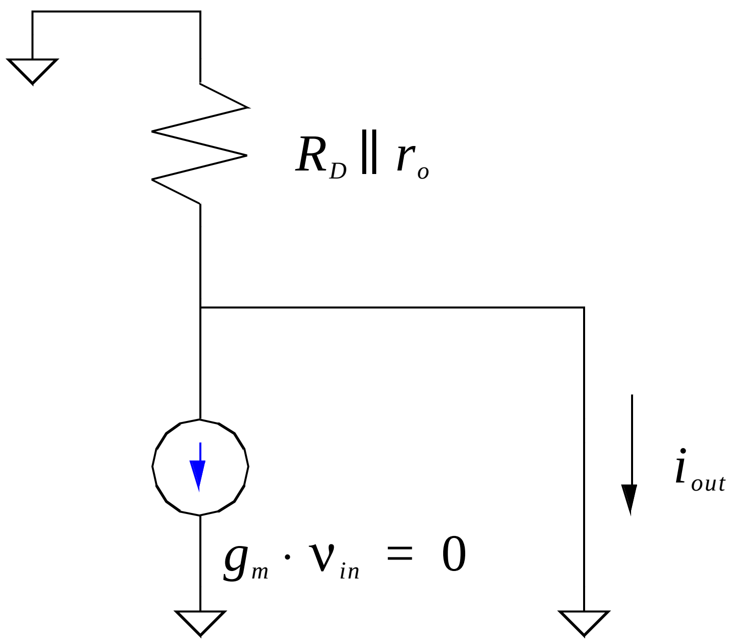

$$i_{out} = -g_m \cdot v_{in}$$

$$GM = \frac{i_{out}}{v_{in}} = -g_m$$

S-10

$$a_v = GM \cdot R_{out} \qquad a_v = -g_m \cdot (R_D \| r_o) \qquad r_0 = \frac{1}{\lambda \cdot I_{DS}}$$

$$g_m \propto I_{DS}^{\frac{1}{2}} \qquad GM \propto I_{DS}^{\frac{1}{2}} \qquad R_D \gg r_o$$

$$a_{v,\,max} = -g_m \cdot r_0 \propto \frac{I_{DS}^{\frac{1}{2}}}{I_{DS}} \sim \frac{1}{I_{DS}^{\frac{1}{2}}}$$

For Bipolar

$$g_m = \frac{I_C}{V_{THERMAL}}$$

$$(g_m \cdot r_o)_{MAX} = \frac{V_{EARLY}}{V_{THERMAL}}$$

$$r_o = \frac{V_{EARLY}}{I_C}$$

Common Source with Source Degeneration

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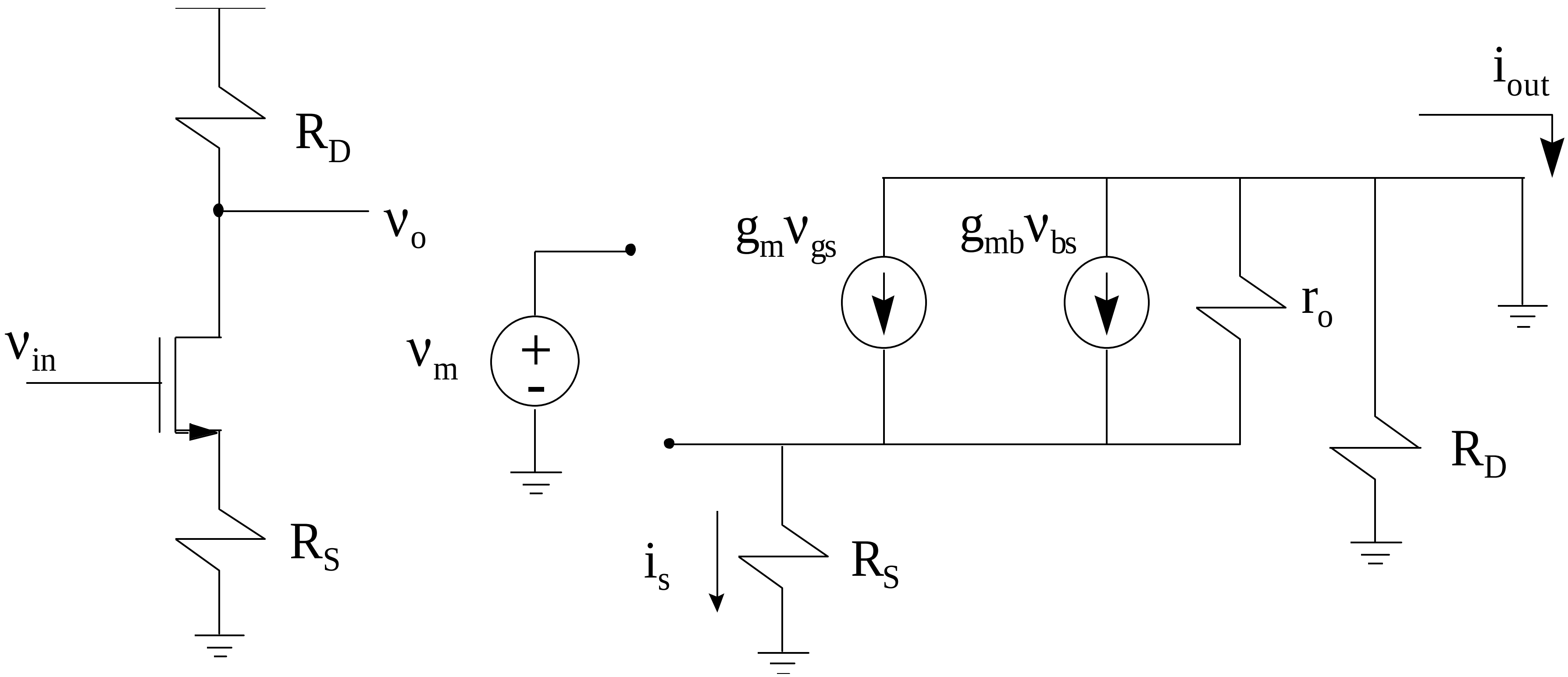


Common Source with Source Degeneration (Cont.)

S-12

Calculate circuit transconductance GM :

$$G_m = \left.\frac{i_{out}}{v_{in}}\right|_{R_L = 0\Omega} = -\frac{i_s}{v_{in}}$$

$$i_s = g_m \cdot (v_{in} - v_s) + g_{mb} \cdot (-v_s) - \frac{v_s}{r_o}$$

$$i_s = v_{in} \cdot g_m - v_s\left(g_m + g_{mb} + \underbrace{\frac{1}{r_o}}_{\text{small compared to } g_m}\right)$$

$$v_s = i_s \cdot R_S$$

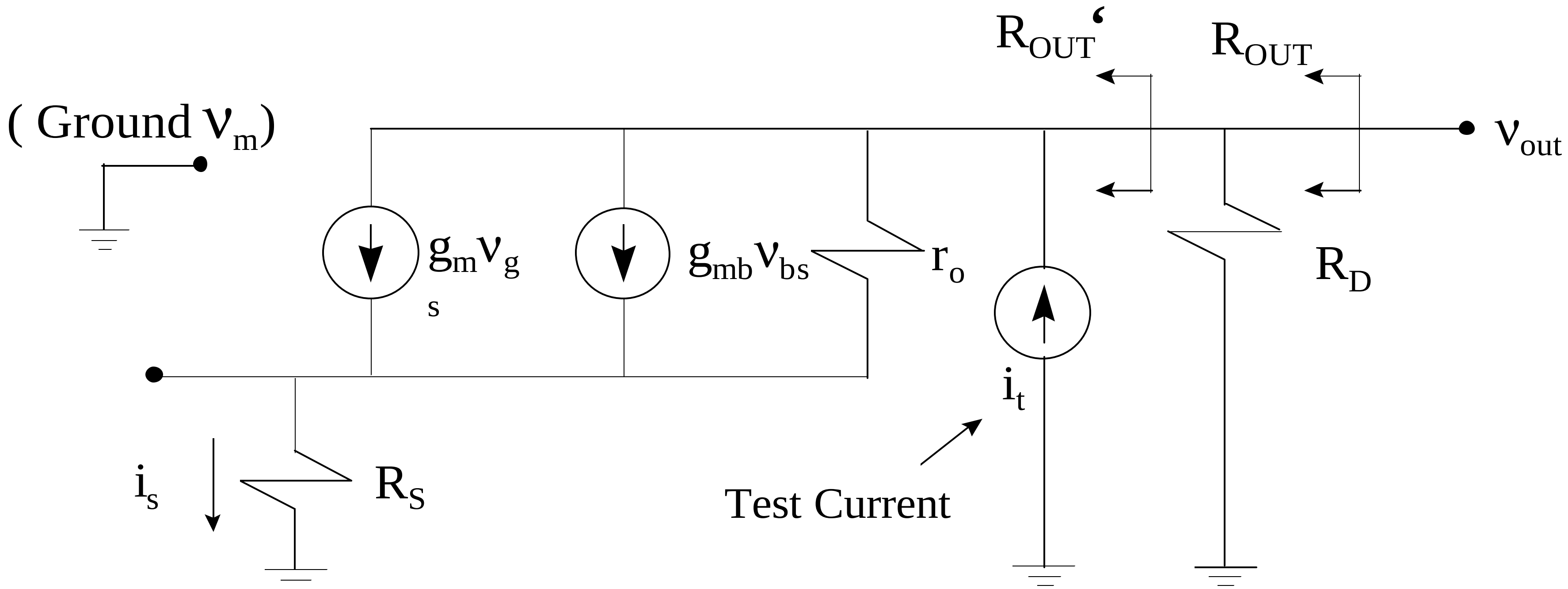
$$GM = \frac{i_s}{v_{in}} = \frac{-g_m}{1 + R_S \cdot \underbrace{(g_m + g_{mb})}} = \frac{-g_m}{1 + R_S \cdot g_m \cdot (1 + \chi)}$$

$$\left(\chi = \frac{g_{mb}}{g_m}\right)$$

Common Source with Source Degeneration (Cont.)

S-13

Rout Calculation :



Common Source with Source Degeneration (Cont.)

S-14

$$R_{OUT} = R'_{OUT} \parallel R_D$$

$$i_s = i_t \qquad v_s = i_t \cdot R_S \qquad v_g = v_b = 0$$

$$v_t = i_{ro} \cdot r_o + v_s$$

$$= (i_t - g_m \cdot v_{gs} - g_{mb} \cdot v_{bs}) \cdot r_o + i_t \cdot R_S$$

$$= (i_t - (g_m + g_{mb}) \cdot \underbrace{v_s}_{i_t \cdot R_S}) \cdot r_o + i_t \cdot R_S$$

$$v_t = i_t \cdot [R_S + r_o \cdot \{1 + (g_m + g_{mb}) \cdot R_S\}]$$

$$R_{OUT} = \frac{v_t}{i_t} = \{R_S + r_o \cdot [1 + (g_m + g_{mb}) \cdot R_S]\} \parallel R_D$$

$$A_v = -GM \cdot R_{OUT} = \frac{g_m \cdot R_S}{1 + R_S \cdot (g_m + g_{mb})} + \underbrace{g_m \cdot r_o}_{\approx unchanged} \quad \underline{\underline{\text{if}}} \quad R_D \rightarrow x$$

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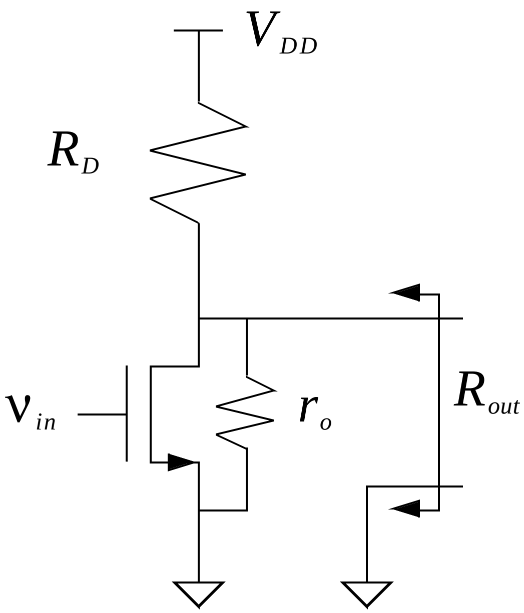
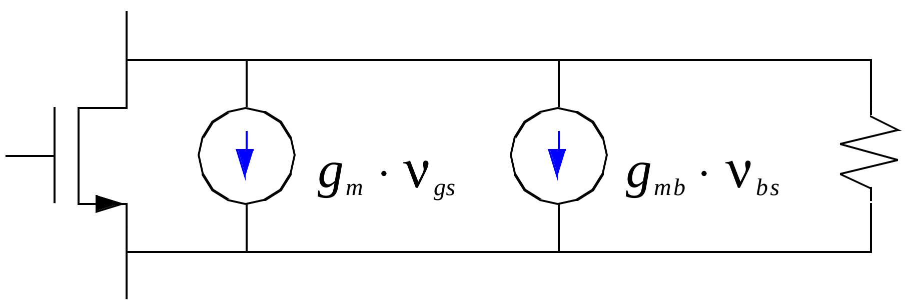


$$GM = \frac{i_{out}}{v_{in}} = -g_m$$

$$R_{OUT} = R_D \parallel r_o$$

$$R_{IN} = \infty$$

$$A_v = GM \cdot R_{OUT}$$



$R'_{out} = r_o$

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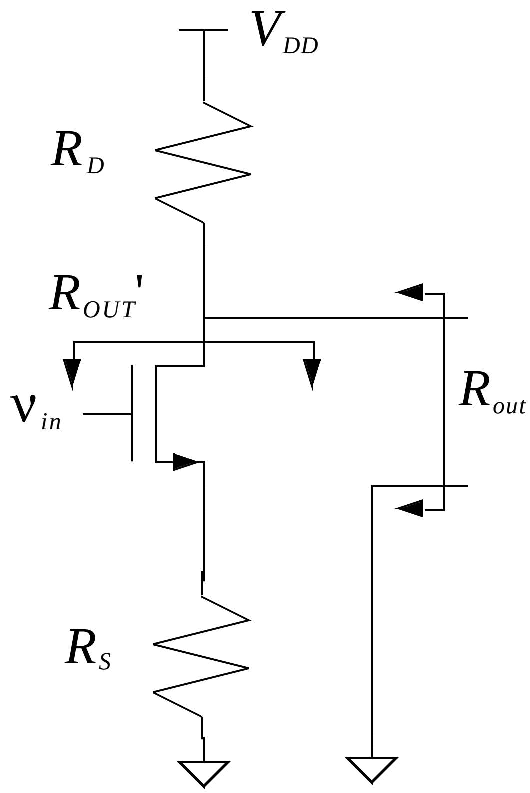

$$\chi = \frac{g_{mb}}{g_m}$$

$$R_{OUT}' = R_S + r_o \cdot \{1 + (1 + \chi) \cdot g_m \cdot R_S\}$$

$$R_{out} = R_D \| R_{OUT}'$$

$$g_m \cdot R_s » 1 \qquad R_{OUT}' \approx r_o \cdot (1 + \chi) \cdot g_m \cdot R_S$$

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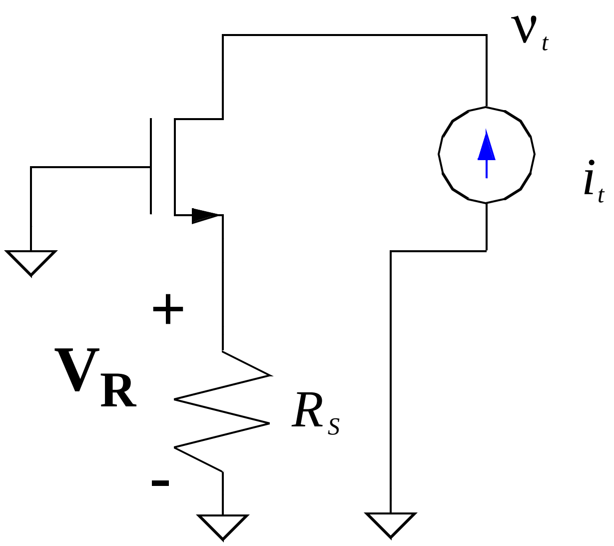
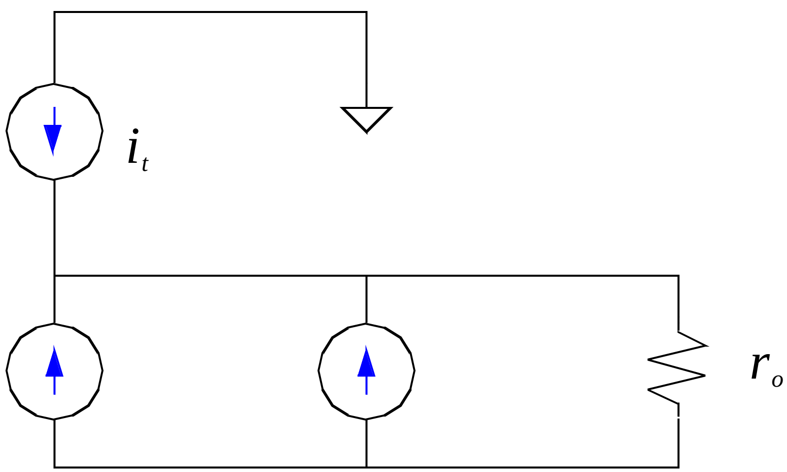


$$R_{out}' = \frac{v_t}{i_t}$$

$$V_R = R_S \cdot i_t$$

$$V_{GS} = V_G - V_S = -R_S \cdot i_t$$

$$V_{BS} = V_B - V_S = -R_S \cdot i_t$$



More current gets shoved into r_o because of polarity of V_{GS} and V_{BS} due to R_S

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$$GM = \frac{-g_m}{1 + R_S \cdot g_m \cdot (1 + \chi)}$$

$$a_v = GM \cdot R_{OUT}$$

$$= \frac{-g_m}{1 + R_S \cdot g_m \cdot (1 + \chi)} \cdot \{ r_o \cdot [1 + (g_m + g_{mb}) \cdot R_S] \} \parallel R_D$$

$$R_D \rightarrow \infty$$

$$a_v \rightarrow -g_m \cdot r_o$$

Common Gate (High Gain, Non-Inverting)

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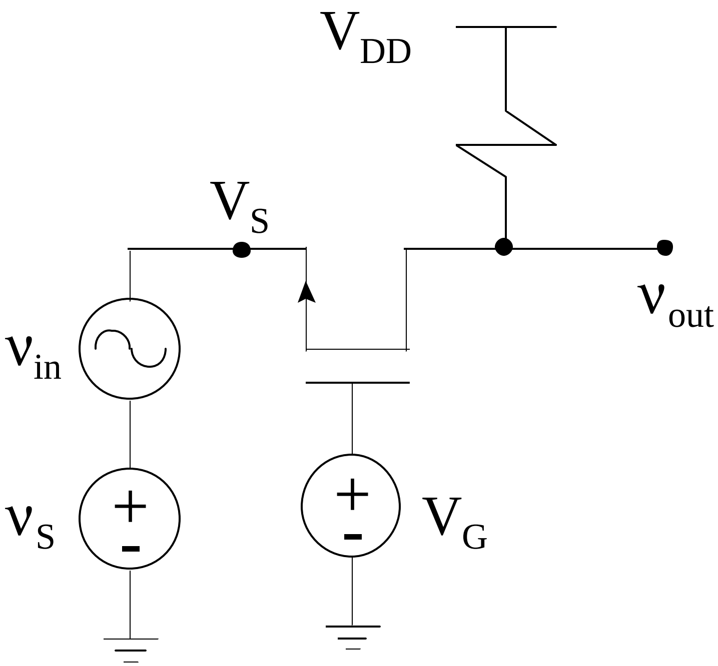


$$\chi = \frac{\gamma}{2(2\phi_f + V_{BS})}$$

$$V_T = V_{TO} + \gamma \cdot [(2 \cdot \phi_f + V_{BS})^{\frac{1}{2}} - (2 \cdot \phi_f)^{\frac{1}{2}}]$$

For Saturation :

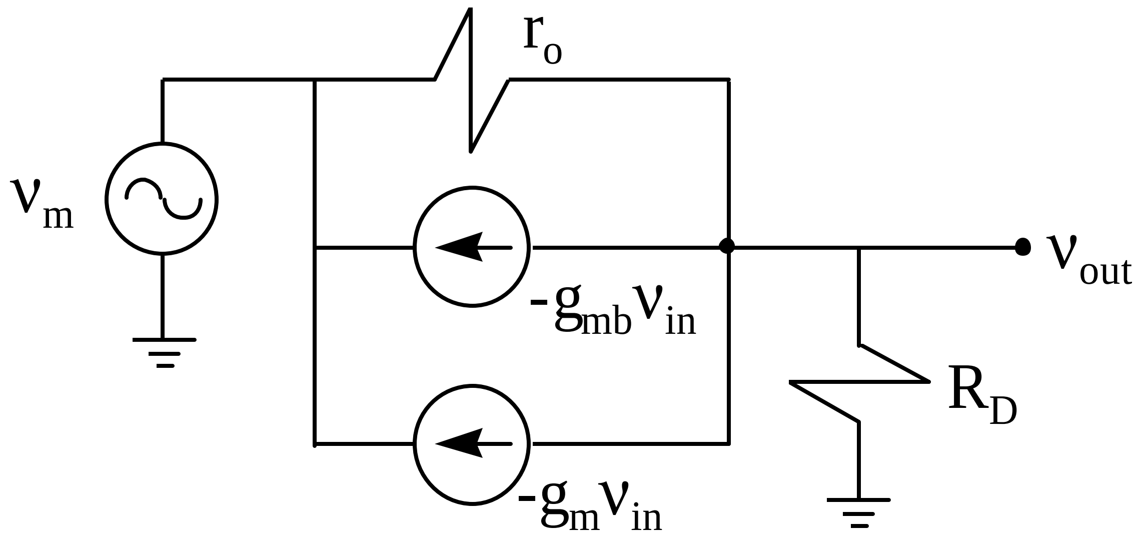
$$V_{OUT} > V_G - V_S - V_T$$

$$g_m = k' \cdot \frac{W}{L} \cdot (V_{GS} - V_T)$$

$$I_{DS} = \frac{k'}{2} \cdot \frac{W}{L} \cdot (V_{GS} - V_T)^2$$

Common Gate (Cont.)

S-20



$$v_{gs} = 0 - v_m$$
$$v_{bs} = 0 - v_m$$

$$v_o = (g_m + g_{mb}) \cdot R_D \cdot v_{in} + \frac{v_{in} - v_{OUT}}{r_o} \cdot R_D$$

$$A_v = \frac{v_o}{v_{in}} = \frac{[1 + (g_m + g_{mb}) \cdot r_o] \cdot R_D}{r_o + R_D}$$

$$G_m = \frac{1}{r_o} + g_m \cdot (1 + \chi) \qquad R_{OUT} = r_o \| R_D \qquad A_v = G_m \cdot R_{OUT}$$

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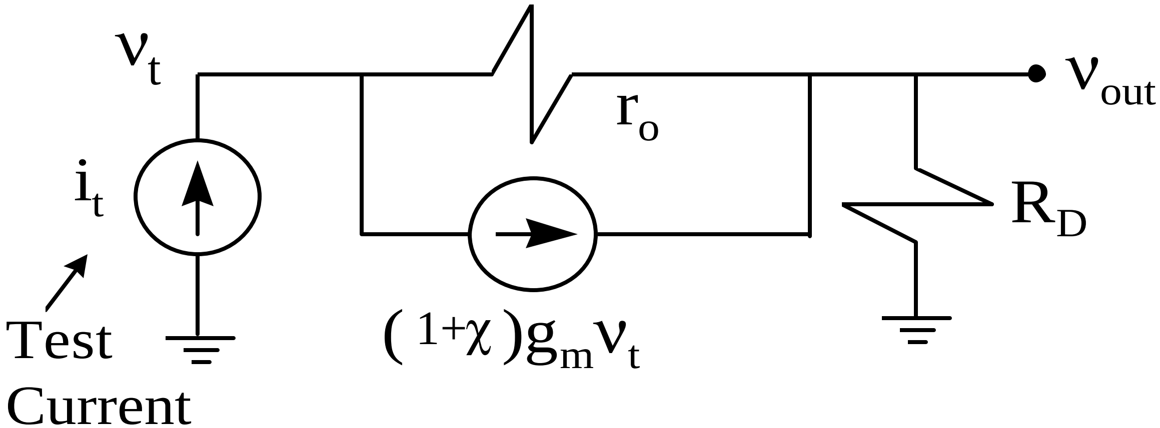
Common Gate (Cont.)

Rin for common gate :



$$i_t = -\frac{v_t - v_{OUT}}{r_o} - g_m \cdot (1+\chi) \cdot v_t = 0$$

$$v_{OUT} = i_t \cdot R_D \qquad \frac{v_t}{i_t} = R_{in} = \frac{r_o + R_D}{1 + (1+\chi) \cdot g_m \cdot r_o}$$

$$R_D < r_o \qquad r_o \rightarrow \infty \qquad R_{in} \rightarrow \frac{1}{(1+x) \cdot g_m}$$

$$R_D > r_o \qquad R_D \rightarrow \infty \qquad R_{in} \rightarrow \frac{R_D}{(1+x) \cdot g_m \cdot r_o}$$

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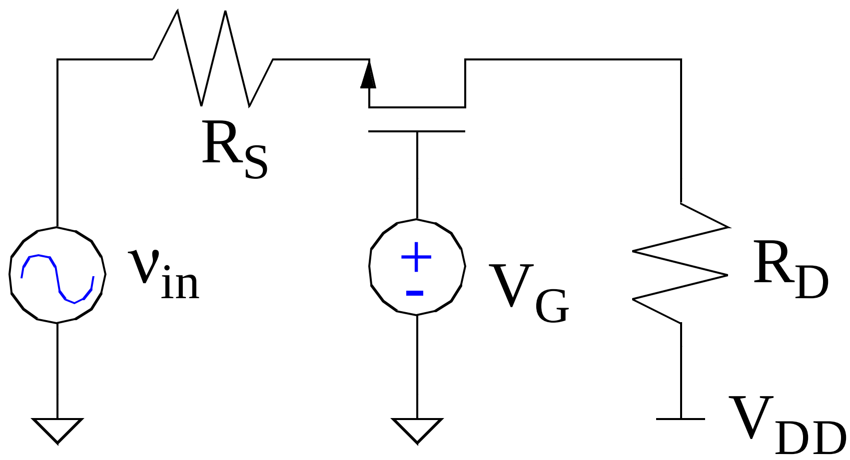
Common Gate with R_S



$$R_{OUT} = (R_S + r_o \cdot [1 + (g_m + g_{mb}) \cdot R_S]) \parallel R_D$$

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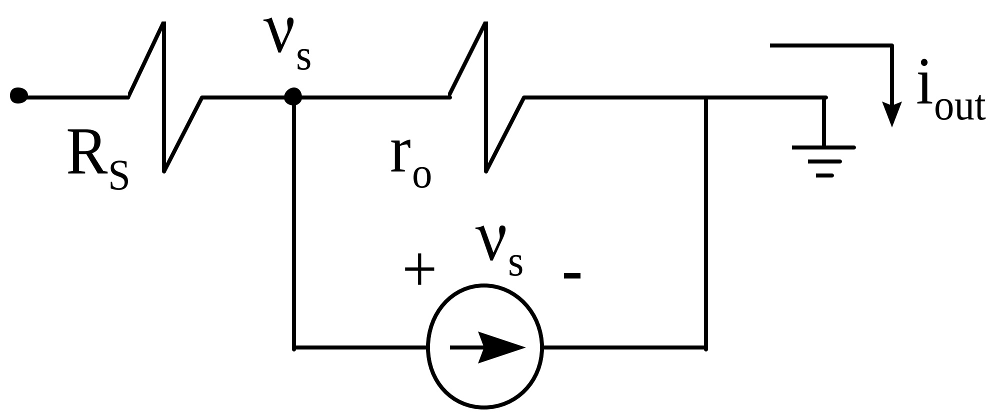
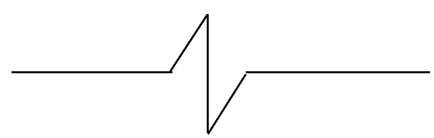
Common Gate with R_S (Cont.)



$$\underbrace{(1+\chi)\cdot g_m \cdot v_s}$$



$$R_{EQ} = \frac{1}{(1+\chi)\cdot g_m}$$

Common Gate with R_S (Cont.)

$$i_{OUT} = \frac{v_{in}}{R_S + \frac{1}{(1+\chi)\cdot g_m}}$$

$$G_m = \frac{i_{OUT}}{v_{in}} = \frac{1}{R_S + \frac{1}{(1+\chi)\cdot g_m}}$$

$$= \frac{(1+\chi)\cdot g_m}{1 + (1+\chi)\cdot g_m \cdot R_S}$$

$$A_v = GAIN = G_M \cdot R_{OUT}$$

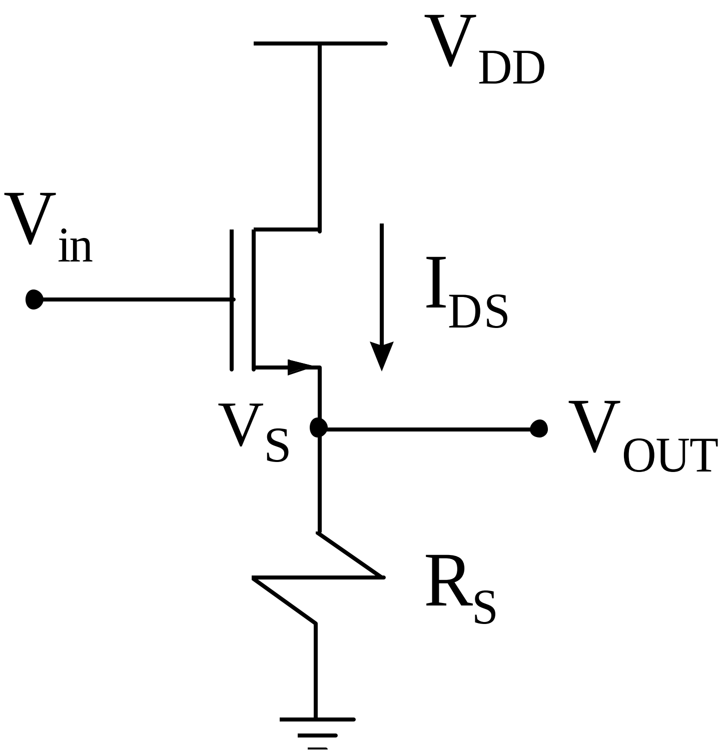
$$A_v = g_m \cdot (1+\chi)\cdot r_o$$

for R_D *LARGE* $\gg r_o$

R_S *typical* $\ll r_o$

Source Follower

DC Analysis :



Good for Buffering & Impedance Transformation

Voltage $GAIN \sim 1$

High $R_{in} = \infty$

Low $R_{OUT} \sim 10 - 1k\Omega$

Source Follower (Cont.)

$$I_{DS} = \frac{k'}{2} \cdot \frac{W}{L} \cdot (V_{IN} - V_{OUT} - V_T)^2$$

$$V_{OUT} = I_{DS} \cdot R_S$$

$$V_{OUT} = \left(R_S \cdot \frac{k'}{2} \cdot \frac{W}{L}\right) \cdot (V_{IN} - V_{OUT} - V_T)^2$$

$$V_{OUT} + V_T + \left(\frac{V_{OUT} \cdot 2}{R_S \cdot k' \cdot \frac{W}{L}}\right)^{\frac{1}{2}} = V_{IN}$$

Solve iteratively !

Source Follower (Cont.)

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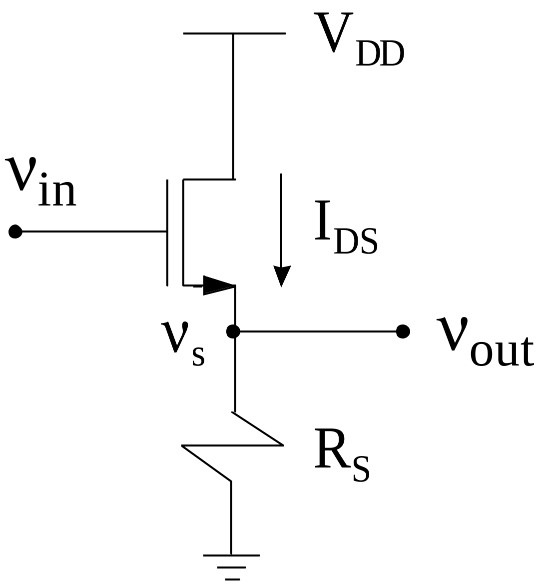
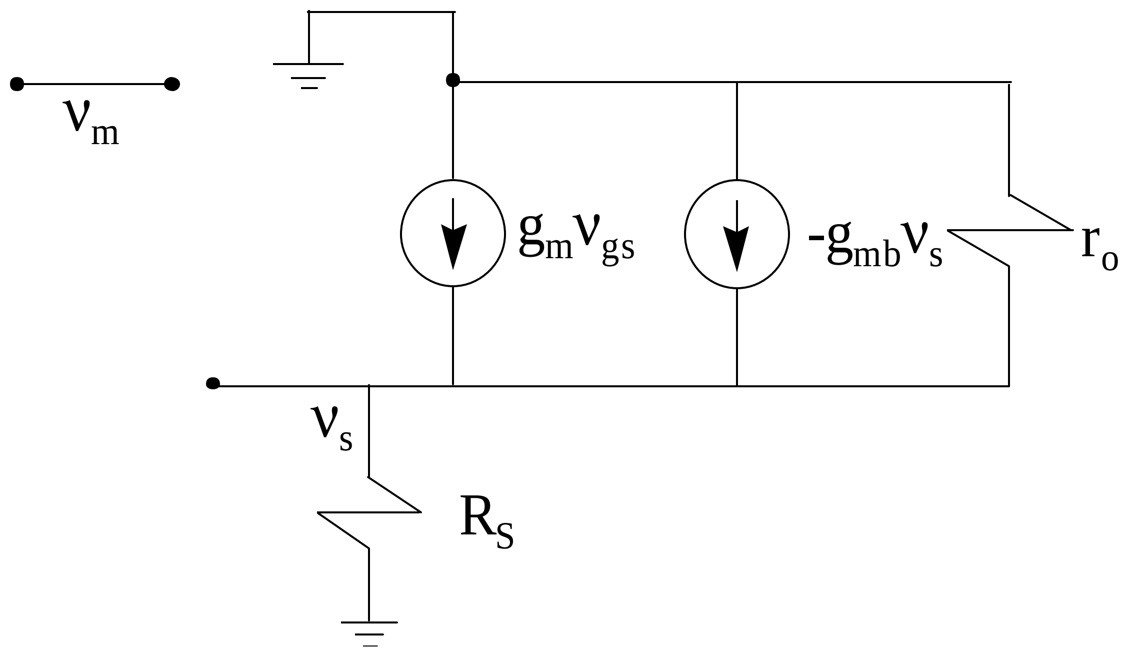
Small Signal :





$$v_{gs} = v_{in} - v_s \qquad v_{OUT} = v_s$$

$$v_s = (g_m \cdot v_{gs} - g_{mb} \cdot v_s) \cdot (r_o \parallel R_S)$$

$$= g_m \cdot R_S \cdot v_{in} - (1 + \chi) \cdot g_m \cdot R_S \qquad r_o \gg R_S$$

$$v_s = \frac{g_m \cdot R_S \cdot v_{in}}{1 + (1 + \chi) \cdot g_m \cdot R_S} = v_{OUT}$$

Source Follower (Cont.)

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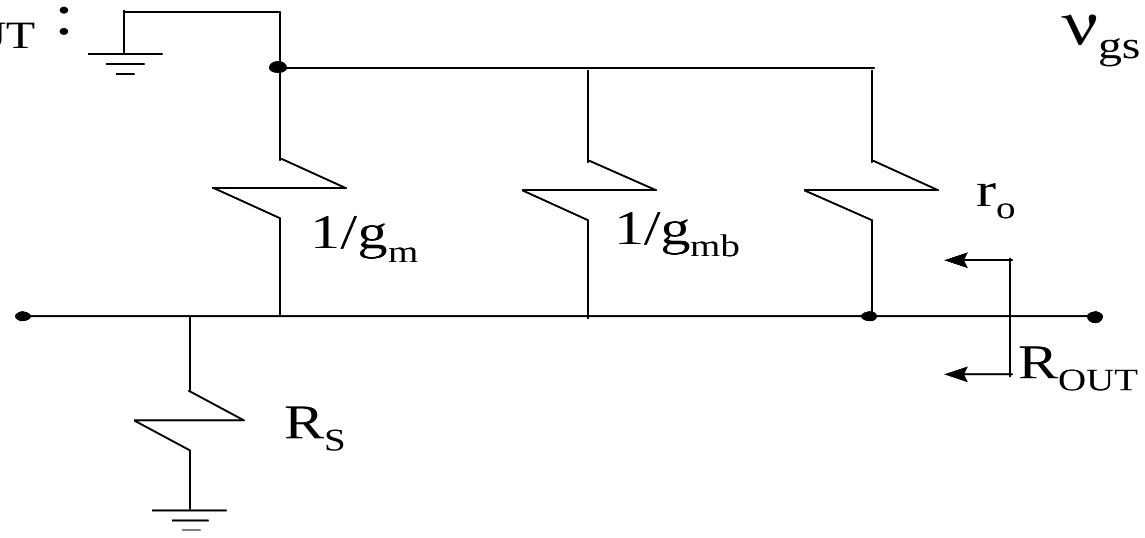
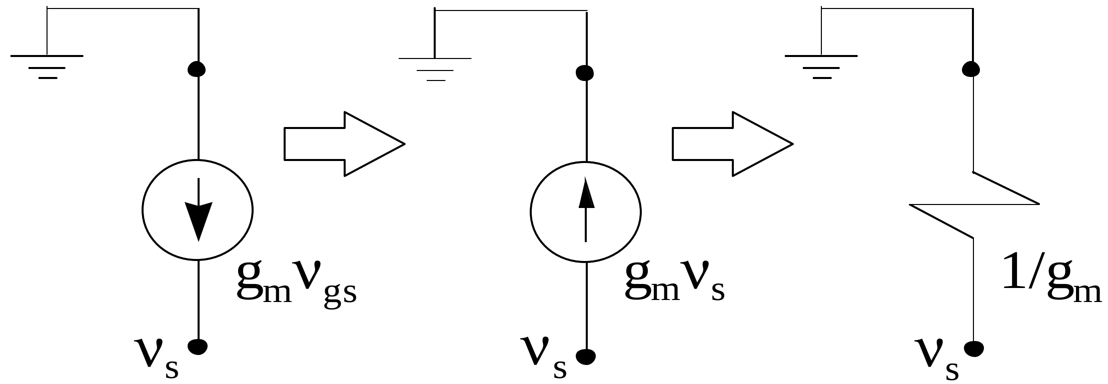
R_{OUT} :



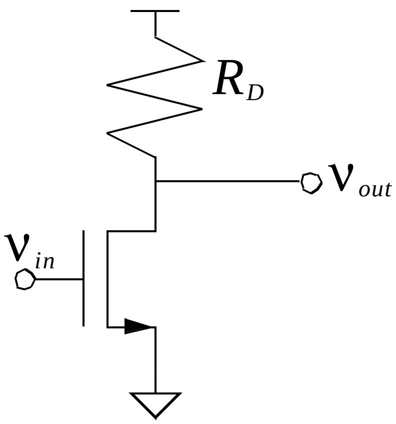
$v_{gs} = -v_s \Big|_{v_{in}=0}$

$$R_{OUT} = \frac{1}{g_m} \| \frac{1}{g_{mb}} \| r_o \| R_S$$



Single Transistor Circuits Summary

Common Source :



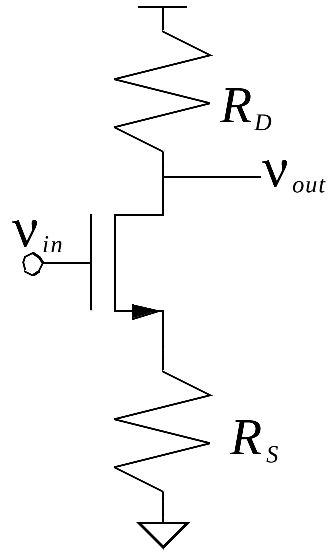
$$A_v = GM \cdot R_{out}$$

$$GM = -g_m$$

$$R_{OUT} = r_o \parallel R_D$$

$$A_{v,max} = -\frac{2}{V_{DSAT}} \cdot \frac{1}{\lambda};(R_D \gg r_o)$$

Common Source with Source Degeneration :



$$GM = \frac{-g_m}{1 + g_m \cdot (1+\chi) \cdot R_S}$$

$$R_{OUT} \approx r_o \cdot [1 + g_m \cdot (1+\chi) \cdot R_S] \parallel R_D$$

Single Transistor Circuits Summary

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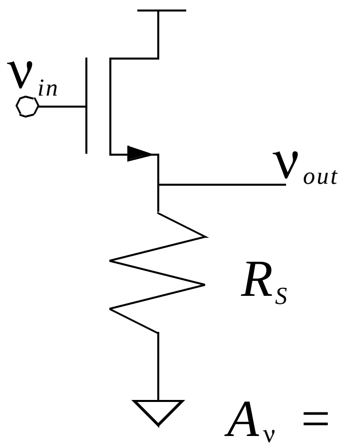
Common Drain (Source Follower) :

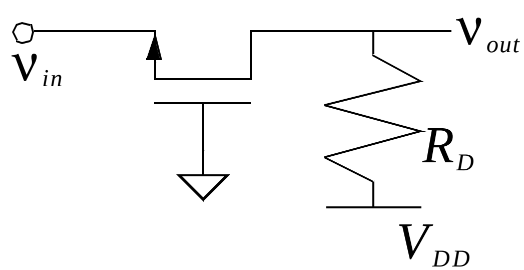


$$A_v = \frac{g_m \cdot R_S}{(1+\chi) \cdot g_m \cdot R_S + 1}$$

$$R_{OUT} = \frac{1}{(1+\chi) \cdot g_m} \parallel R_S$$

$$A_{v,\,max} = \frac{1}{1+\chi};\left(R_S \gg \frac{1}{g_m}\right)$$

Common Gate :



$$A_v = g_m \cdot (1+\chi) \cdot R_{OUT}$$

$$R_{OUT} = R_D \parallel r_o$$

$$R_{in} = \frac{r_o + R_D}{1 + (1+\chi) \cdot g_m \cdot r_o}$$

Common Source $A_{v,max}$ Calculation

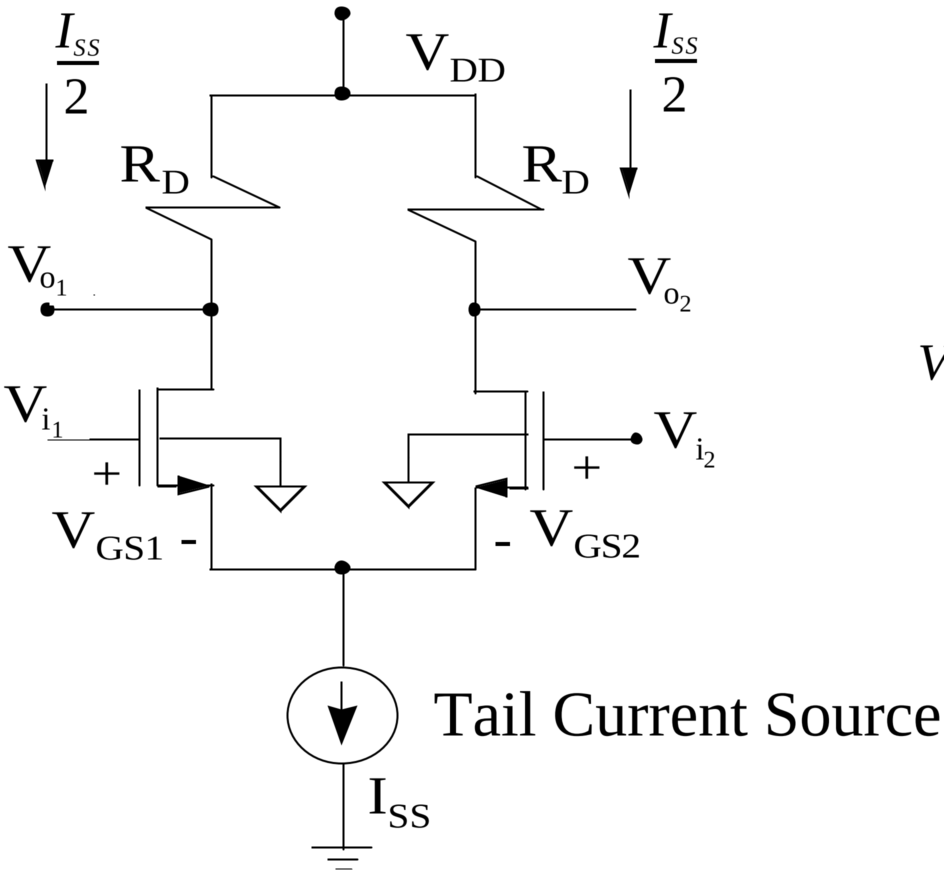
$$A_{v,MAX} = -g_m \cdot R_{OUT}|_{R_D \gg r_o} = -g_m \cdot r_o$$

$$= k' \cdot \frac{W}{L} \cdot (V_{GS} - V_T) \cdot \frac{1}{\lambda \cdot I_{DS}} = \frac{k' \cdot \frac{W}{L} \cdot (V_{GS} - V_T)}{\lambda \cdot \frac{k'}{2} \cdot \frac{W}{L} \cdot (V_{GS} - V_T)^2}$$

$$= -\frac{2}{(V_{GS} - V_T)} \cdot \frac{1}{\lambda} = -\frac{2}{\lambda \cdot V_{DSAT}}$$

Differential Pair

DC Analysis :



$$V_{GS} = V_T + \left(\frac{I_{SS}}{k' \cdot \frac{W}{L}} \right)^{\frac{1}{2}}$$

Differential Pair (Cont.)

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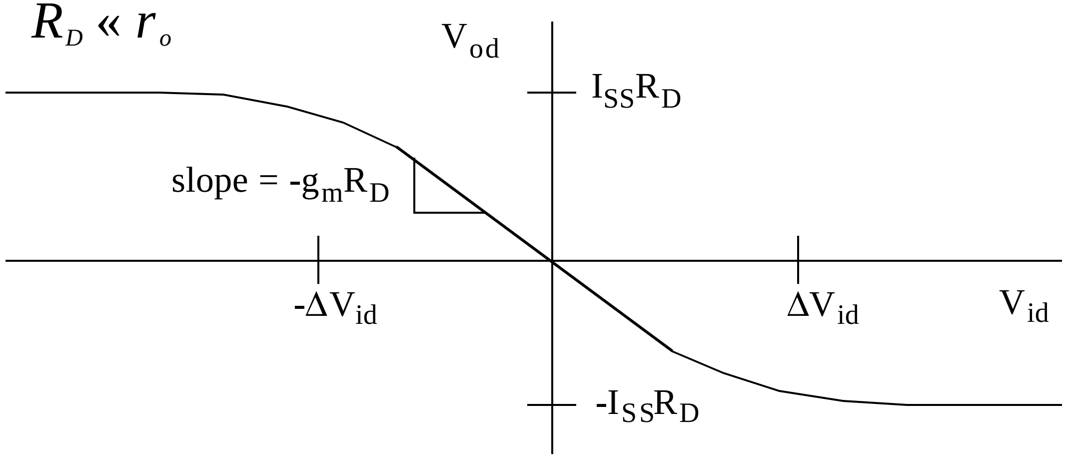
$R_D \ll r_o$



$$V_{od} = V_{o1} - V_{o2}$$

$$V_{id} = V_{i1} - V_{i2}$$

Solve eqns : $V_{i1} - V_{GS1} + V_{GS2} - V_{i2} = 0$

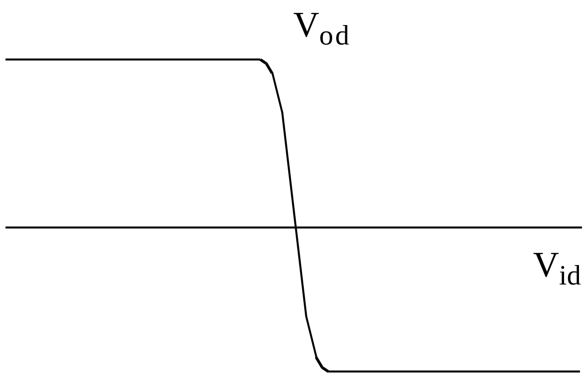
$$I_{DS1} + I_{DS2} = I_{SS}$$

$$V_{GS1} = V_T + \left(\frac{2 \cdot I_{DS1}}{k' \cdot \frac{W}{L}} \right)^{\frac{1}{2}} \qquad g_m = \left(2 \cdot k' \cdot \frac{W}{L} \cdot \overbrace{I_{DS}}^{= \frac{I_{SS}}{2}} \right)^{\frac{1}{2}}$$

also $$\Delta V_{GS} \approx \frac{I_{SS} \cdot R_D}{SLOPE} = \frac{I_{SS} \cdot R_D}{g_m \cdot R_D} = \left(\frac{I_{SS}}{k' \cdot \frac{W}{L}} \right)^{\frac{1}{2}}$$

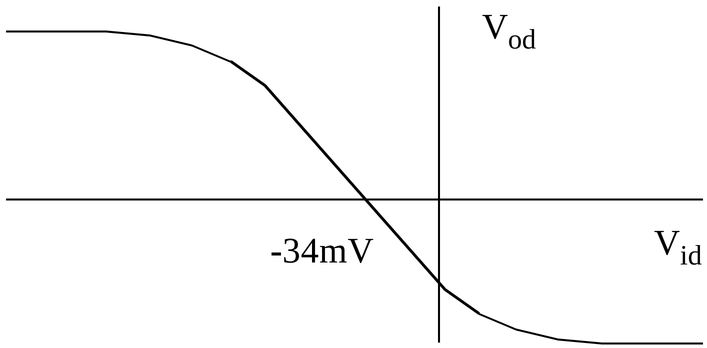
Differential Pair (Cont.)

If we have a lot of gain, low current and a large R_D, then



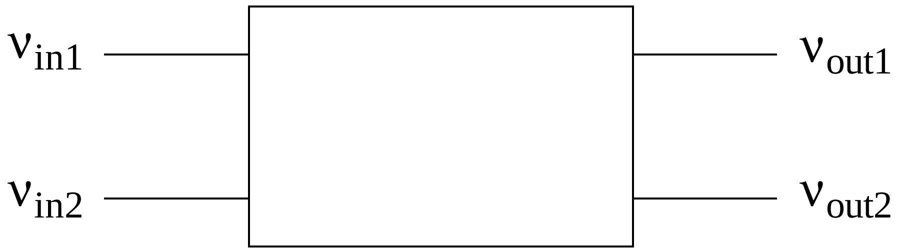
This can quickly become unbiased

To make sure your output voltage is at zero when your input voltage is zero, do a DC sweep in spice.



Differential Pair (Cont.)

Define New Variables for Differential 2-Port :



Differential Mode :

$$v_{od} = v_{OUT1} - v_{OUT2}$$

$$v_{id} = v_{in1} - v_{in2}$$

Common Mode :

$$v_{oc} = \frac{v_{OUT1} + v_{OUT2}}{2}$$

$$v_{ic} = \frac{v_{in1} + v_{in2}}{2}$$

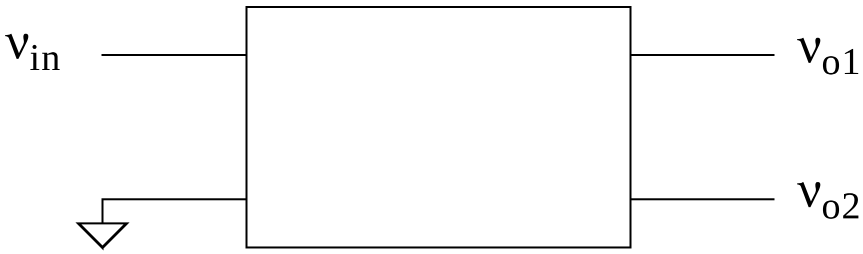
Then the differential mode gain is $A_{DM} = \dfrac{v_{od}}{v_{id}}$

and the common mode gain is $A_{CM} = \dfrac{v_{oc}}{v_{ic}}$

Differential Pair (Cont.)

$$v_{o1} = v_{oc} + \frac{v_{od}}{2} \qquad v_{i2} = v_{ic} - \frac{v_{id}}{2}$$

$$v_{o2} = v_{oc} - \frac{v_{od}}{2} \qquad v_{i1} = v_{ic} + \frac{v_{id}}{2}$$



$$v_{id} = v_{in} - 0 = v_{in}$$

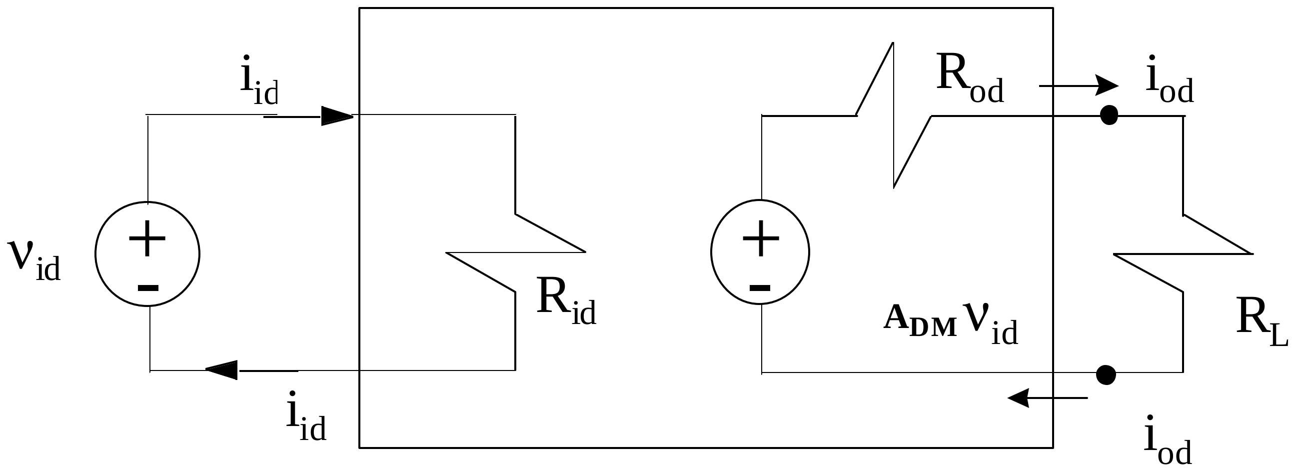
$$v_{ic} = \frac{v_{in} + 0}{2} = \frac{v_{in}}{2}$$

$$v_{o1} = v_{oc} + \frac{v_{od}}{2} = A_{CM} \cdot v_{in} + \frac{A_{DM}}{2} \cdot v_{in}$$

So for our example, $$v_{o1} = A_{CM} \cdot \frac{v_{in}}{2} + \frac{A_{DM}}{2} \cdot v_{in} = v_{in} \cdot \left(\frac{A_{CM}}{2} + \frac{A_{DM}}{2} \right)$$

Differential Pair (Cont.)

Differential Mode 2-Port :



$$A_{DM} = \frac{v_{od}}{v_{id}} \qquad R_{id} = \frac{v_{id}}{i_{id}} \qquad R_{od} = \frac{v_{od}}{i_{od}}$$

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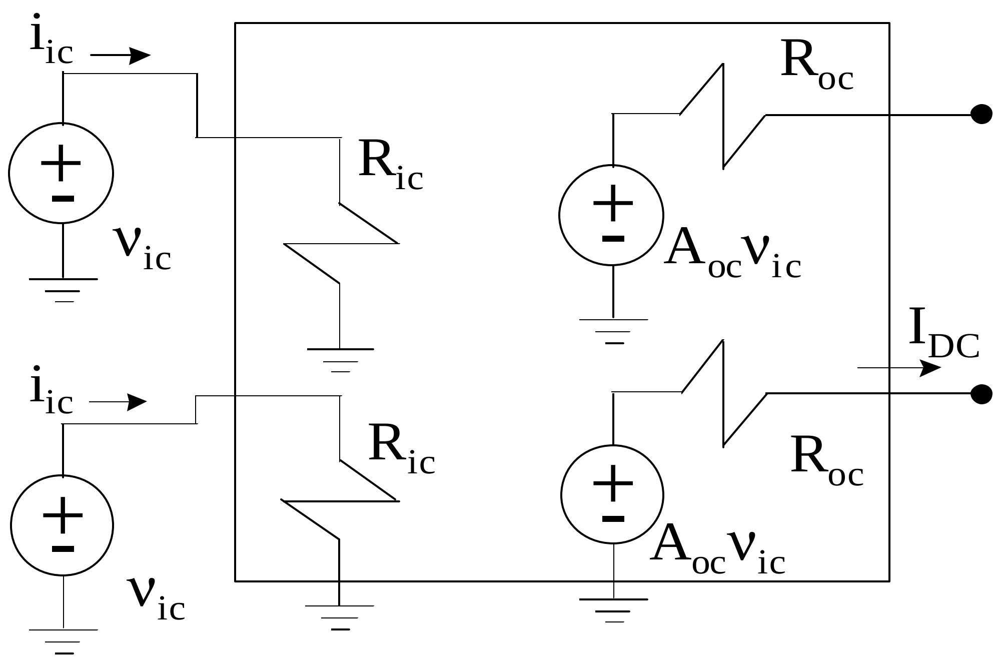
Differential Pair (Cont.)

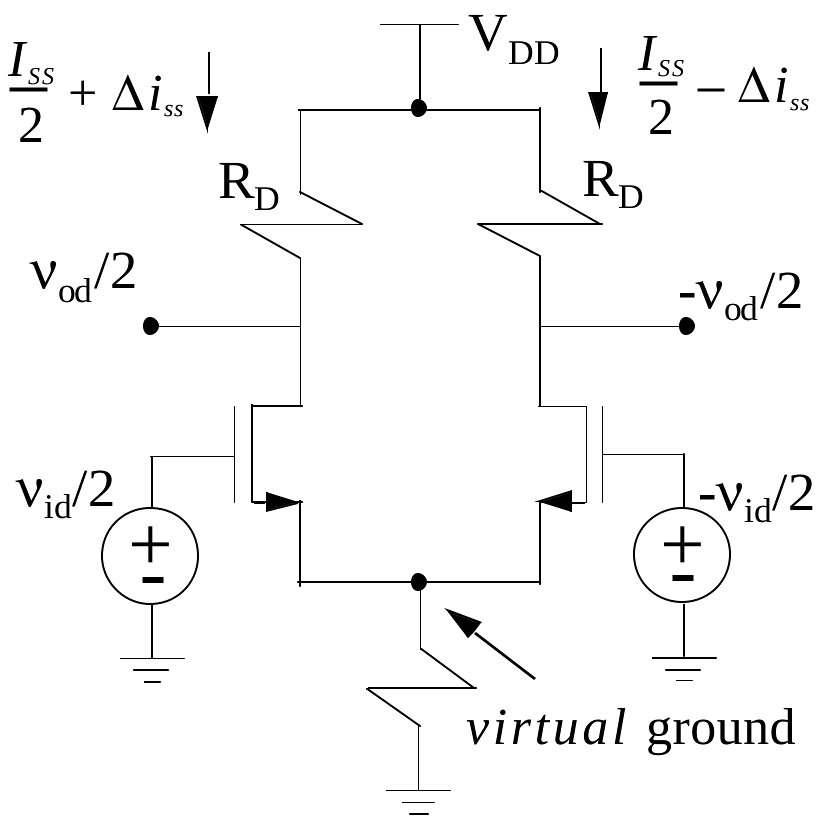
Common Mode 2-Port :



$$A_{CM} = \frac{v_{oc}}{v_{ic}} \qquad R_{ic} = \frac{v_{in}}{i_{ic}} \qquad R_{oc} = \frac{v_{in}}{i_{ic}}$$

Differential Pair - Half Circuits

Differential Mode Operation :



Differential Pair - Half Circuits (Cont.)

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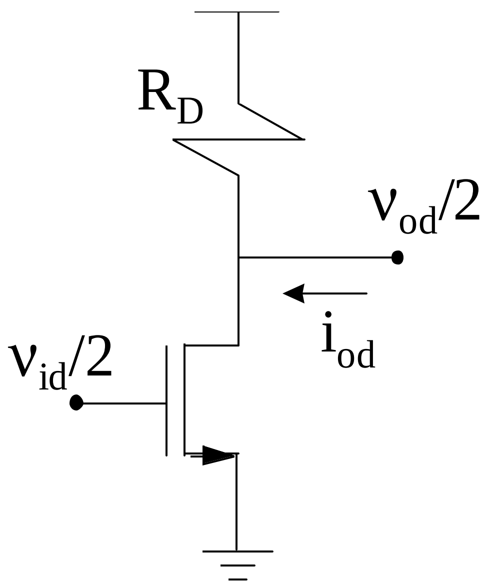
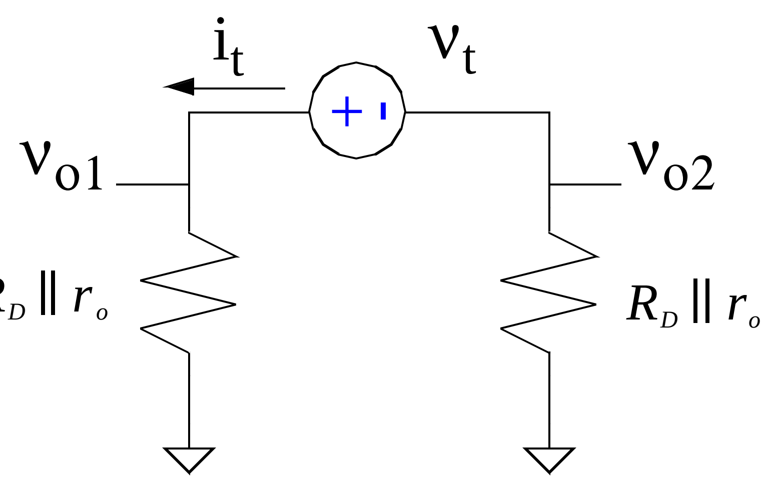
Half Circuit :



$$A_v = \frac{\frac{v_{od}}{2}}{\frac{v_{id}}{2}} = \frac{v_{od}}{v_{id}} = -g_m(R_D \| r_o)$$

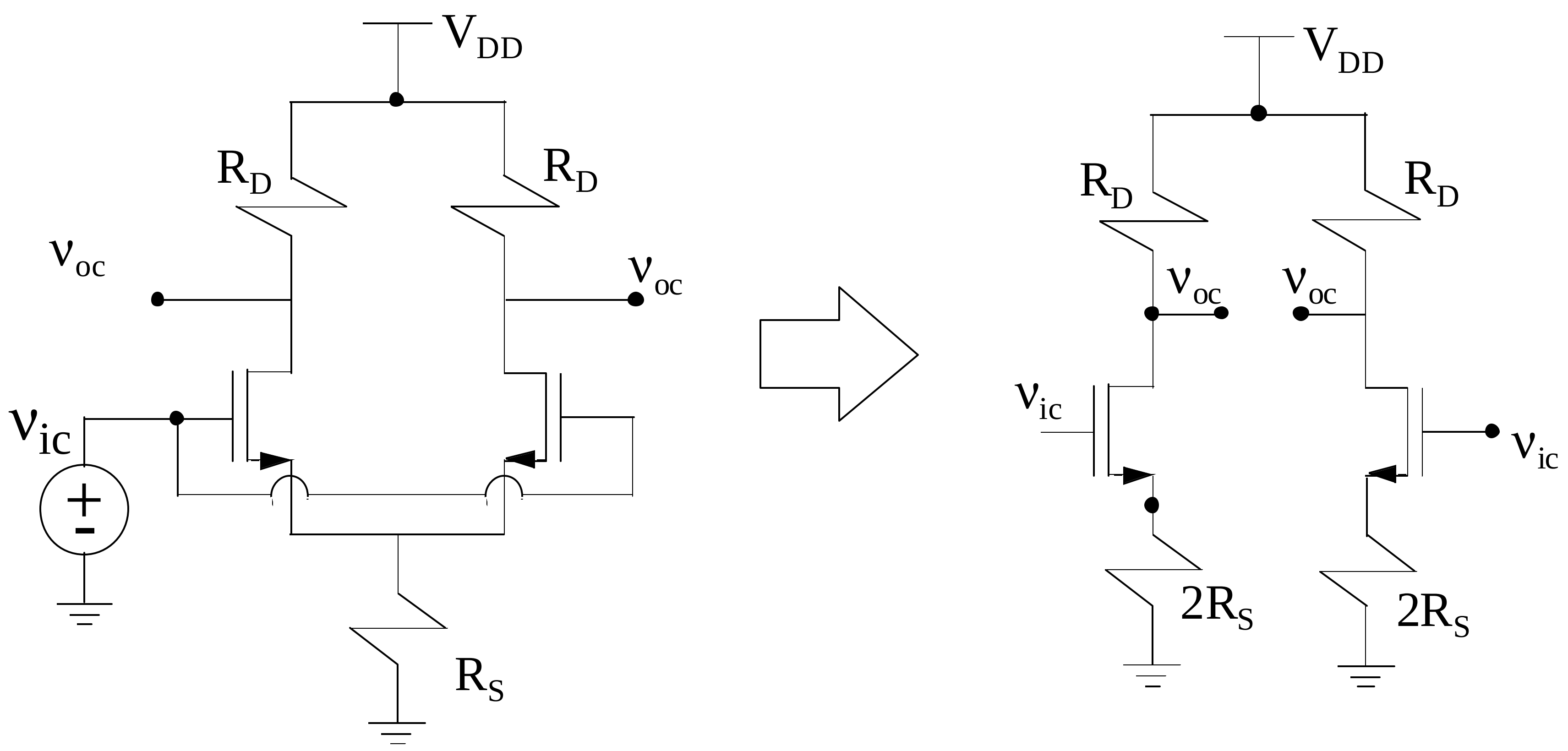
$$\frac{\frac{v_{od}}{2}}{i_{od}} = R_D \| r_o \qquad R_{id} = \infty$$



$$R_{od} = \frac{v_{od}}{i_{id}} = 2 \cdot (R_D \| r_o)$$

Differential Pair - Half Circuits (Cont.)

Common Mode Operation :



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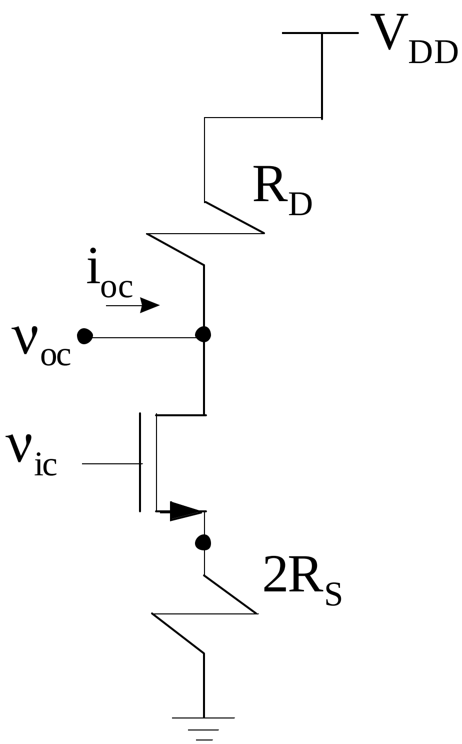
Differential Pair - Half Circuits (Cont.)

Half circuit :



$$A_{CM} = \frac{\nu_{oc}}{\nu_{ic}} = \left[\frac{g_m}{\underbrace{1 + 2 \cdot R_S \cdot (1+\chi) \cdot g_m}_{GM}}\right] \cdot [R_{OUT}]$$

$$R_{OUT} \approx r_o \cdot [1 + g_m \cdot (1+\chi) \cdot R_S] \parallel R_D \approx R_{OC}$$

CMRR and Single vs Double Ended Diff. Pairs

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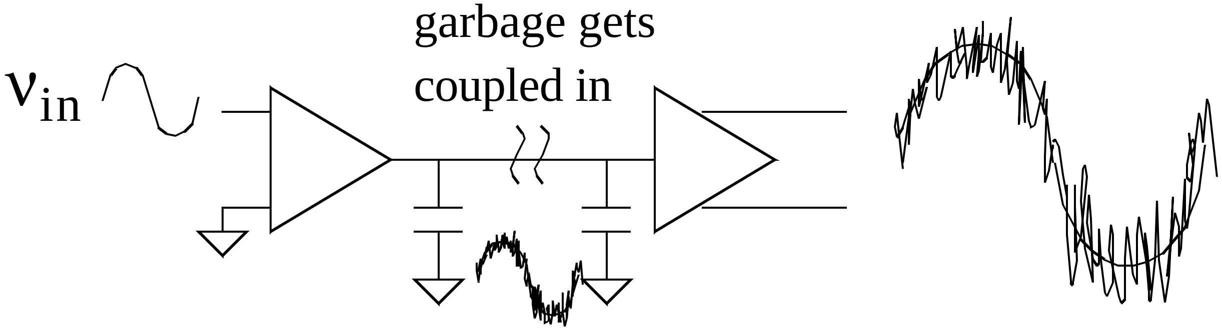
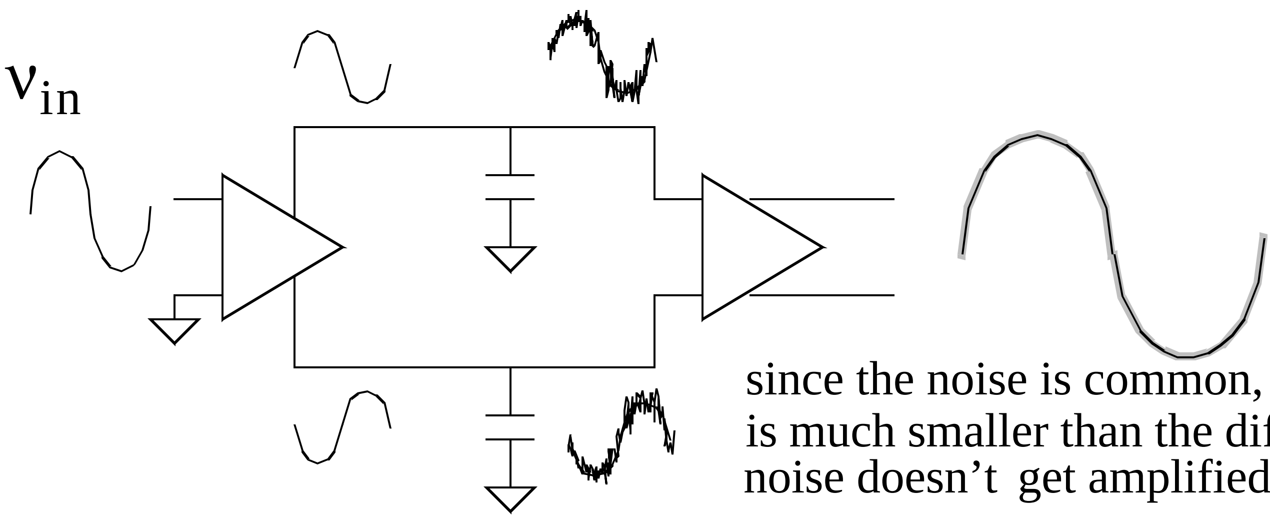
$$CMRR \equiv \text{ Common Mode Rejection Ratio (dB)} = 20 \cdot \log \frac{A_{CM}}{A_{DM}}$$

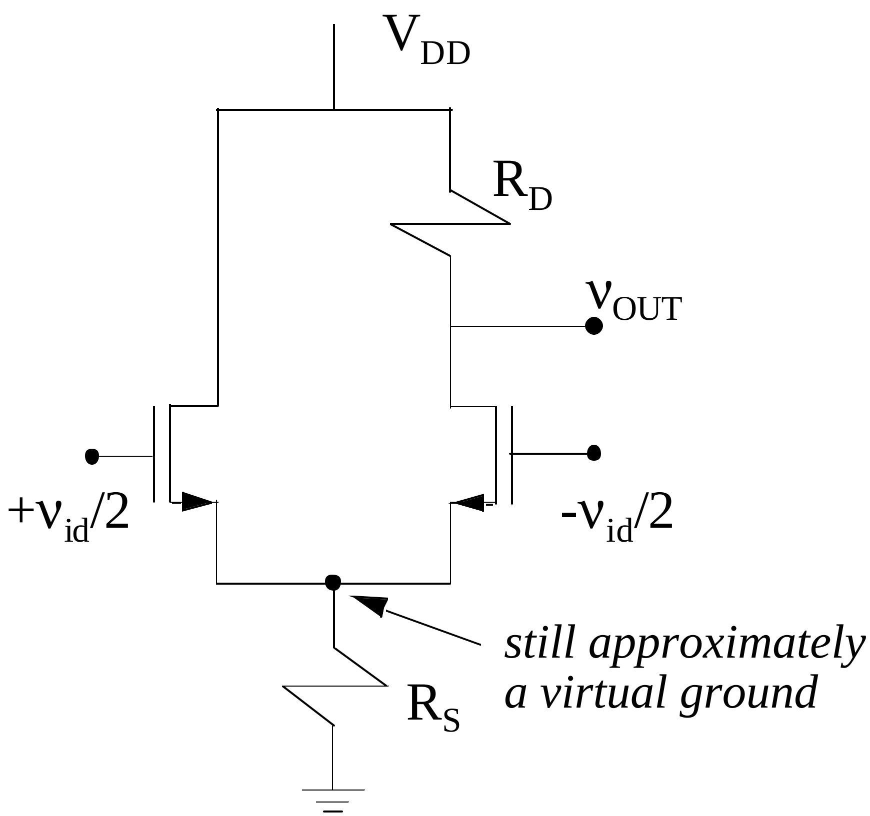


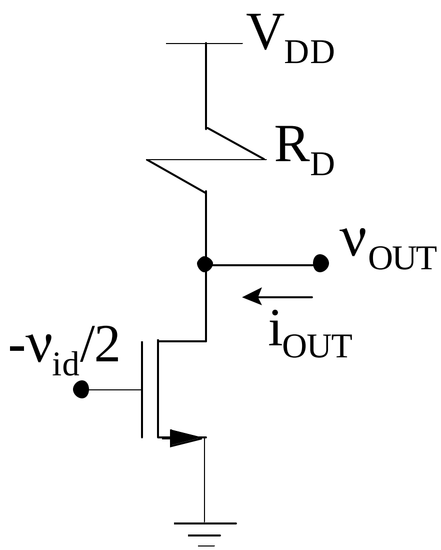
noise gets amplified
along with signal



since the noise is common, and the common mode gain is much smaller than the differential mode gain, the noise doesn't get amplified as much as the signal

Single Ended Differential Pair





$$\frac{v_{OUT}}{-\frac{v_{id}}{2}} = -g_m \cdot (R_D \| r_o)$$

$$\frac{v_{OUT}}{v_{id}} = \frac{g_m \cdot (R_D \| r_o)}{2}$$

$$R_{OUT} = \frac{v_{OUT}}{i_{OUT}} = R_D \| r_o$$