University of California
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College of Engineering
Department of Electrical Engineering
and Computer Science

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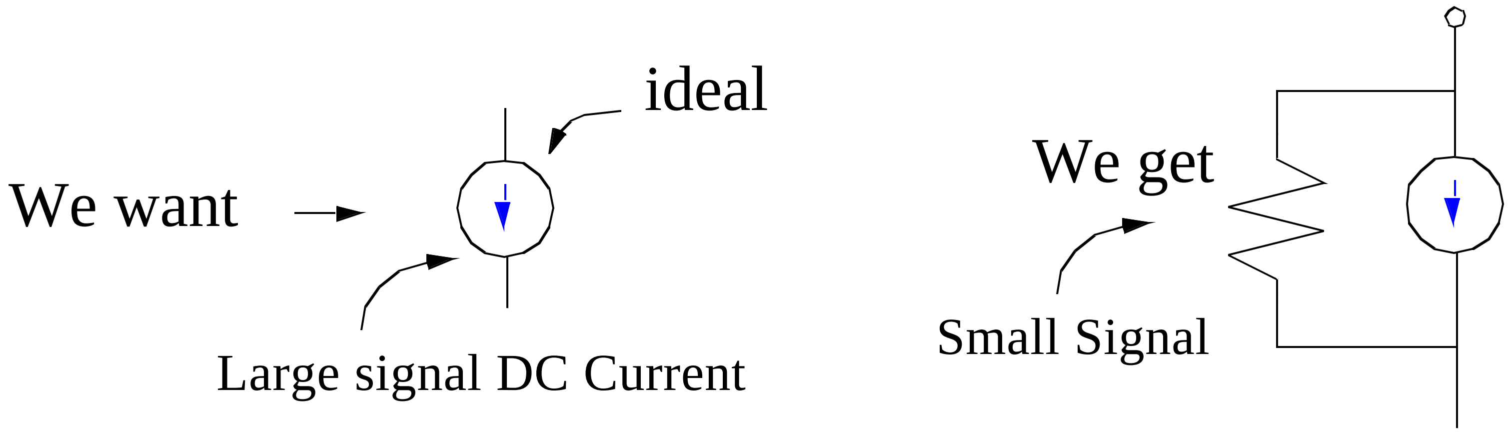
EECS140

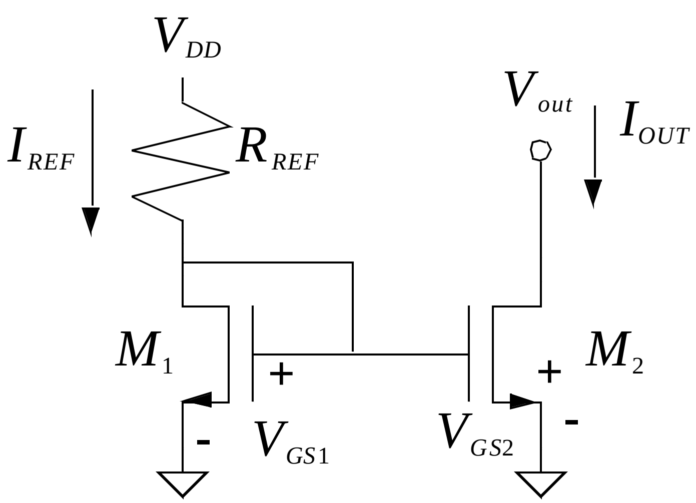
Analog Circuit Design

# Lectures on CURRENT SOURCES

# Current Sources




# Simple Source

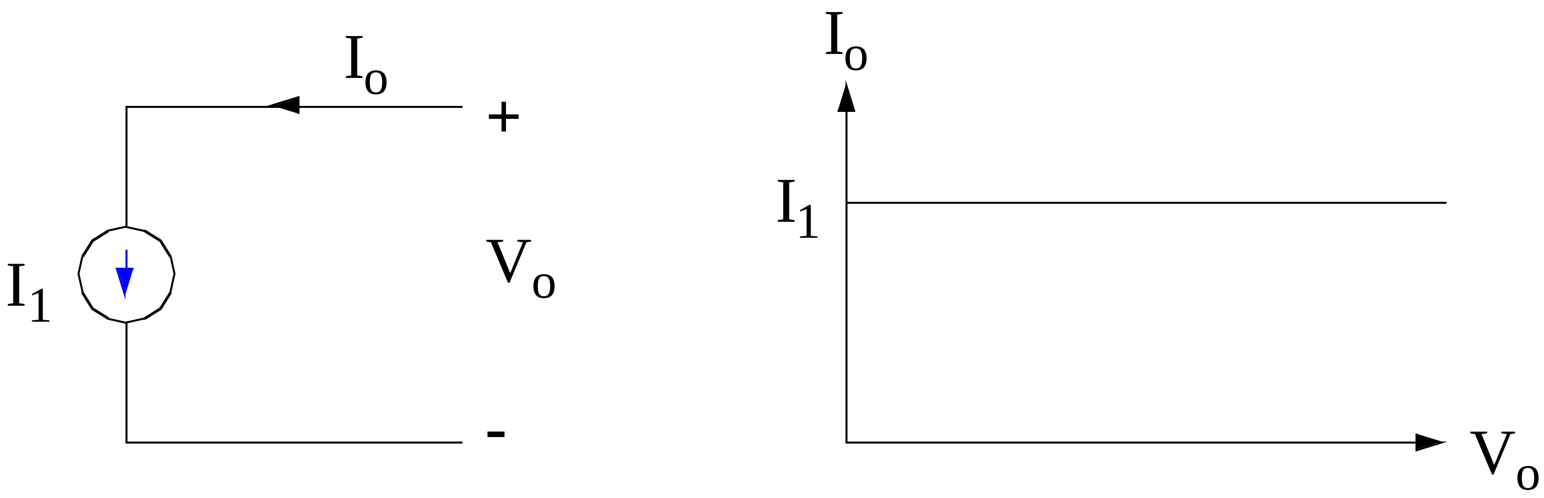



$$V_{OUT} > V_{GS} - V_T$$

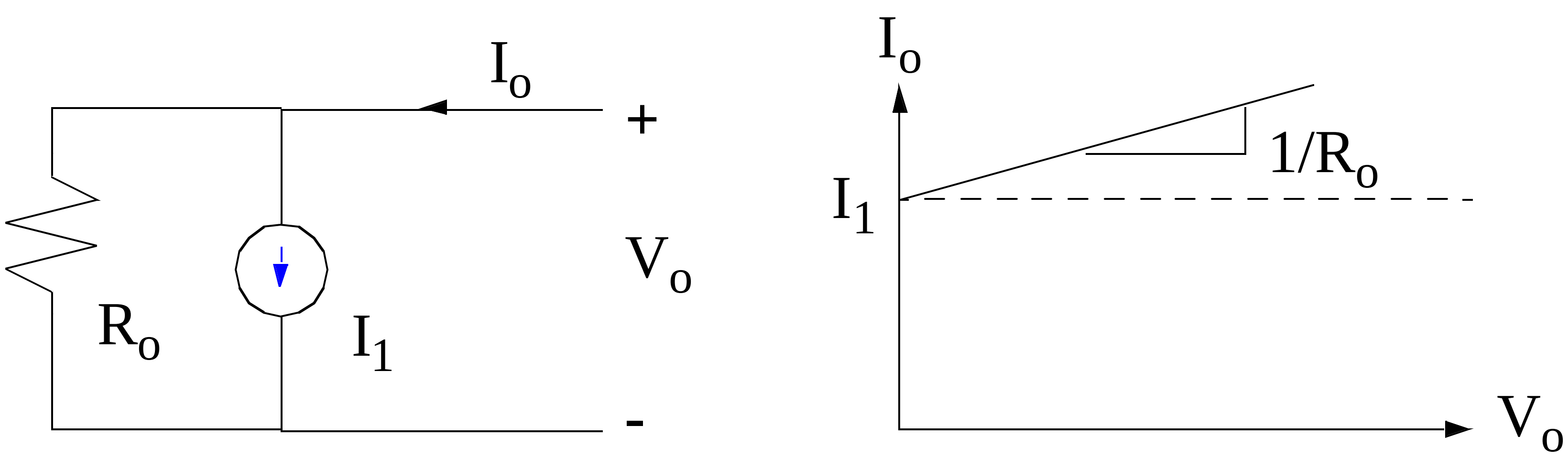
$$= V_{DSAT}$$

$$V_{DSAT} = \left( \frac{2 \cdot I_{DS}}{k' \cdot \frac{W}{L}} \right)^{\frac{1}{2}} \approx 0.1$$

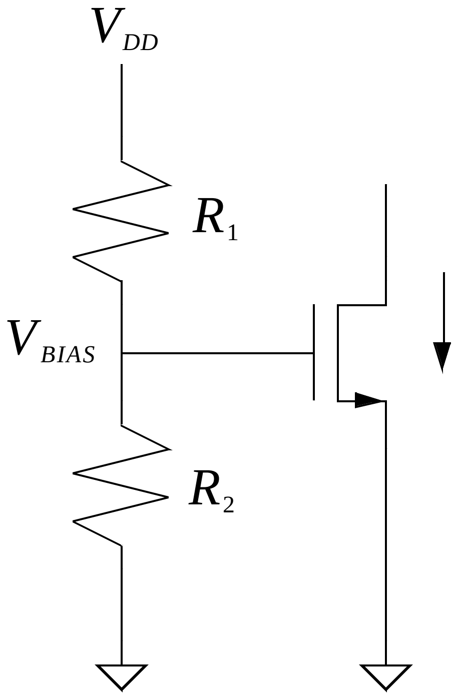
# Ideal Current Source

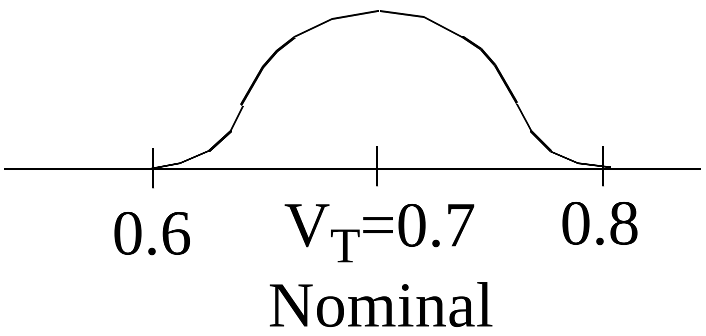



# Real Current Source



CS-2



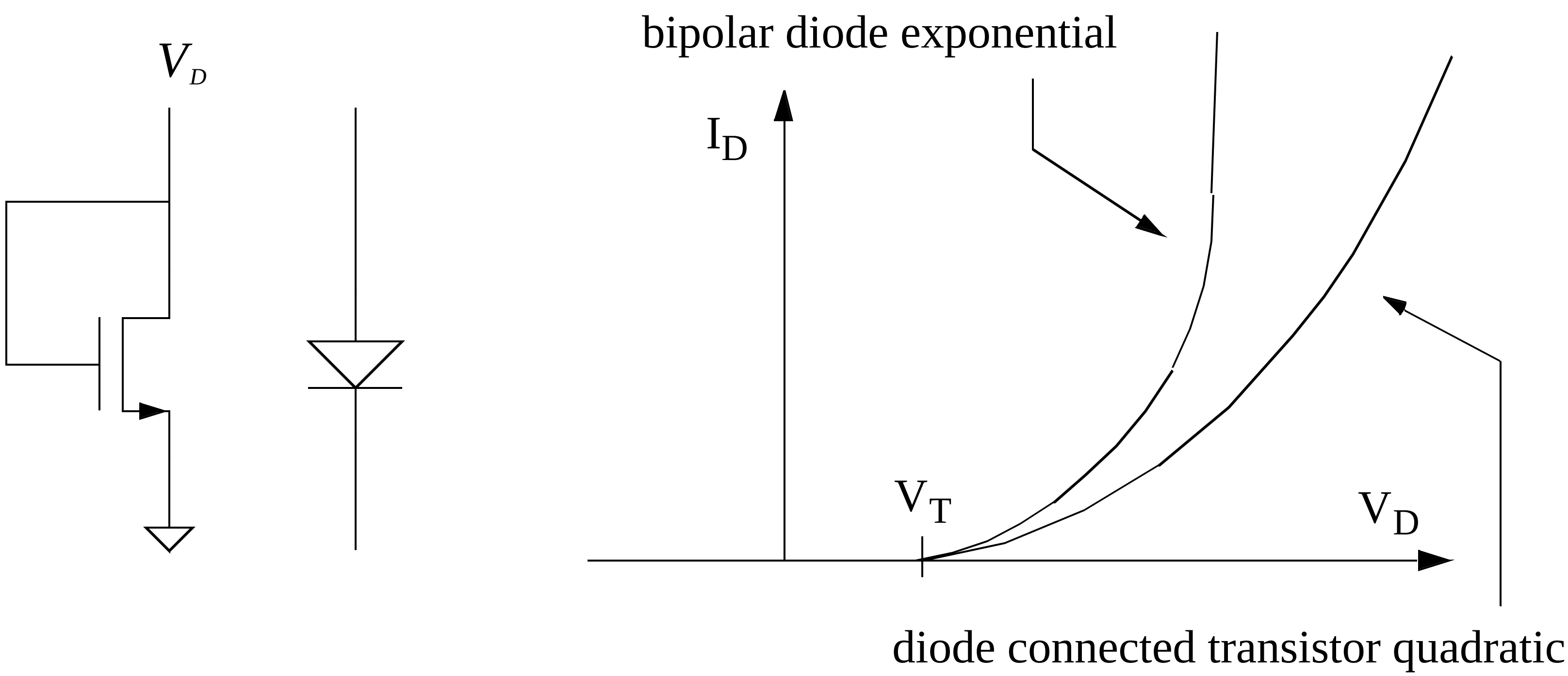




$$I_{DS1} = \frac{W}{L} \cdot \frac{k'}{2} \cdot (V_{BIAS} - V_T)^2$$

# Simple Source (Cont.)



## Diode Connected Transistor :




$V_{DS} > V_{GS} - V_T \}$ After we reach the point $V_D > V_T$, the transistor will always be in Sat.

# Simple Source (Cont.)



$$I_{DS1} = \left(\frac{W}{L}\right)_1 \cdot \frac{k'}{2} \cdot (V_{GS1} - V_T)^2$$

$$I_{DS2} = \left(\frac{W}{L}\right)_2 \cdot \frac{k'}{2} \cdot (V_{GS2} - V_T)^2$$

if $\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2$, then $I_{DS1} = I_{DS2}$

otherwise $I_{OUT} = I_{REF} \cdot \left[\frac{\left(\frac{W}{L}\right)_2}{\left(\frac{W}{L}\right)_1}\right]$

$I_{REF} = I_{DS1}$ , $I_{OUT} = I_{DS2}$

# Simple Source (Cont.)



## Current Calculation :

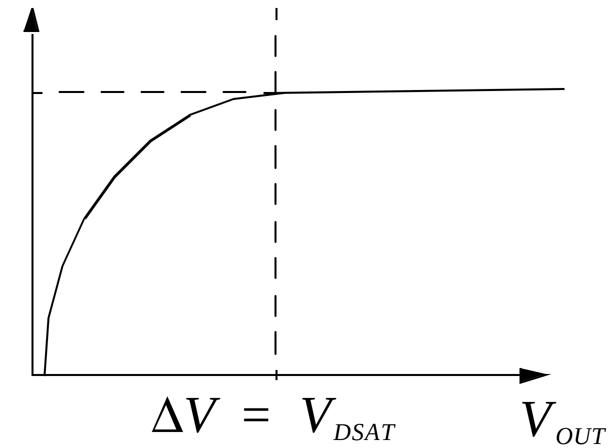
**Analysis :**

$$I_{DS1} = I_{REF} = \frac{V_{DD} - V_{GS1}}{R_{REF}}$$

$$V_{GS1} = V_T + \left( \frac{2 \cdot I_{DS1}}{k' \cdot \frac{W}{L}} \right)^{\frac{1}{2}} = V_T + \Delta V$$

$$I_{REF} = V_{DD} - V_T - \left( \frac{2 \cdot I_{REF}}{k' \cdot \frac{W}{L}} \right)^{\frac{1}{2}}$$

Iterative Solution !



# Simple Source (Cont.)

**Design :**

$$R_{REF} = \frac{V_{DD} - V_{GS1}}{I_{REF}} = \frac{V_{DD} - V_T - \left( \frac{2 \cdot I_{REF}}{k' \cdot \frac{W}{L}} \right)^{\frac{1}{2}}}{I_{REF}}$$

$$I_{REF} = 10\mu A\ ,\ V_{DD} = 5\ ,\ V_T = 0.7$$

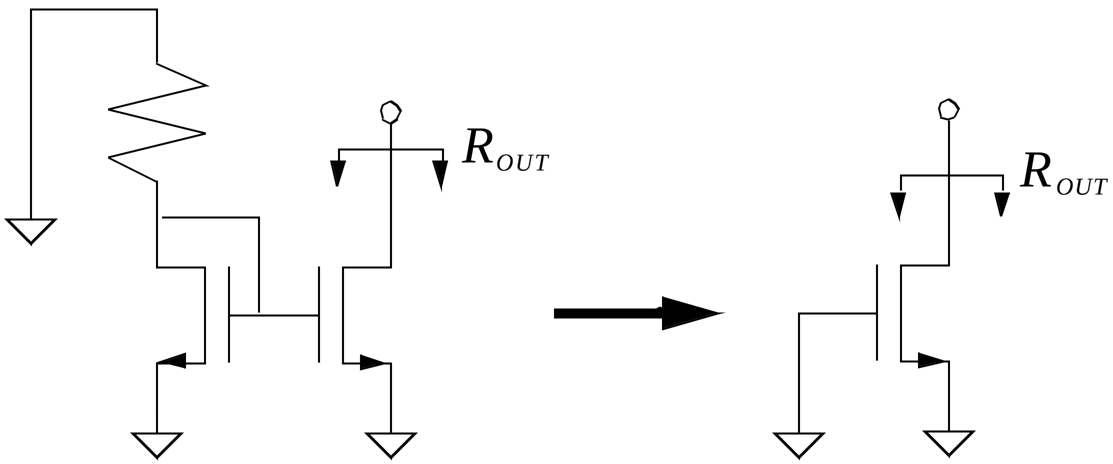
$$k' = 90 \times 10^{-6}$$

$R_{REF} = 415k\Omega$ ( Pretty Big! )

# Simple Source (Cont.)



## Small Signal :



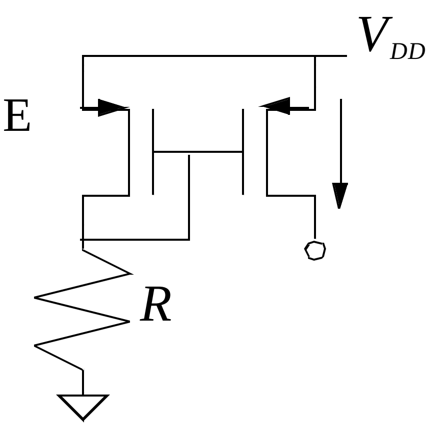

$$R_{OUT} = r_o = \frac{1}{\lambda \cdot I_{OUT}}$$

$$I_{OUT} = 10\mu A$$

$$\lambda = 0.01$$

$$R_{OUT} = 10M\Omega$$

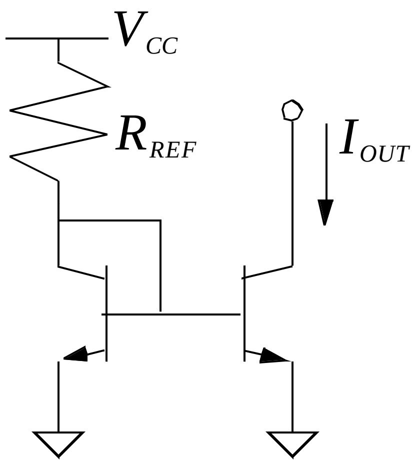
NMOS → current SINK
PMOS → current SOURCE



# Simple Source (Cont.)



## Bipolar :




$$I_{OUT} \approx \frac{V_{CC} - V_{BE(ON)}}{R_{REF}}$$

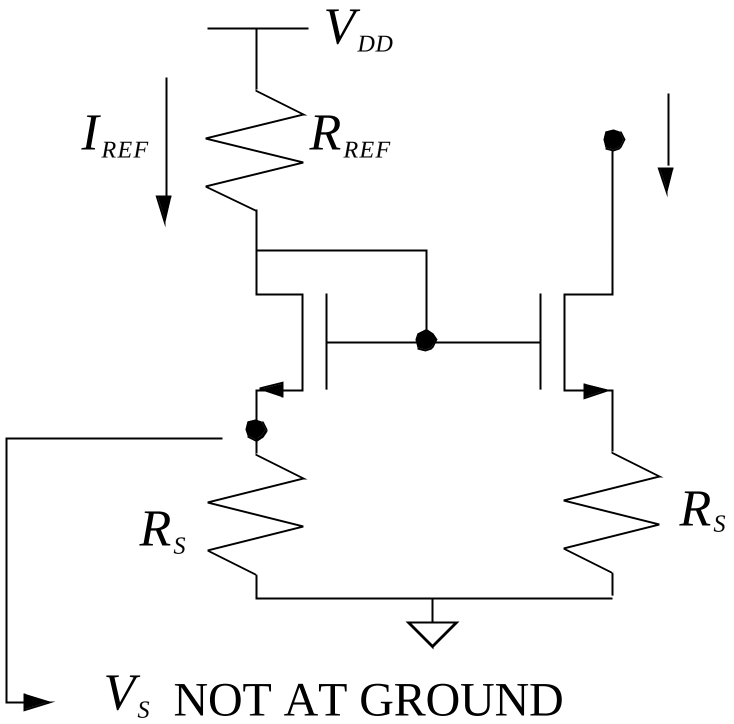
$$V_{BE(ON)} \approx 0.6$$

$$R_{OUT} = \frac{V_A}{I_{OUT}}$$

# Simple Source (Cont.)

How to make $R_{OUT}$ better (ie. larger) ?

Degeneration?




$$V_{OUT} > V_{DSAT} + I_{OUT} \cdot R_S$$

$$R_{OUT} = r_o \cdot [1 + (1 + x) \cdot g_m \cdot R_S]$$

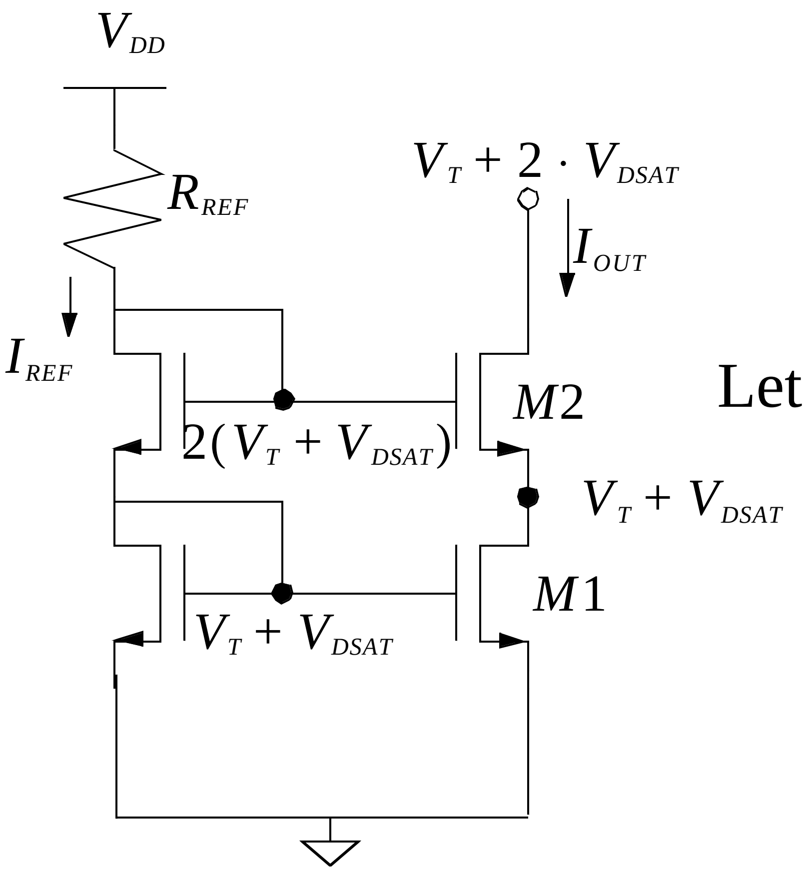
$$V_T = V_{To} + \gamma \cdot [(V_{SB} + 2 \cdot \phi_f)^{\frac{1}{2}} - (2 \cdot \phi_f)^{\frac{1}{2}}]$$

Not a very efficient way to get high $R_{OUT}$ too much area for resistor.

Better method is to use transistors instead of resistors.

# Cascode Source






Let $\gamma = 0$ or tie all wells to sources

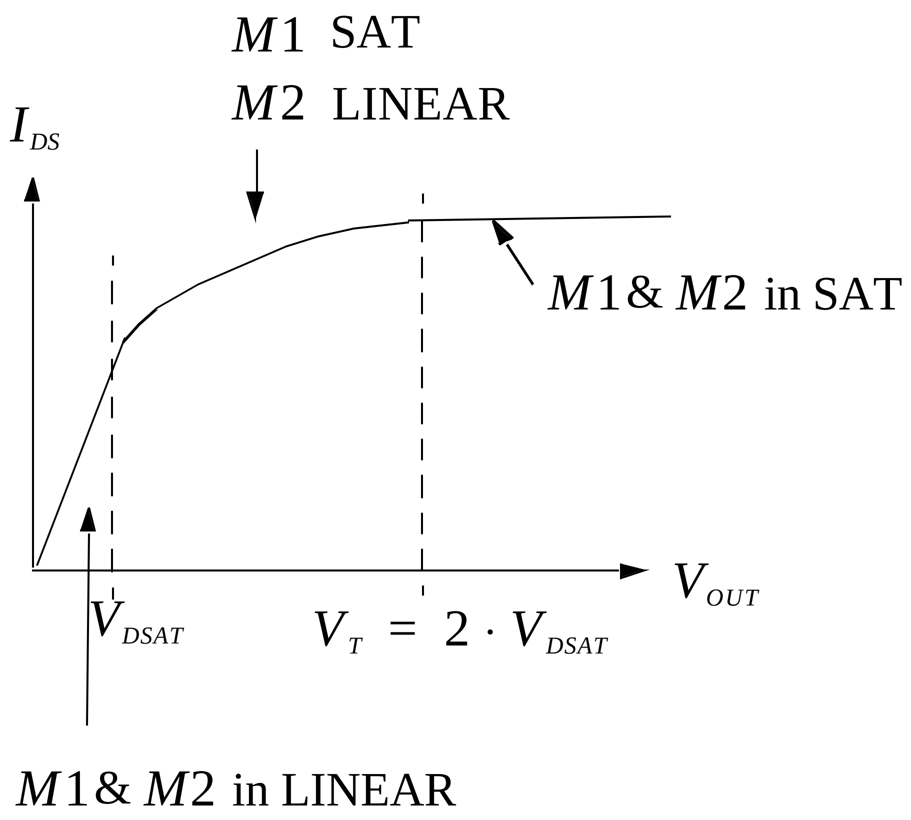
for EOS

# Cascode Source (Cont.)

$$V_{GS} = V_T + V_{DSAT}$$

$$\Delta V = \left( \frac{2 \cdot I_{DS}}{k' \cdot \frac{W}{L}} \right)^{\frac{1}{2}}$$

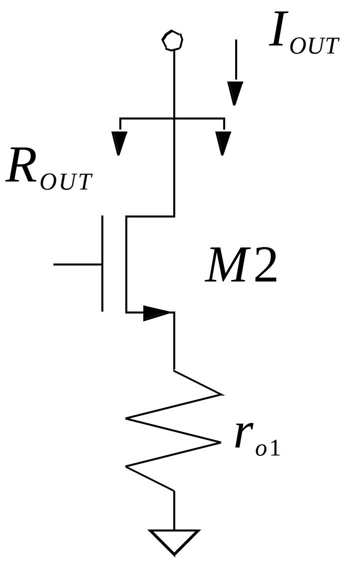
$$I_{REF} = \frac{V_{DD} - 2 \cdot (V_T - V_{DSAT})}{R_{REF}}$$



# Cascode Source (Cont.)



## Rout for Cascode Source :




$\lambda = 0.01$

$\gamma = 0$

$I_{DS} = 10\mu A$

$\frac{W}{L} = 5$

$$R_{OUT} = r_{o2} \cdot [1 + (1 + \chi_2) \cdot g_{m2} \cdot r_{o1}]$$

$$\approx (1 + \chi_2) \cdot g_{m2} \cdot r_{o1} \cdot r_{o2}$$

$$r_o = \frac{1}{\lambda \cdot I_{DS}} \qquad g_{m2} = \left(2 \cdot k' \cdot \frac{W}{L} \cdot I_{DS}\right)^{\frac{1}{2}} \approx 10^{-4}$$

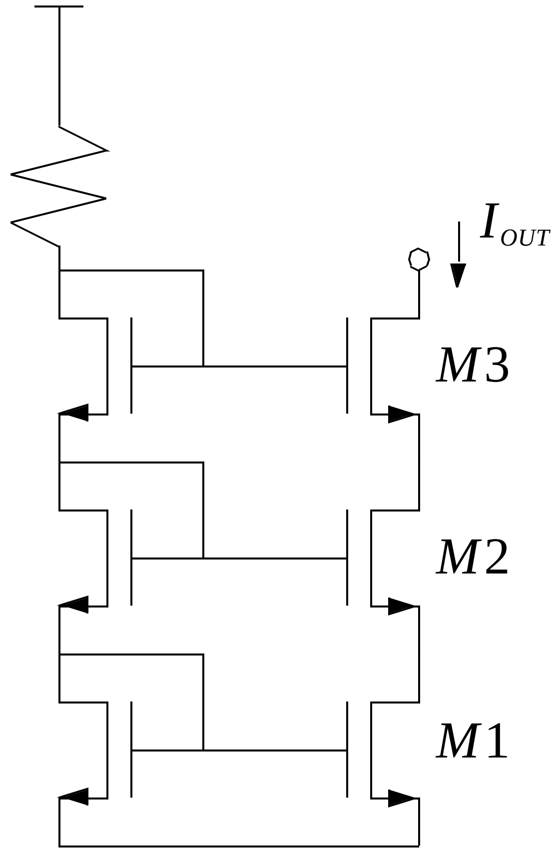
$$R_{OUT} \approx 10^{-4} \cdot \left(\frac{1}{0.1 \cdot 10^{-5}}\right)^2 \approx 10^{10}\Omega$$

$$g_{m2} = \frac{2 \cdot I_{OUT}}{V_{DSAT}} \qquad V_{DSAT} \approx 0.2V$$

$$R_{OUT} = \frac{(1 + \chi_2)}{\lambda^2} \cdot \frac{2}{(V_{DSAT}) \cdot (I_{OUT})}$$

# Cascode Source (Cont.)

## Triple Cascode :




$$R_{OUT} = (1 + \chi_2) \cdot (1 + \chi_3) \cdot g_{m2} \cdot g_{m3} \cdot r_{o1} \cdot r_{o2} \cdot r_{o3}$$

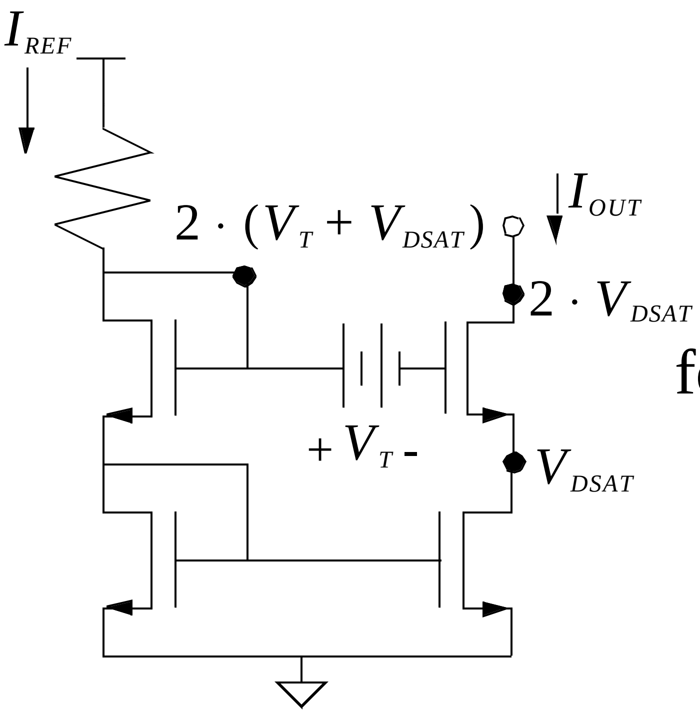
HUGE!!

# Cascode Source (Cont.)

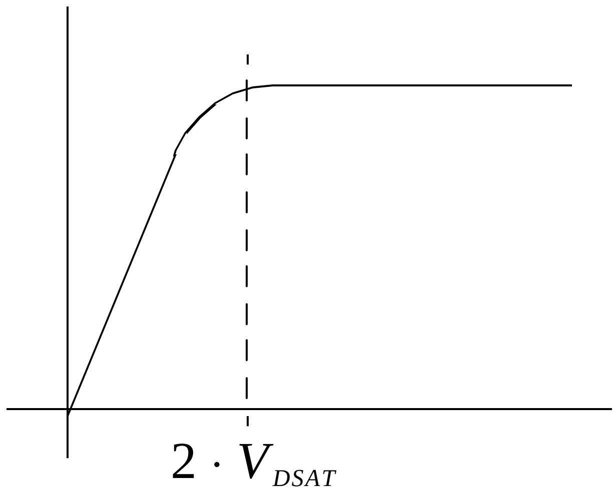


Problem with Cascode is the $V_T + 2 \cdot V_{DSAT}$ drop required for saturation.

High Swing Cascode is the solution

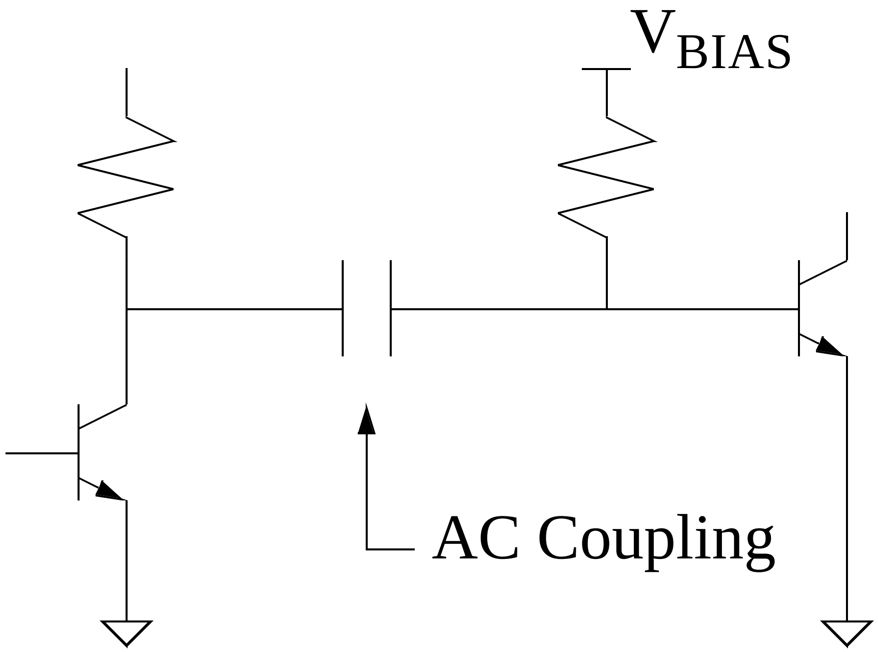



for EOS



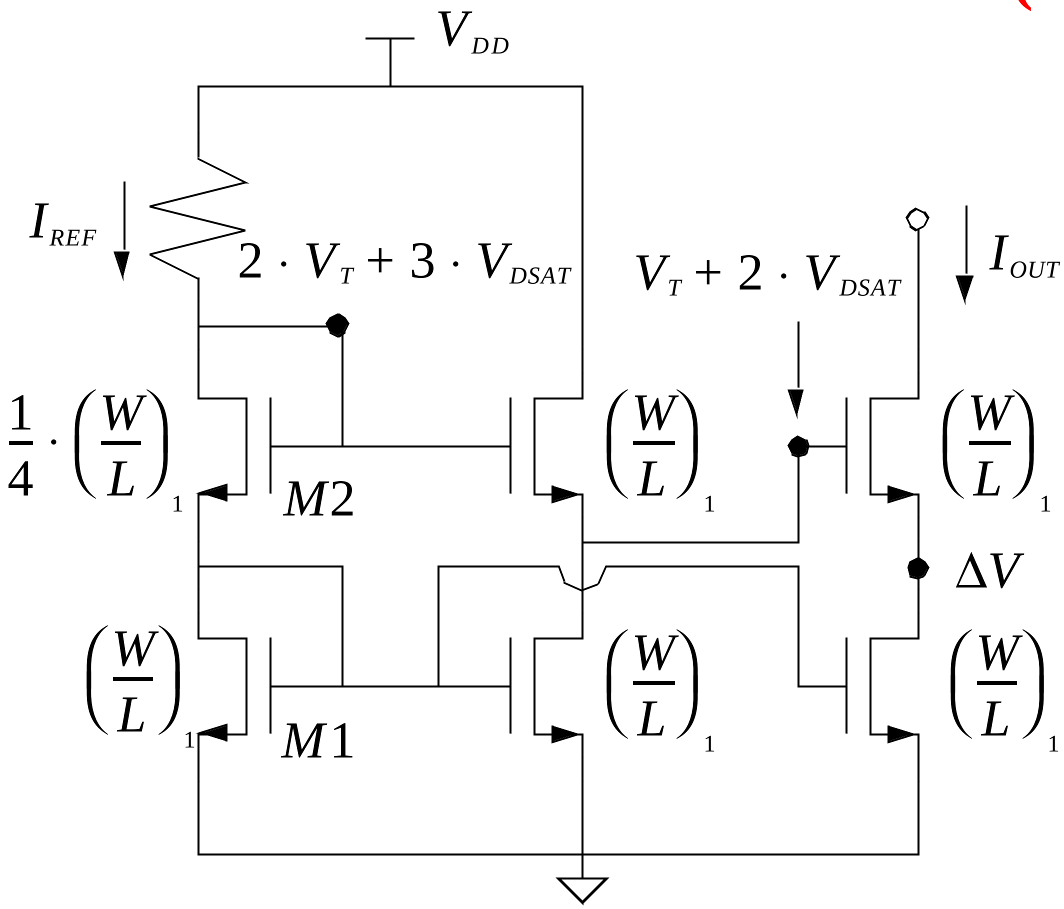
# Cascode Source (Cont.)

## How to implement Battery?



# Cascode Source (Cont.)






$$V_{DSAT1} = \left(\frac{2 \cdot I_{REF}}{k' \cdot \left(\frac{W}{L}\right)_1}\right)^{\frac{1}{2}}$$

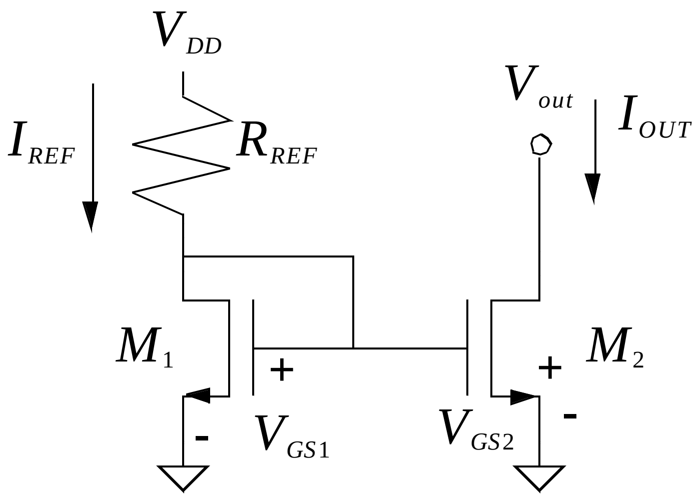
$$V_{DSAT2} = \left(\frac{2 \cdot I_{REF}}{k' \cdot \frac{\left(\frac{W}{L}\right)_1}{4}}\right)^{\frac{1}{2}}$$

$$= 2 \cdot V_{DSAT1}$$

$$\approx 2 \cdot V_{DSAT}$$

# Cascode Source (Cont.)

## Power Reduction :




If we want $I_{out}$ = 1mA,

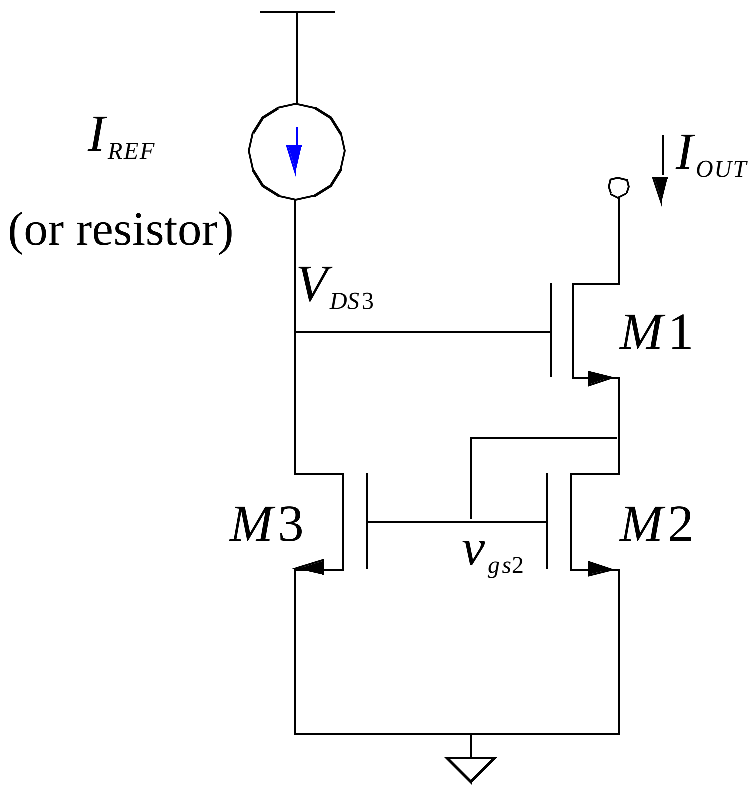
then we can either:

a) Set $(W/L)_1 = (W/L)_2$ and $I_{REF} = I_{out}$

b) Ratio $(W/L)_1$ and $(W/L)_2$ so that $I_{REF} < I_{out}$, e.g. set $(W/L)_2 = 100(W/L)_1$ so that $I_{REF} = 10\mu A$

The tradeoff is between power and area.

# Wilson Source




*Assume* $\gamma = 0$

$$I_{OUT} = I_{REF}$$

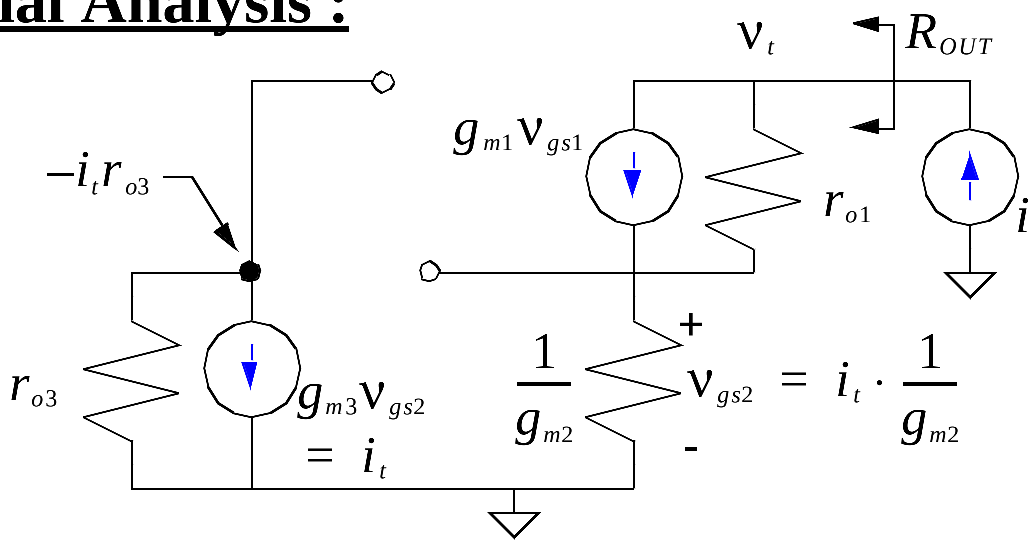
$$V_{DS3} = 2 \cdot (V_T + V_{DSAT})$$

Equivalent to Cascode

# Wilson Source (Cont)

CS-19

## Small Signal Analysis :




$$g_{m1} = g_{m2} = g_{m3} = g_m$$

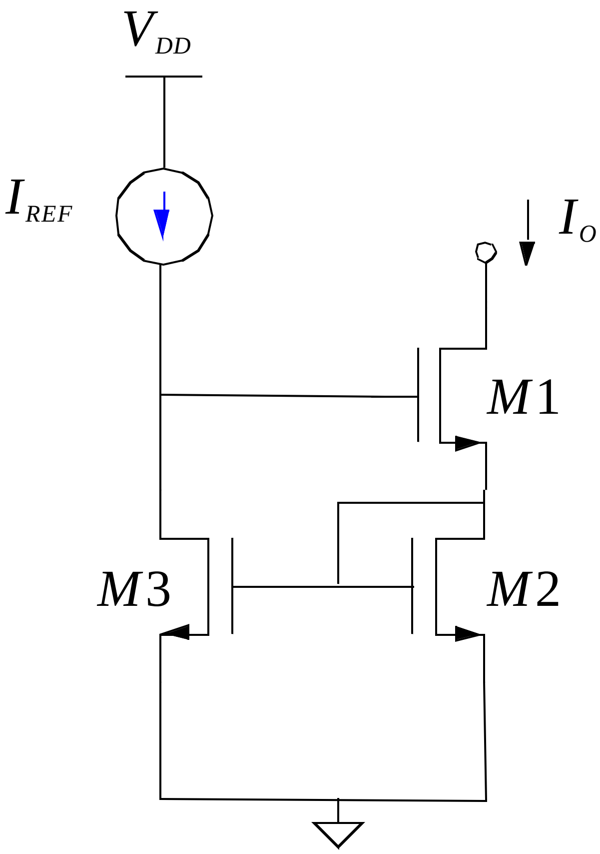
$$v_t = i_t \cdot r_o - g_{m1} \cdot v_{gs1} \cdot r_{o1} + \cancel{i_t \cdot \frac{1}{g_{m2}}}$$

$$v_{gs1} = -i_t \cdot r_{o3} - \cancel{i_t \frac{1}{g_{m2}}}$$

$$v_t = i_t \cdot (r_{o1} + g_m \cdot r_{o1} \cdot r_{o3}) \quad ; \quad R_{OUT} = r_{o1} \cdot (1 + g_m \cdot r_{o3})$$

# Wilson Current Source






$$I_{DS1} = \frac{1}{2} \cdot \mu \cdot C_{OX} \cdot \frac{W}{L} \cdot (V_{GS} - V_T)^2 \cdot (1 + \lambda \cdot V_{DS})$$

with $\lambda = 0$

$$I_O = I_{REF}$$

assuming same W/L and

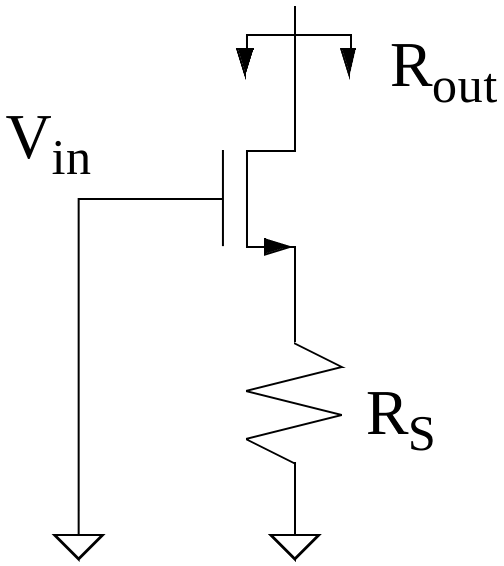
$$V_{GS3} = V_{GS2}$$

# Wilson Current Source (Cont)



## Rout :

For this we must apply the small signal analysis, it is not correct to calculate $R_{out}$ as for a common source with degeneration :

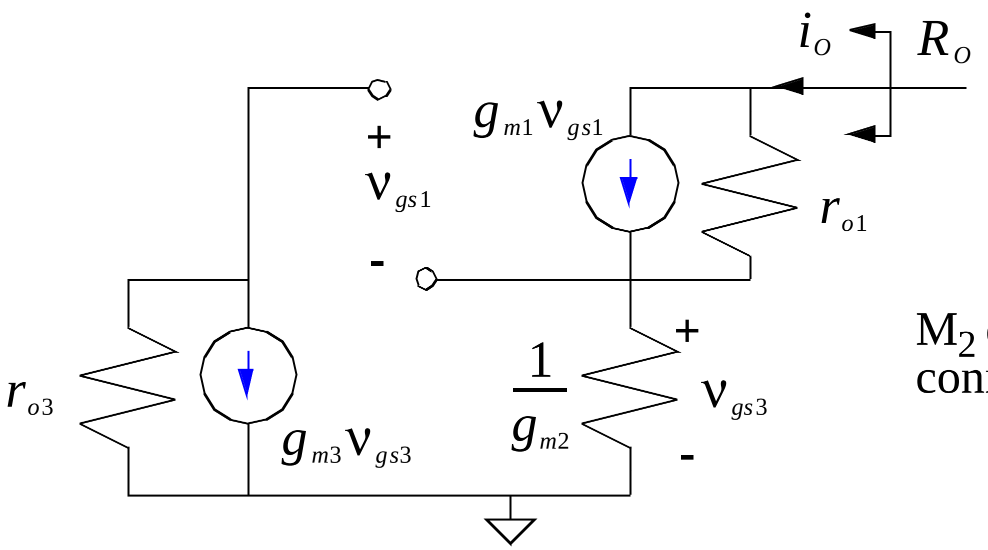



NO!

$$R_{OUT} = r_o \cdot (1 + g_m \cdot R_S)$$

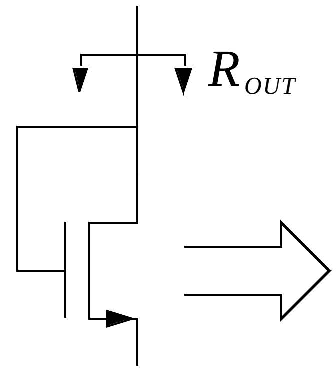
# Wilson Current Source (Cont)






assume $g_{m1} = g_{m2} = g_{m3} = g_m$

$M_2$ diode connected :



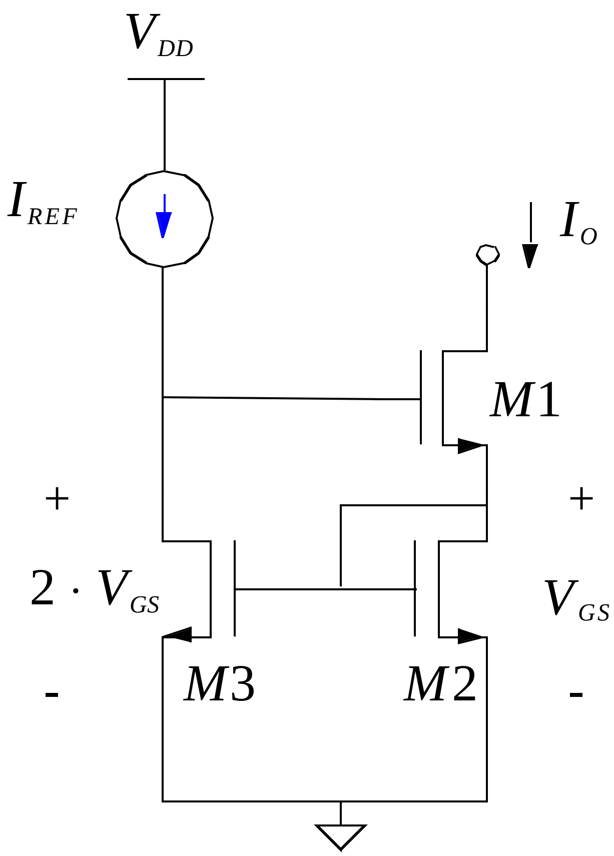

$$\Rightarrow \frac{1}{g_m}$$

$$i_O = g_m \cdot v_{gs1} + \frac{v_O - v_{gs3}}{r_{o1}}$$

$$g_m \cdot v_{gs1} + \frac{v_O - v_{gs3}}{r_{o1}} = v_{gs3} \cdot g_{m2} \quad \Rightarrow \quad \frac{v_O}{i_O} = r_{o1} \cdot (2 + g_m \cdot r_{o3})$$

$$\frac{v_{gs1} + v_{gs3}}{r_{o3}} + g_m \cdot v_{gs3} = 0$$

# Wilson Current Source (Cont)




$$I_{DS1} = \frac{1}{2} \cdot \mu \cdot C_{OX} \cdot \frac{W}{L} \cdot (V_{GS} - V_T)^2 \cdot (1 + \lambda \cdot V_{DS})$$

with $\lambda \neq 0$

$$V_{DS2} = V_{GS2} = V_{GS}$$

$$V_{DS3} = 2 \cdot V_{GS}$$

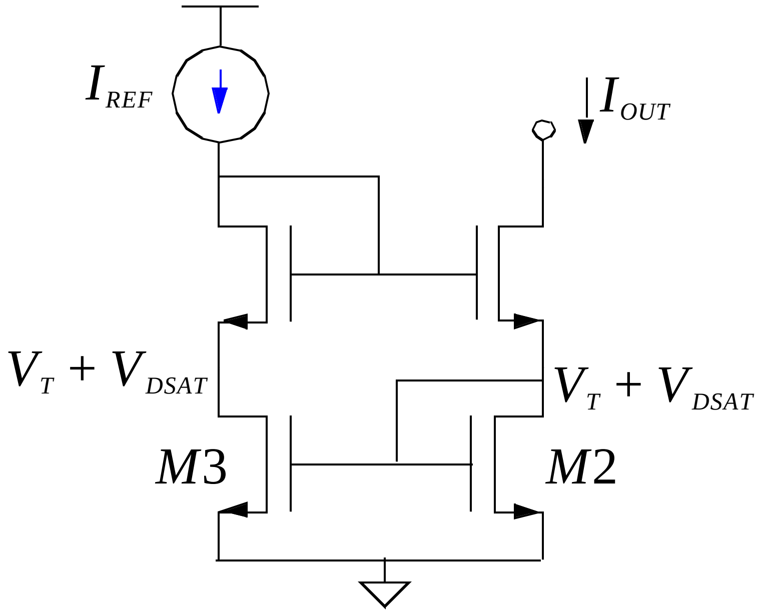
$$\frac{I_{DS2}}{I_{DS3}} = \frac{1 + \lambda \cdot V_{DS2}}{1 + \lambda \cdot V_{DS3}} = \frac{1 + \lambda \cdot V_{GS}}{1 + 2 \cdot \lambda \cdot V_{GS}}$$

$$I_O \neq I_{REF}$$

# Wilson Current Source (Cont)



To solve the matching problem,

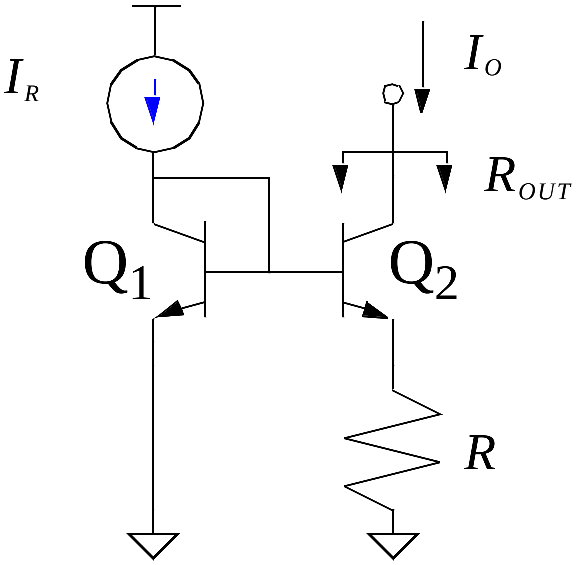



$$I_O = I_{REF}$$

$$R_O = \text{ same as before}$$

# Widlar Current Source






$R_{OUT} =$

for emitter degenerated BJT

$$R_{OUT} = r_o \cdot [1 + g_m \cdot (R \| r_\pi)]$$

$I_O =$

$$V_{BE1} = V_{BE2} + I_O \cdot R$$

for BJT

$$V_{BE} = V_T \cdot \ln\frac{I_C}{I_S} \quad \text{so,}$$

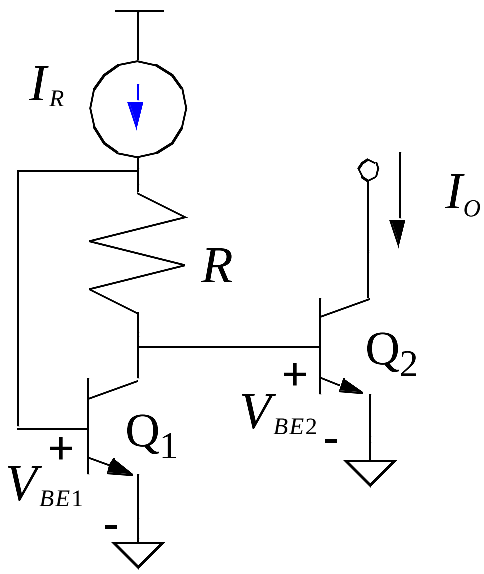
$$V_T \cdot \ln\frac{I_R}{I_{S1}} = V_T \cdot \ln\frac{I_O}{I_{S2}} + I_O \cdot R$$

$$I_O \cdot R = V_T \cdot \ln\left(\frac{I_R}{I_O} \cdot \frac{I_{S2}}{I_{S1}}\right)$$

$$I_O = \frac{V_T}{R} \cdot \ln\frac{I_R}{I_O} \quad \text{solve iteratively}$$

# Low Current Bias Circuit (bipolar)






Very low $I_O$ ~ nanoamps

$$V_{BE1} = V_{BE2} + I_R \cdot R$$

$$V_T \cdot \ln\frac{I_R}{I_{S1}} = V_T \cdot \ln\frac{I_O}{I_{S2}} + I_R R$$

let $I_{S1} = I_{S2}$

$$I_O = I_R \cdot \exp\left(-\frac{I_R \cdot R}{V_T}\right)$$

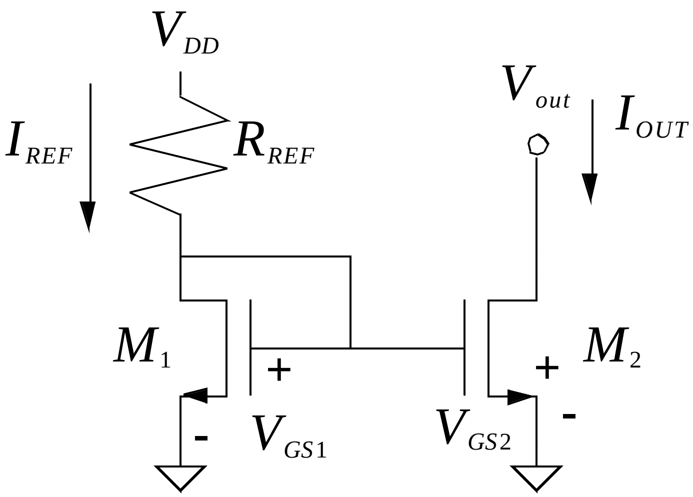
for $I_R = 10\mu A \quad R = 12k\Omega$

$$I_O = 100nA$$

allows low $I_o$ with reasonable values of R

$$R_{OUT} = r_{o2}$$

# Supply Independent Biasing




$$I_{OUT} = I_{REF}$$

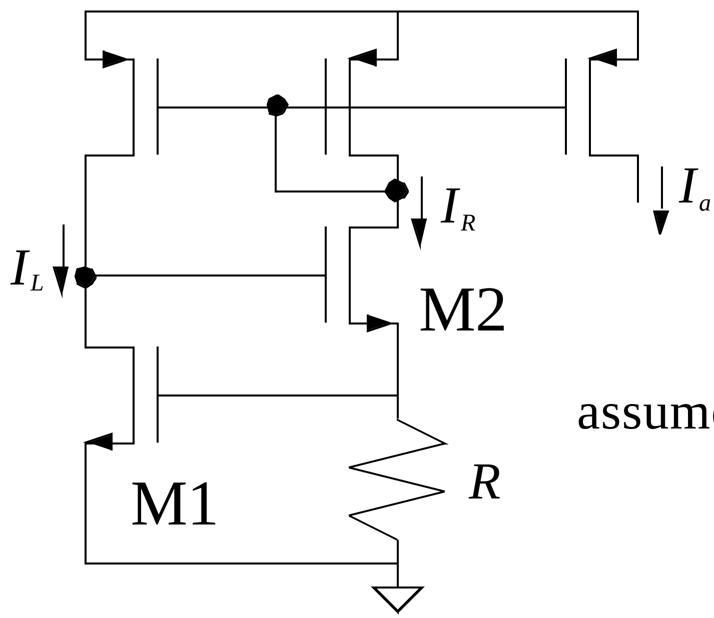
$$I_{REF} = \frac{V_{DD} - V_{GS}}{R}$$

Hence, $I_{REF}$ depends on the supply voltage $V_{DD}$. If the supply is a battery or similar device, then this will change over time, causing the reference current to also vary with time

# Supply Independent Biasing (Cont.)



## Vt - Referenced Self-Biased Circuit :




$$V_{GS1} = I_R \cdot R = V_{DSAT}$$

$$= \sqrt{\frac{2 \cdot I_L}{\mu \cdot C_{OX} \cdot \left(\frac{W}{L}\right)_1}} + V_{T1}$$

assume $\left(\frac{W}{L}\right)_1 >> 1$ , Vdsat is then negligible

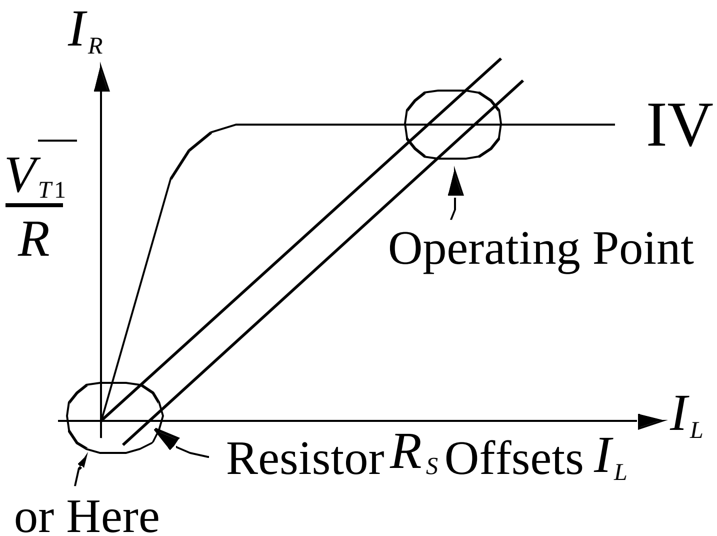
$$I_L = I_R$$

$$I_R \approx \frac{V_{T1}}{R}$$

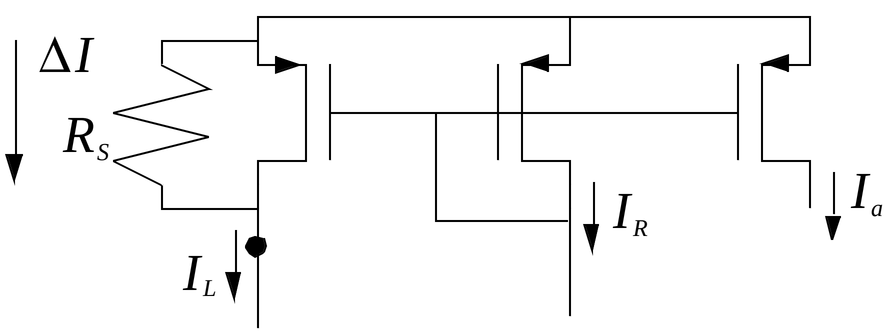
$$I_O = I_R = \frac{V_{T1}}{R}$$

# Supply Independent Biasing (Cont.)

For most self-biased circuits, there is a startup problem:



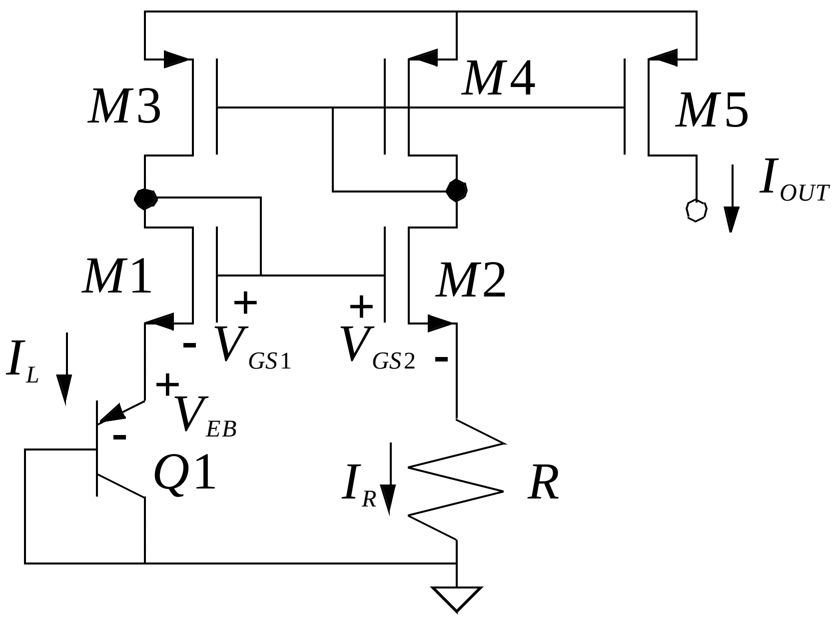

Soln : add $R_S$ to top mirror




$$I_L = I_R + \Delta I$$

# Supply Independent Biasing (Cont.) Vbe referenced

CS-31




$$I_{OUT} = I_L$$

$$V_{EB1} + V_{GS1} = V_{GS2} + I_R \cdot R$$

$M1$ & $M2$ matched

$$V_{GS1} = V_{GS2}$$

$$V_{EB1} = I_R \cdot R$$

$$I_R = I_{OUT}$$

$$I_{OUT} = \frac{V_{EB}}{R} = \frac{\left(V_{thermal} \ln \frac{I_{out}}{I_S}\right)}{R} \approx \frac{.5}{R}$$

# Supply Independent Biasing (Cont.)



## With Temperature Fluctuation

$$TC = \text{parts per million/degree C} = \text{ppm/}°C = \frac{\left(\frac{\Delta I}{I}\right)}{\Delta T} = \left(\frac{1}{I_{OUT}} \cdot \frac{\partial I_{OUT}}{\partial T}\right)$$

$$I_{OUT} = \frac{V_{EB1}}{R}$$

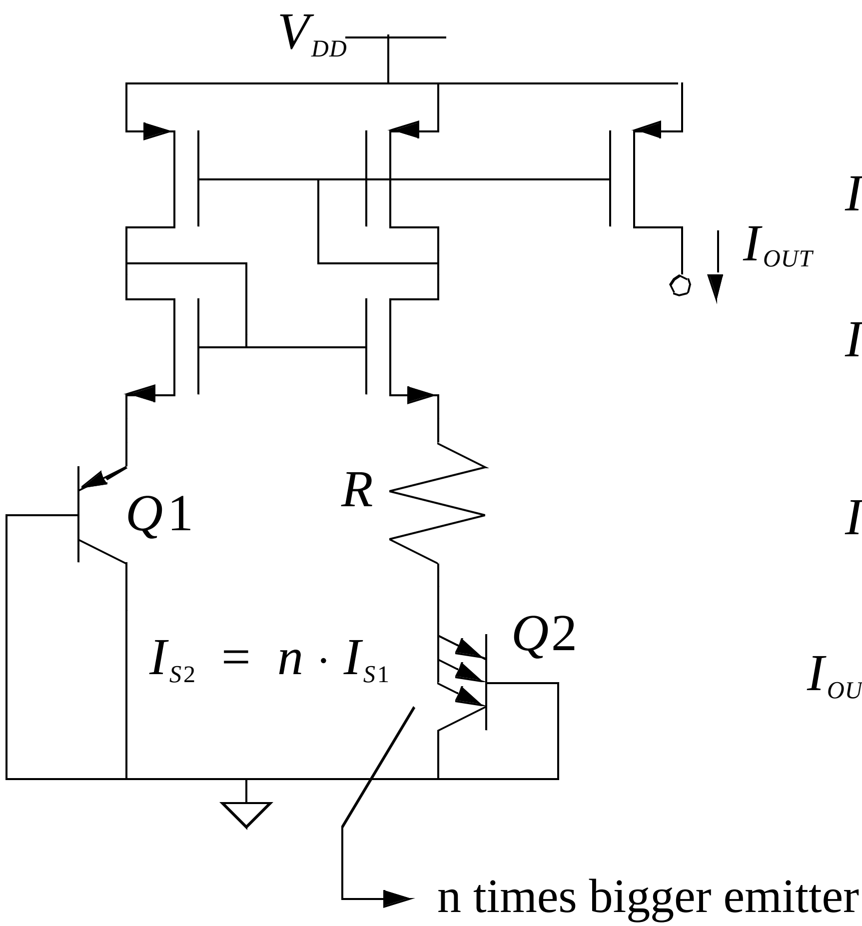
$$\frac{\partial I_{OUT}}{\partial T} = \frac{1}{R} \cdot \frac{\partial V_{EB}}{\partial T} - \frac{V_{EB}}{R^2} \cdot \frac{\partial R}{\partial T}$$

$$TC = \frac{1}{V_{EB}} \cdot \frac{\partial V_{EB}}{\partial T} - \frac{1}{R} \cdot \frac{\partial R}{\partial T}$$

# Supply Independent Biasing (Cont.)



## $V_{thermal}$ referenced Self-Biased Circuit :




$$I_{OUT} \cdot R + V_{EB2} = V_{EB1}$$

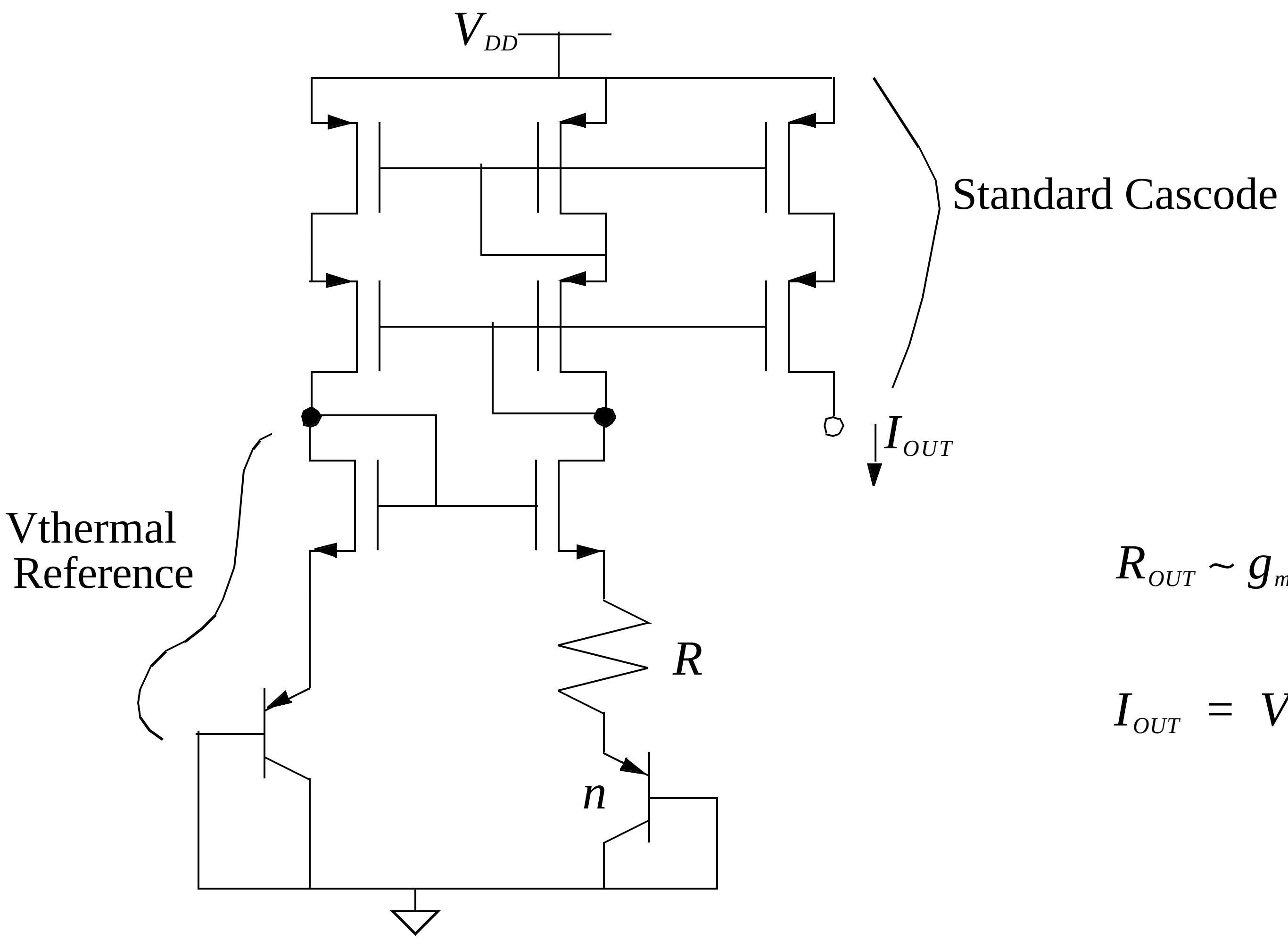
$$I_{OUT} \cdot R + V_T \cdot \ln\left(\frac{I_{OUT}}{n \cdot I_{S1}}\right) = V_T \cdot \ln\left(\frac{I_{OUT}}{I_{S1}}\right)$$

$$I_{OUT} = \frac{V_T}{R} \cdot \ln(n)$$

$$I_{OUT} \cdot R = V_{Thermal} \cdot \ln(n)$$

# Cascode - Self-Biased Source



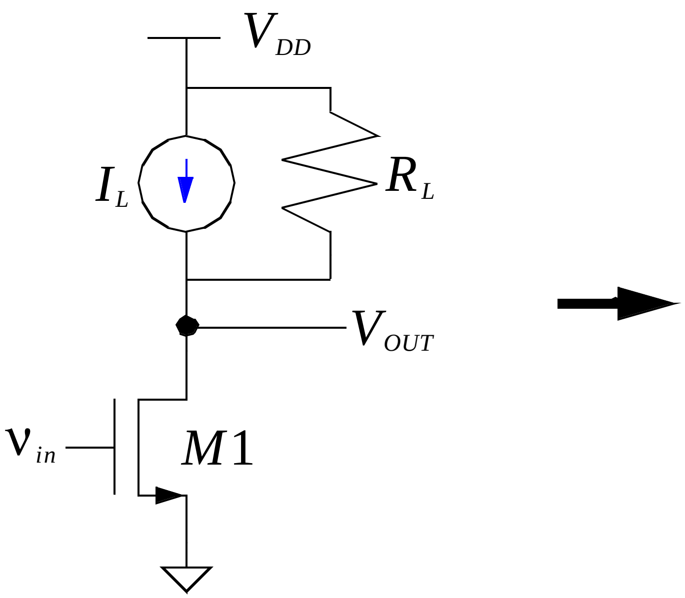


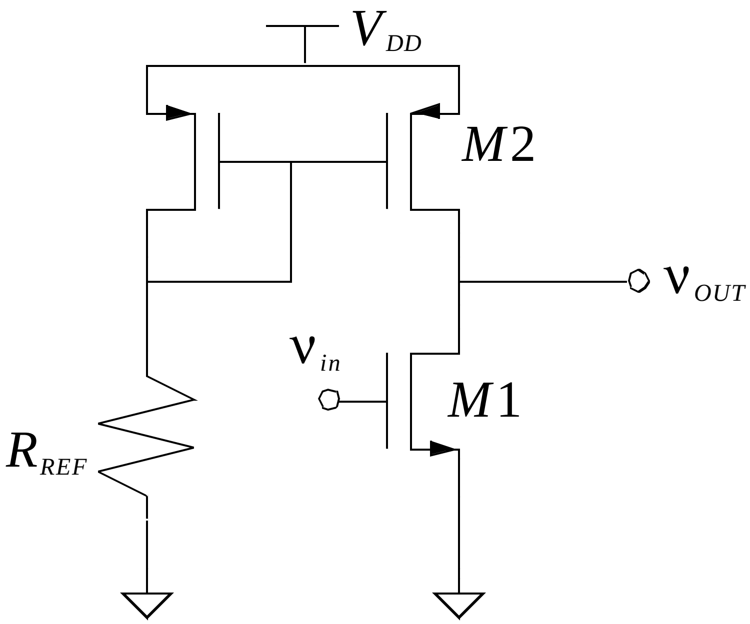

$$R_{OUT} \sim g_m \cdot r_o^2$$

$$I_{OUT} = V_T \cdot \frac{\ln(n)}{R}$$

# Current Source Load






or any other current source

# Current Source Load (Cont.)



$$R_L = r_{o2} = \frac{1}{\lambda_P \cdot I_L} \qquad r_{o1} = \frac{1}{\lambda_n \cdot I_L}$$

$$R_{OUT} = R_L \parallel r_{o1} = r_{o2} \parallel r_{o1}$$

$$A_v = -g_m \cdot (r_{o1} \parallel r_{o2}) \qquad \text{If } \lambda_n = \lambda_p$$

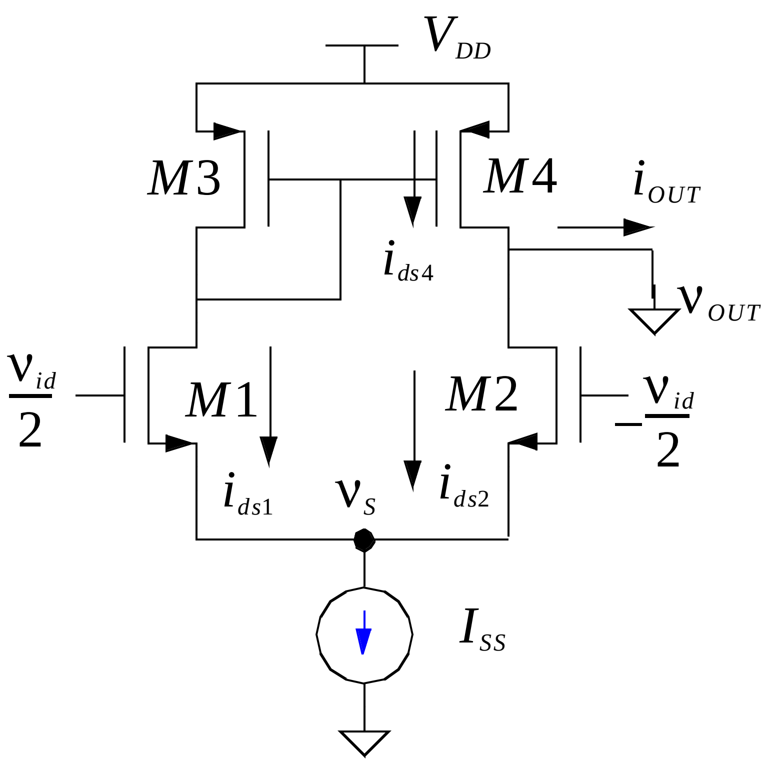
$$g_m = \frac{2 \cdot I_{DS}}{V_{DSAT}} \longleftarrow \text{Handy Formula}$$

$$A_v = -\frac{2 \cdot I_L}{V_{DSAT}} - \frac{1}{2 \cdot \lambda \cdot I_L} = -\frac{1}{\lambda \cdot V_{DSAT}} \propto \frac{1}{I_L^{\frac{1}{2}}}$$

# Differential Pair with Current Source Load



## Double to single ended conversion without loss :




Calculate $GM$

by current source connection

$$\left(\frac{v_{id}}{2} - v_S\right) \cdot g_m = i_{ds1} = i_{ds3} = i_{ds4}$$

$$i_{ds2} = \left(-\frac{v_{id}}{2} - v_S\right) \cdot g_m$$

$$i_{OUT} = i_{ds4} - i_{ds2} = \left(\frac{v_{id}}{2} - v_S\right) \cdot g_m - \left(-\frac{v_{id}}{2} - v_S\right)$$

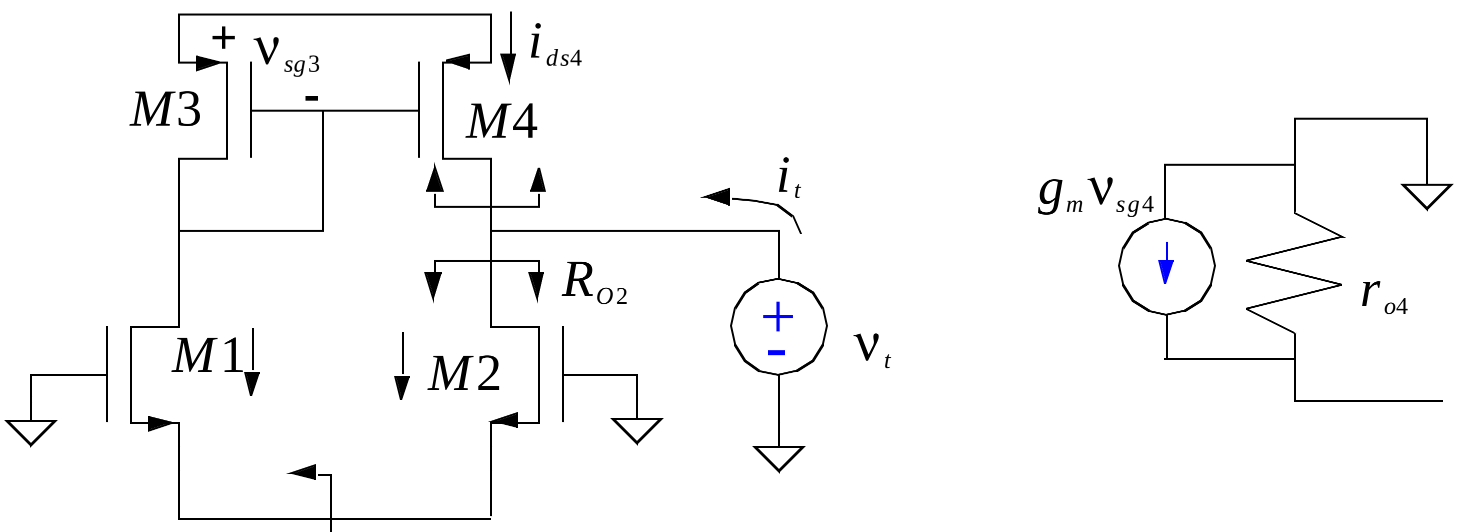
$$i_{OUT} = g_m \cdot v_{id}$$

$$GM = \frac{i_{OUT}}{v_{id}} = g_m$$

# Differential Pair with Current Source Load (Cont.)



## Rout for Differential Pair :




$$R_{S2} = \frac{1}{g_m(1+\chi)}$$

$$R_{O2} = r_{o2} \cdot [1 + (1+\chi) \cdot g_m \cdot R_{S2}] = r_{o2} \cdot [2]$$

# Differential Pair with Current Source Load (Cont.)



$$i_t = i_{ds2} - i_{ds4}$$

$$i_{ds4} = g_m \cdot v_{sg4} - \frac{v_t}{r_{o4}}$$

$$v_{sg4} = v_{sg3} = \left(\frac{1}{g_m}\right) \cdot i_{ds1} = -i_{ds2} \cdot \left(\frac{1}{g_m}\right)$$

$$-g_m \cdot v_{sg4} = -\cancel{g_m} \cdot \frac{v_t}{2 \cdot r_{o2}} \cdot \frac{1}{\cancel{g_m}} = -\frac{v_t}{2 \cdot r_{o2}}$$

$$i_{ds4} = -\frac{v_t}{2 \cdot r_{o2}} - \frac{v_t}{r_{o4}}$$

$$i_t = i_{ds2} - i_{ds4} = \frac{v_t}{2 \cdot r_{o2}} + \frac{v_t}{2 \cdot r_{o2}} + \frac{v_t}{r_{o4}}$$

$$R_{OUT} = \frac{v_t}{i_t} = r_{o2} \parallel r_{o4}$$