University of California
Berkeley

College of Engineering
Department of Electrical Engineering
and Computer Science

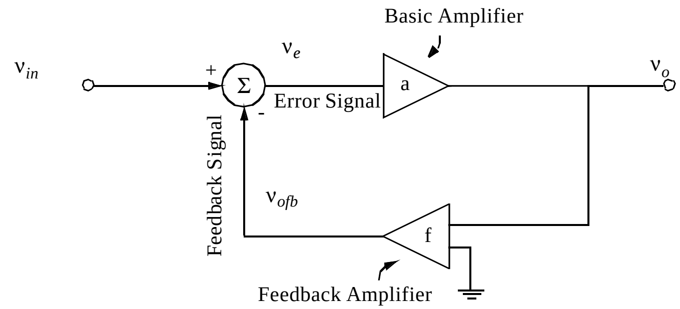
Robert W. Brodersen
EECS140

Analog Circuit Design

Lectures
on
FEEDBACK

Feedback

FB-1



f = gain of feedback amplifier

a = gain of basic amplifier

Feedback (Cont.)

FB-2

$v_\varepsilon = v_{in} - v_{ofb}$

$v_{ofb} = f \cdot v_o$

$v_\varepsilon = v_{in} - f \cdot v_o$

$v_o = a \cdot v_\varepsilon$

$A_v = v_o / v_{in} = \dfrac{a}{(1 + a \cdot f)} = \dfrac{1}{f} \cdot \left(\dfrac{T}{1+T}\right) \leftarrow$ Closed loop gain

$T \equiv$ Loop Gain $= a \cdot f$

a = Open Loop Gain

f = Feedback Factor

$A_v|_{a \to \infty} = \dfrac{1}{f} \qquad T \gg 1$

Feedback (Cont.)

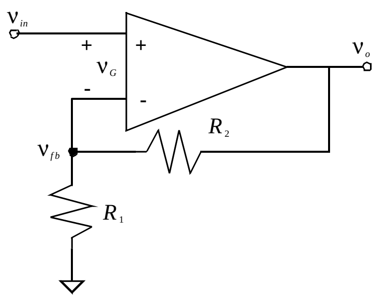
FB-5

There are 4 basic kinds of Feedback Circuits :

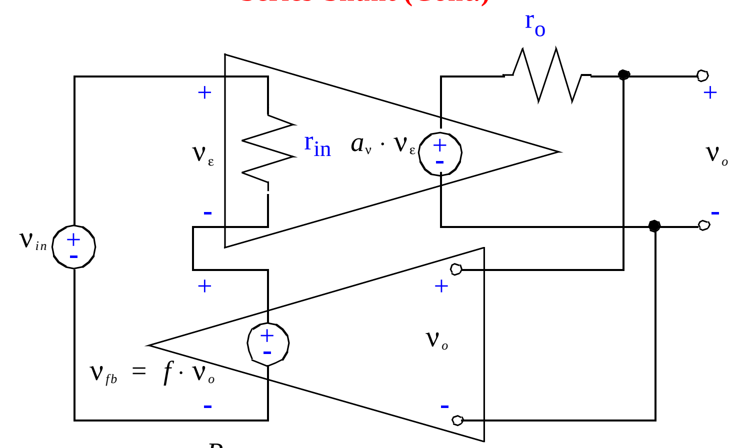
	(Type of Feedback)		(Type of Sensing)
1)	Series (Voltage)	-	Shunt (Voltage)
2)	Shunt (Current)	-	Shunt (Voltage)
3)	Shunt (Current)	-	Series (Current)
4)	Series (Voltage)	-	Series (Current)

Series-Shunt (v_{out}/v_{in}) FB-6

This is a voltage amplifier, a typical example is the well known configuration shown below :



Series-Shunt (Cont.) FB-7



$$f = \frac{R_1}{R_1 + R_2}$$

Series-Shunt (Cont.) FB-8

Gain Calculation :

$$v_o = a_v \cdot v_\varepsilon$$

$$v_{fb} = f_v \cdot v_o$$

$$v_{in} = v_\varepsilon + v_{fb} = \frac{v_o}{a_v} + f_v \cdot v_o$$

$$\frac{v_o}{v_{in}} = \frac{1}{f_v} \cdot \left(\frac{T}{1+T}\right) = A_v \quad \text{Closed Loop Gain}$$

$$v_o = \frac{v_{in} \cdot a_v}{1 + a_v \cdot f_v}$$

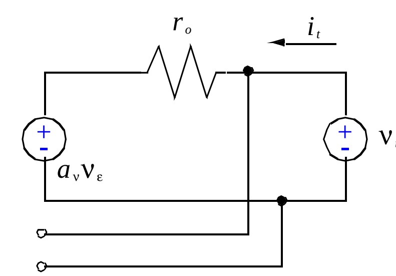
$$v_{in} = v_\varepsilon \cdot (1 + a_v \cdot f_v)$$

Series-Shunt (Cont.) FB-9

Rout Calculation (Closed Loop Output Resistance) :

$$R_{out}\big|_{v_{in}=0} = \frac{v_t}{i_t}$$

Drive output with v_t, measure i_t



$$i_t = \frac{v_t - a_v \cdot v_\varepsilon}{r_o}$$

$$v_\varepsilon + f_v \cdot v_t = v_{in} = 0 \; ; \; v_\varepsilon = -f_v \cdot v_t$$

$$v_{in} = 0$$

$$i_t = \frac{v_t + a_v \cdot f_v \cdot v_t}{r_o}$$

$$\frac{v_t}{i_t} = R_{out} = \frac{r_o}{1 + a_v \cdot f_v} = \frac{r_o}{1+T}$$

$$A_v = 10 \quad f_v = 0.1 \quad a_v = 50,000$$

$$T = 0.1 \cdot 50,000 = 5000 \quad r_o = 100\Omega$$

$$R_{OUT} = \frac{r_o}{1+T} = 0.02\,\Omega$$

Series-Shunt (Cont.) FB-10

Rin Calculation :

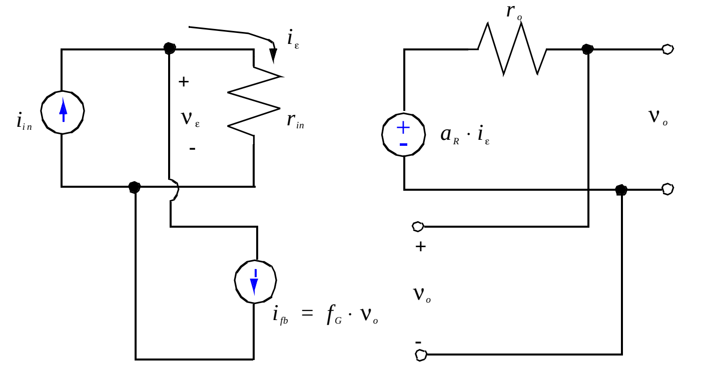
$$R_{in} = \frac{v_{in}}{i_{in}}$$

$$v_{in} = (1 + T) \cdot v_{\varepsilon}$$

$$i_{in} = \frac{v_{\varepsilon}}{r_{in}} = \frac{v_{in}}{(1+T) \cdot r_{in}}$$

$$R_{in} = \frac{v_{in}}{i_{in}} = (1+T) \cdot r_{in}$$

Shunt-Shunt (Transresistance v_{out}/i_{in}) FB-11



Shunt-Shunt (Cont.) FB-13

Gain Calculation :

$$i_{fb} = f_G \cdot v_o$$

$$f_G = \frac{i_{fb}}{v_o}$$

$$i_{\varepsilon} = i_{in} - i_{fb}$$

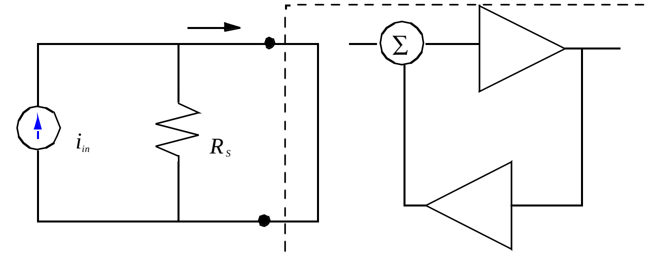
$$v_o = a_R \cdot i_{\varepsilon}$$

a_R has units of resistance

$$v_o = a_R \cdot (i_{in} - \underbrace{f_G \cdot v_o}_{i_{fb}})$$

$$\frac{v_o}{i_{in}} = \frac{1}{f_G} \cdot \left(\frac{T}{1+T}\right)$$

Shunt-Shunt (Cont.) FB-14



Shunt-Shunt (Cont.)

FB-15

Rin Calculation :

$$R_{IN} = \frac{v_\varepsilon}{i_{in}}$$

$$i_\varepsilon = i_{in} - f_G \cdot v_o = i_{in} - a_R \cdot f_G \cdot i_\varepsilon$$

$$\therefore i_\varepsilon = \frac{i_{in}}{1 + a_R \cdot f}$$

$$R_{IN} = \frac{v_\varepsilon}{i_{in}} = \frac{r_{in}}{1 + T}$$

Shunt-Shunt (Cont.)

FB-16

Rout Calculation (Closed Loop Output Resistance) :

$$R_{OUT} = \left.\frac{v_t}{i_t}\right|_{i_{in} = 0}$$

$$(v_o = v_t)$$

$$i_{in} = 0 = i_t + i_{fb}$$

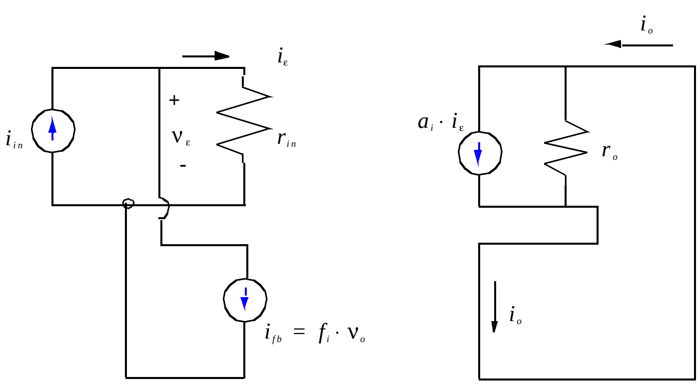
$$i_\varepsilon = -i_{fb} = -f_G \cdot v_o$$

$$v_o = i_t \cdot r_o + a_R \cdot i_\varepsilon = i_t \cdot r_o - a_R \cdot f_G \cdot v_o$$

$$\frac{v_o}{i_t} = \frac{r_o}{1 + T} = R_{OUT}$$

Shunt-Series (Current Amp i_{out}/i_{in})

FB-17



Shunt-Series (Cont.)

FB-18

$$T = a_i \cdot f_i$$

$$f_i = \frac{i_{fb}}{i_o}$$

Gain Calculation :

$$\frac{i_o}{i_{in}} = \frac{1}{f_i} \cdot \left(\frac{T}{1 + T}\right)$$

Rin Calculation :

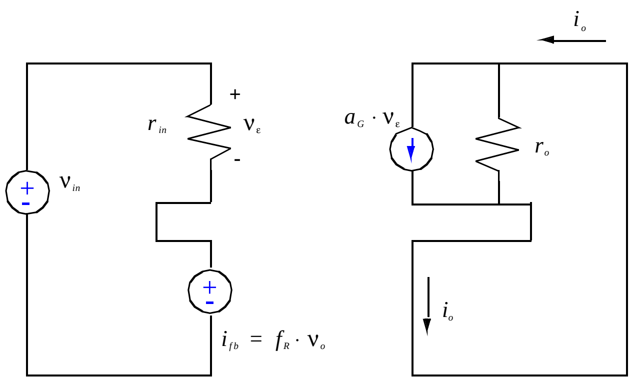
$$R_{IN} = \frac{r_{in}}{1 + T}$$

Rout Calculation (Closed Loop Output Resistance) :

$$R_{OUT} = r_o \cdot (1 + T)$$

Series-Series (Transconductance i_{out}/v_{in})

FB-19



Series-Series (Cont.)

FB-20

$$T = a_G \cdot f_R$$

$$f_R = \frac{v_{fb}}{i_o}$$

Gain Calculation :

$$A_G = \frac{i_o}{v_{in}} = \frac{1}{f_R} \cdot \left(\frac{T}{1+T}\right)$$

Rin Calculation :

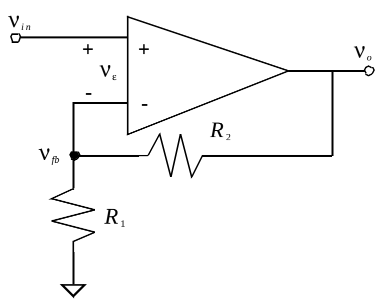
$$R_{IN} = r_{in} \cdot (1+T)$$

Rout Calculation (Closed Loop Output Resistance) :

$$R_{OUT} = r_o \cdot (1+T)$$

Series-Shunt Example (Without Loading)

FB-21



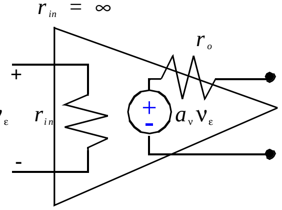
$a_v = 10^5$

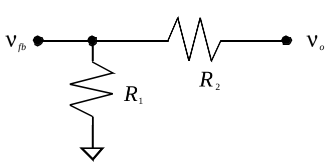
$R_1 = 1k\Omega$

$R_2 = 9k\Omega$

$r_o = 10 \cdot k\Omega$

$r_{in} = \infty$





$$f_v = \frac{v_{fb}}{v_o}$$

$$\frac{v_{fb}}{v_o} = \frac{R_1}{R_1+R_2} = f_v = 0.1$$

Series-Shunt Example (Cont.)

FB-22

$$A_v = \frac{v_o}{v_{in}} = \frac{1}{f_v} \cdot \frac{T}{1+T} \qquad T = a_v \cdot f_v = a_v \cdot \left(\frac{R_1}{R_1+R_2}\right)$$

$$T = 10^5 \cdot \left(\frac{1k}{1k+9k}\right) = 10^4$$

(No loading means we don‘t taken into account the drop across r_o due to R_1 &R_2)

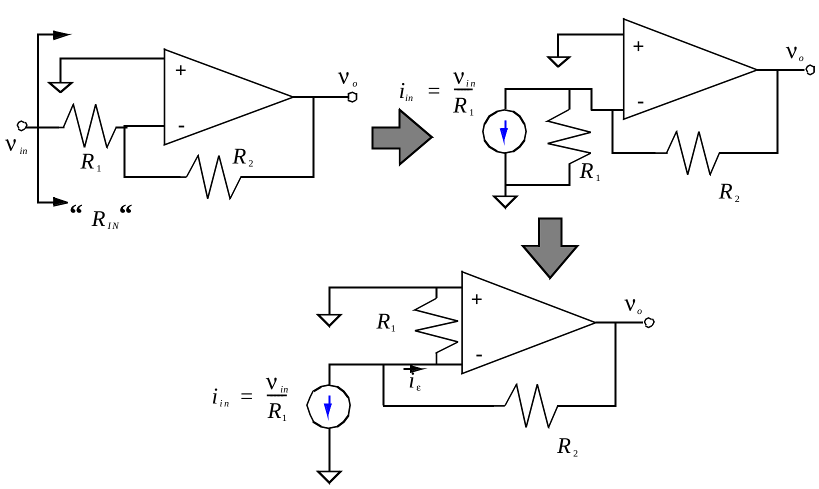
$$A_v = \frac{1}{0.1} \cdot \left(\frac{10^4}{1+10^4}\right) = 10.000$$ precisely 10 to 4 decimal places

$$R_{in} = r_{in} \cdot (1+T) = \infty \cdot (1+10^4) \to \infty$$

$$R_{OUT} = \frac{r_o}{1+T} = \frac{10^4}{1+10^4} = 1\Omega$$

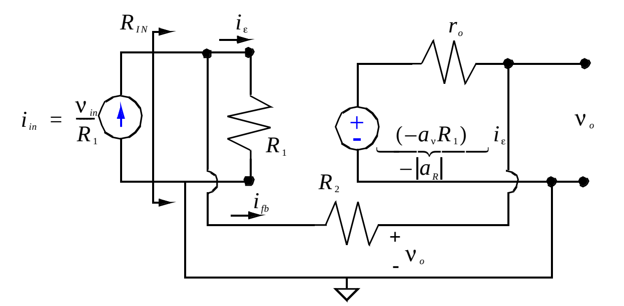
Shunt-Shunt Example (Without Loading)

FB-24



Shunt-Shunt Example (Cont.)

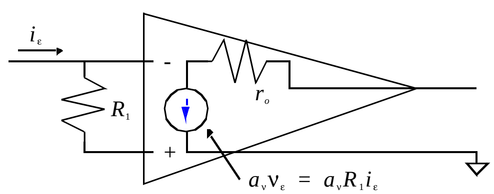
FB-25



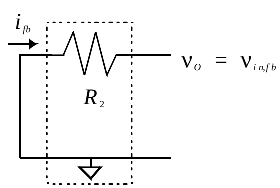
$$i_\varepsilon = i_{in} - i_{fb}$$

Shunt-Shunt Example (Cont.)

FB-26



$$a_v v_\varepsilon = a_v R_1 i_\varepsilon$$
$$v_\varepsilon = -i_\varepsilon R_1$$



$$v_O = v_{in,fb}$$

$$f = \frac{i_{fb}}{v_O}$$

$$i_{fb} = -\frac{v_O}{R_2}$$

$$f = -\frac{1}{R_2}$$

Shunt-Shunt Example (Cont.)

FB-27

$$T = (-a_R) \cdot \left(-\frac{1}{R_2}\right) = a_v \cdot \left(\frac{R_1}{R_2}\right) = 10^5 \cdot \left(\frac{1}{9}\right) = 1.1 \times 10^4$$

$$A_R = \frac{v_o}{i_{in}} = \frac{1}{f} \cdot \frac{T}{1+T} = -R_2 \cdot \left(\frac{1.1 \times 10^4}{1 + 1.1 \times 10^4}\right) = -R_2 = \underline{\underline{-9k\Omega}}$$

But we want $\frac{v_o}{v_{in}}$

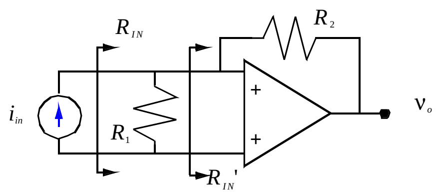
$$i_{in} = \frac{v_{in}}{R_1}$$

$$\frac{v_o}{v_{in}} = \frac{v_o}{i_{in}} \cdot \frac{1}{R_1} = -\frac{R_2}{R_1} = -9 = \frac{v_o}{v_{in}}$$

Shunt-Shunt Example (Cont.)

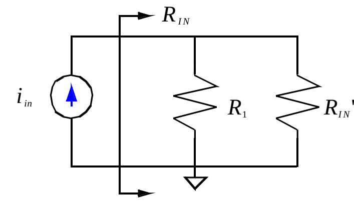
$$R_{OUT} = \frac{r_o}{1+T} = \frac{10k}{1.1\times 10^4} \approx 0.9\Omega$$

$$R_{IN} = \frac{R_1}{1+T} = \frac{R_1}{1 + a_v \cdot \frac{R_1}{R_2}}$$



But R_{IN} is not what we want.

Shunt-Shunt Example (Cont.)



$$R_{IN} = \frac{R_2}{a_v}$$

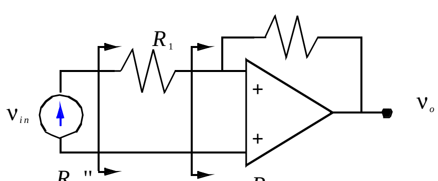
$$R_{IN} = \frac{R_1 \cdot R_{IN}'}{R_1 + R_{IN}'} = \frac{R_1 \cdot R_2}{R_2 + a_v \cdot R_1}$$

$$(\cancel{R_1} \cdot R_{IN}') \cdot (R_2 + a_v \cdot R_1) = (\cancel{R_1} \cdot R_2) \cdot (R_1 \cdot R_{IN}')$$

$$R_{IN}' \cdot (\cancel{R_2} - \cancel{R_2} + a_v \cdot R_1) = R_2 \cdot R_1$$

$$R_{IN}' = \frac{R_2}{a_v}$$

Shunt-Shunt Example (Cont.)



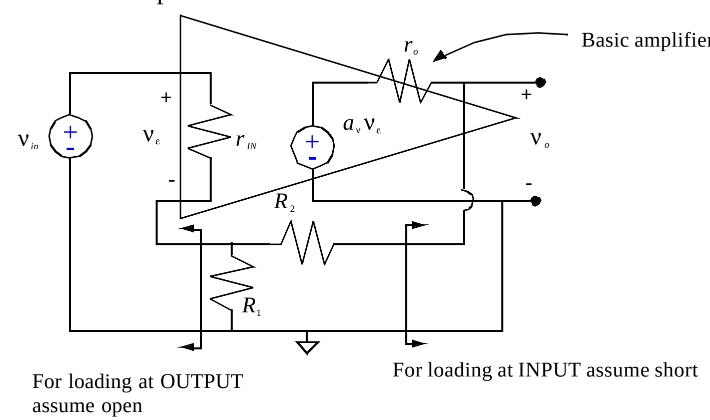
$$R_{IN}' = \frac{R_2}{a_v}$$

$$R_{IN}'' = R_1 + R_{IN}' = R_1 + \frac{R_2}{a_v}$$

R_{IN}'' is the real input resistance we want, but it doesn't come out directly since we used the Norton equvalent circuit at the input

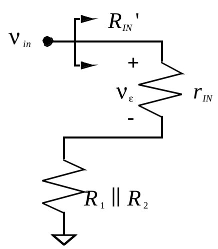
Series-Shunt Example (With Loading)

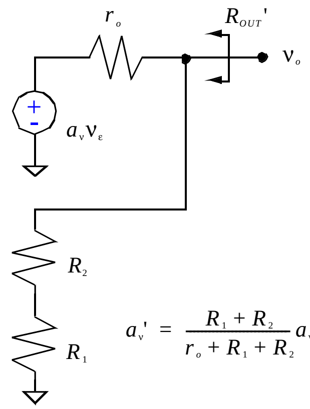
Series - Shunt with loading effects of feecback circuit on the basic amplifier.



Series-Shunt Example (Cont.)

FB-32





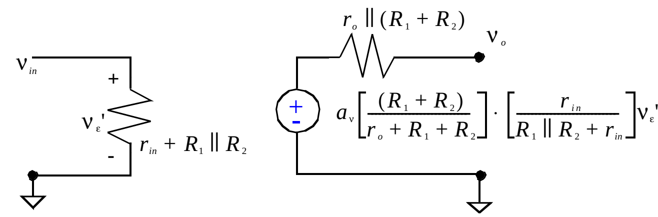
$$v_\varepsilon = \frac{r_{IN}}{R_1 \| R_2 + r_{IN}} v_\varepsilon'$$

$$a_v' = \frac{R_1 + R_2}{r_o + R_1 + R_2} a_v$$

Series-Shunt Example (Cont.)

FB-33

Basic amp with loading :



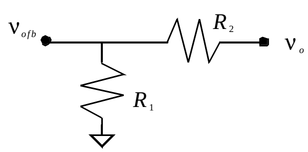
$$a_v' = a_v \left[\frac{(R_1 + R_2)}{r_o + R_1 + R_2}\right] \cdot \left[\frac{r_{in}}{R_1 \| R_2 + r_{in}}\right]$$

$$r_{in}' = r_{in} + (R_1 \| R_2)$$

$$r_o' = r_o \| (R_1 + R_2)$$

Series-Shunt Example (Cont.)

FB-34



$$f_v = \frac{R_1}{R_1 + R_2}$$

$$T' = a_v' \cdot f = a_v \cdot \left(\frac{R_1}{R_1 + R_2}\right) \cdot \left(\frac{r_{in}}{r_{in} + R_1 \| R_2}\right) \cdot \left(\frac{R_1 + R_2}{r_o + R_1 + R_2}\right)$$

$T' < T$ Because of Loading

$T' = 10^4$ ← without loading

$$T' = (10^5) \cdot (0.1) \cdot \underbrace{\left(\frac{\infty}{\infty + 1k \| 9k}\right)}_{1} \cdot \underbrace{\left(\frac{10k}{10k + 10k}\right)}_{1/2}$$

$$= \frac{1}{2} \times 10^4$$ ← with loading

Series-Shunt Example (Cont.)

FB-35

$$R_{IN} = (r_{in} + R_1 \| R_2) \cdot (1 + T')$$

$$T' » 1$$

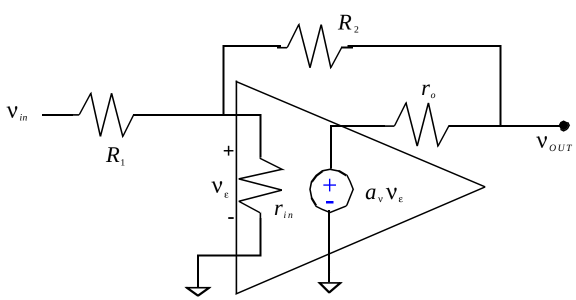
$$R_{IN} = (r_{in} + R_1 \| R_2) \cdot \left(\frac{r_{in} \cdot a_v}{r_{in} + R_1 \| R_2}\right) \cdot \left(\frac{R_1 + R_2}{r_o + R_1 + R_2}\right) \cdot \left(\frac{R_1}{R_1 + R_2}\right)$$

$$R_{IN} = r_{in} \cdot T \cdot \left(\frac{R_1 + R_2}{r_o + R_1 + R_2}\right)$$

$$R_{OUT} = \frac{r_o'}{1 + T'} = \frac{\dfrac{r_o \cdot (R_1 + R_2)}{r_o + R_1 + R_2}}{a_v \cdot \left(\dfrac{r_{in}}{r_{in} + R_1 \| R_2}\right) \cdot \left(\dfrac{R_1}{r_o + R_1 + R_2}\right)}$$

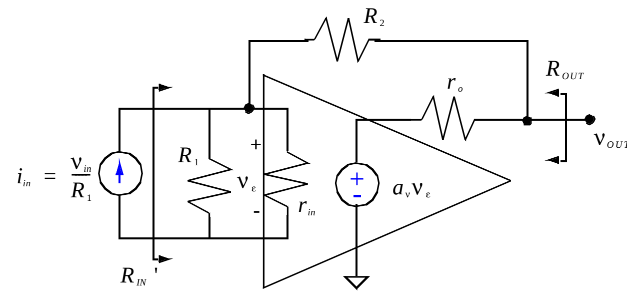
$$R_{OUT} = \frac{r_o}{T \cdot \left(\dfrac{r_{in}}{r_{in} + R_1 \| R_2}\right)}$$

Shunt-Shunt Example (With Loading)



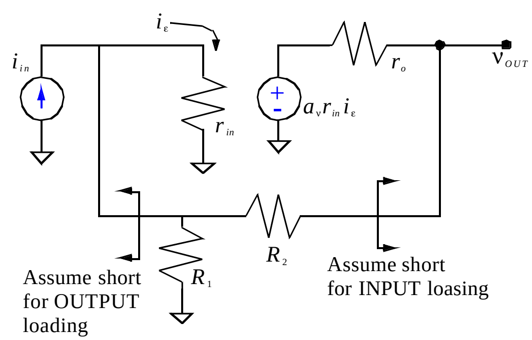
Shunt-Shunt Example (Cont.)

Transforming to current source input :



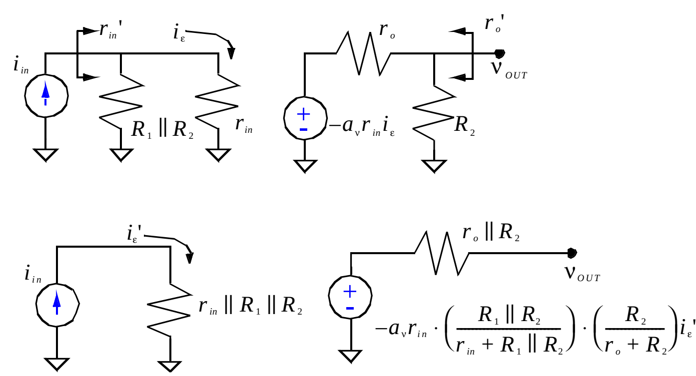
Shunt-Shunt Example (Cont.)

Redrawing :



Shunt-Shunt Example (Cont.)

Basic amplifier with loading :



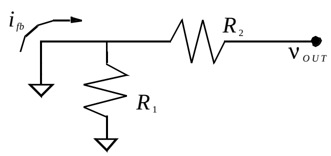
Shunt-Shunt Example (Cont.)

FB-40

$$a_R' = -a_v \cdot r_{in} \cdot \left(\frac{R_1 \parallel R_2}{r_{in} + R_1 \parallel R_2}\right) \cdot \left(\frac{R_2}{r_o + R_2}\right) = -a_v \cdot (r_{in} \parallel R_1 \parallel R_2) \cdot \left(\frac{R_2}{r_o + R_2}\right)$$

$$r_{in}' = r_{in} \parallel R_1 \parallel R_2$$

$$r_{OUT}' = R_2 \parallel r_o$$



$$f_G = \frac{i_{fb}}{v_{OUT}} = -\frac{1}{R_2}$$

EECS140 ANALOG CIRCUIT DESIGN LECTURES ON FEEDBACK

Shunt-Shunt Example (Cont.)

FB-41

Finally we get :

$$\frac{v_{OUT}}{i_{in}} = \frac{1}{f_G} \cdot \left(\frac{T'}{1 + T'}\right)$$

$$T' = a_R' \cdot f_G = a_v \cdot \frac{r_{in}}{R_2} \cdot \left(\frac{R_1 \parallel R_2}{r_{in} + R_1 \parallel R_2}\right) \cdot \left(\frac{R_2}{r_o + R_2}\right)$$

$$\frac{v_{OUT}}{v_{in}} = \frac{1}{R_1} \cdot \left(\frac{v_{OUT}}{i_{in}}\right)$$

$$\frac{v_{OUT}}{v_{in}} = \left(\frac{R_2}{R_1}\right) \cdot \left(\frac{T'}{1 + T'}\right)$$

EECS140 ANALOG CIRCUIT DESIGN LECTURES ON FEEDBACK

Shunt-Shunt Example (Cont.)

FB-42

$$T' \gg 1$$

$$R_{IN}' = \frac{r_{in}'}{1 + T'} = \frac{r_{in} \parallel R_1 \parallel R_2}{a_v \cdot \frac{r_{in}}{R_2} \cdot \left(\frac{R_1 \parallel R_2}{r_{in} + R_1 \parallel R_2}\right) \cdot \left(\frac{R_2}{r_o + R_2}\right)}$$

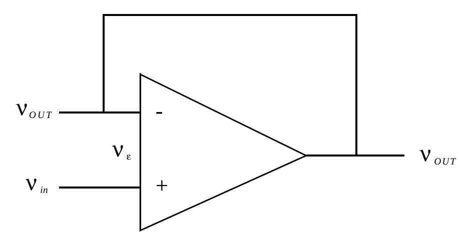
$$R_{IN}' = \frac{r_o + R_2}{a_v}$$ independent of R_1

$$R_{IN}'' = R_1 + R_{IN}' \text{ at } v_{in}$$

$$R_{OUT} = \frac{r_{OUT}'}{1 + T'} = \frac{R_2 \parallel r_o}{1 + T'} = \frac{r_o}{a_v \cdot f_G \cdot (r_{in} \parallel R_1 \parallel R_2)} = R_{OUT}'$$

EECS140 ANALOG CIRCUIT DESIGN LECTURES ON FEEDBACK

FB-43



$$v_\varepsilon = v_{in} - v_{OUT}$$

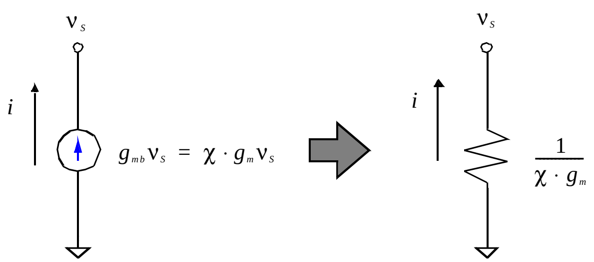
$$= v_{in} - v_{fb} = v_{in} - f \cdot v_{OUT}$$

Series - Shunt

$$f = 1$$

FB-44

Replacing current sources with resistances :

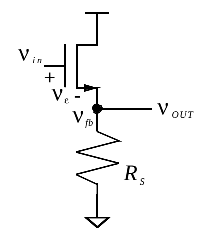


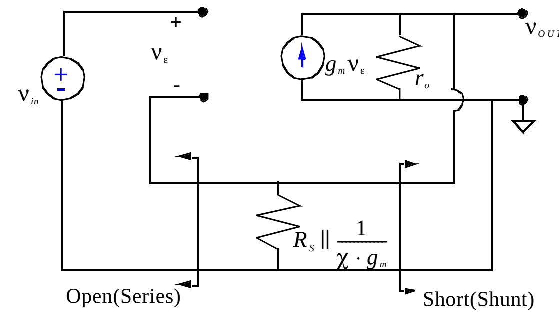
$$i = g_{mb} v_S$$

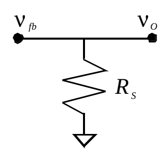
$$\frac{v_S}{i} = \frac{1}{g_{mb}} = \frac{1}{\chi \cdot g_m}$$

Single Transistor : Series - Shunt

FB-45





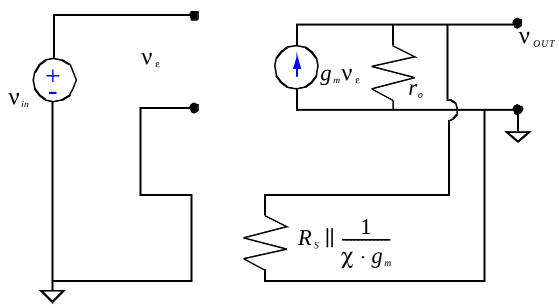


$$f_v = \frac{v_{fb}}{v_o} = 1$$

Single Transistor : Series - Shunt (Cont.)

FB-46

With Loading :



Single Transistor : Series - Shunt (Cont.)

FB-47

$$a_v' = g_m \cdot \left(r_o \parallel R_S \parallel \frac{1}{\chi \cdot g_m} \right) = T' \qquad \text{since } f_v = 1$$

$$r_o \gg R_S$$

$$A_v = \frac{1}{f_v} \cdot \frac{T'}{1+T'} \approx \frac{g_m \cdot \left(R_S \parallel \frac{1}{\chi \cdot g_m} \right)}{1 + g_m \cdot \left(R_S \parallel \frac{1}{\chi \cdot g_m} \right)} = \frac{g_m \cdot R_S}{1 + g_m \cdot R_S \cdot (1+\chi)} = A_v'$$

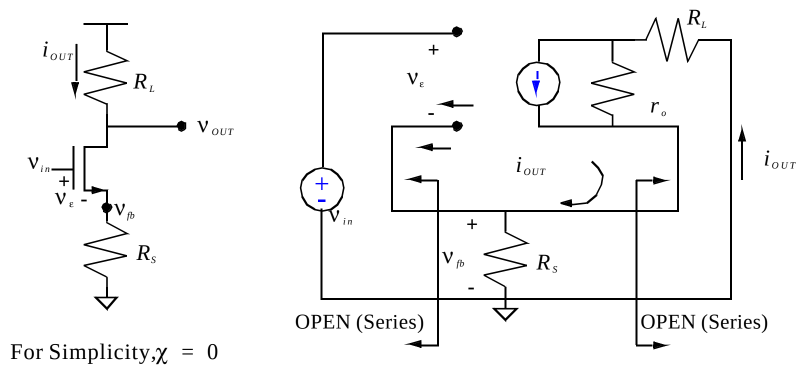
$$r_{OUT}' = r_o \parallel R_S \parallel \frac{1}{\chi \cdot g_m} \approx R_S \parallel \frac{1}{\chi \cdot g_m}$$

$$r_{in}' = \infty$$

$$R_{OUT} = \frac{r_{OUT}'}{1+T'} = \frac{R_S \parallel \frac{1}{\chi \cdot g_m}}{1 + g_m \cdot \left(R_S \parallel \frac{1}{\chi \cdot g_m} \right)} = \frac{R_S}{1 + g_m \cdot (1+\chi) \cdot R_S}$$

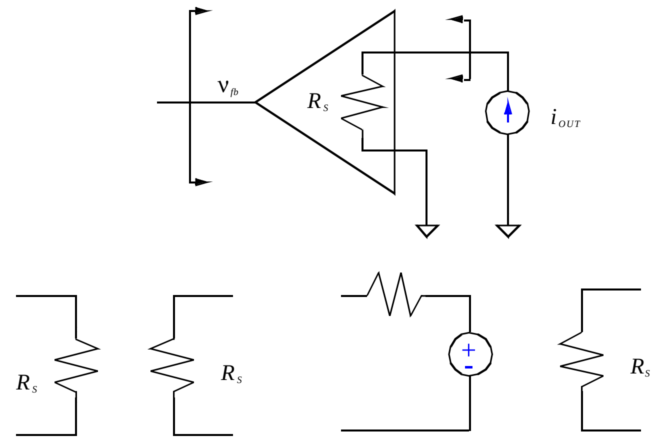
$$= R_S \parallel \frac{1}{g_m \cdot (1+\chi)} = R_{OUT}$$

Series - Series with Degeneration FB-48



For Simplicity, $\chi = 0$

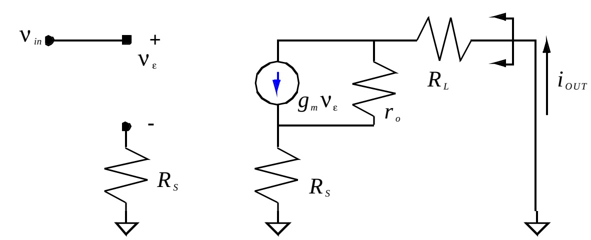
Series - Series Degeneration (Cont.) FB-49



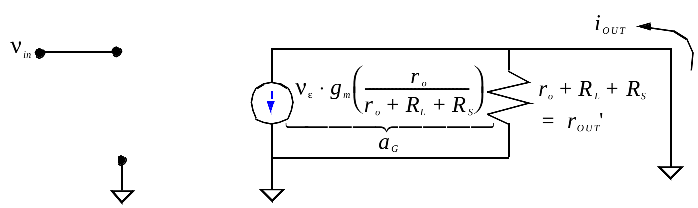
$v_{fb} = i_{OUT} R_S$

Series - Series Degeneration (Cont.) FB-50

Basic amplifier with loading :

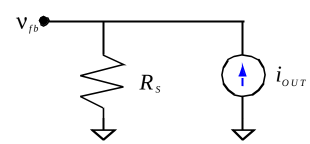


Redrawn :



Series - Series Degeneration (Cont.) FB-51

Feedback Network :



$$f_R = \frac{v_{fb}}{i_{OUT}} = R_S$$

$$T' = a_G \cdot f_G = g_m \cdot \left(\frac{r_o}{r_o + R_L + R_S}\right) \cdot R_S$$

$$R_{OUT}' = r_{OUT}' \cdot (1 + T') = (r_o + R_L + R_S) \cdot \left(1 + g_m \cdot \frac{r_o \cdot R_S}{r_o + R_L + R_S}\right)$$

$$= r_o + R_L + R_S + g_m \cdot r_o \cdot R_S = R_L + R_S + (1 + g_m \cdot R_S) \cdot r_o$$

Series - Series Degeneration (Cont.)

FB-52

$$G_M = \frac{i_{OUT}}{v_{in}} = \frac{1}{f_R} \cdot \frac{T'}{1+T'}$$

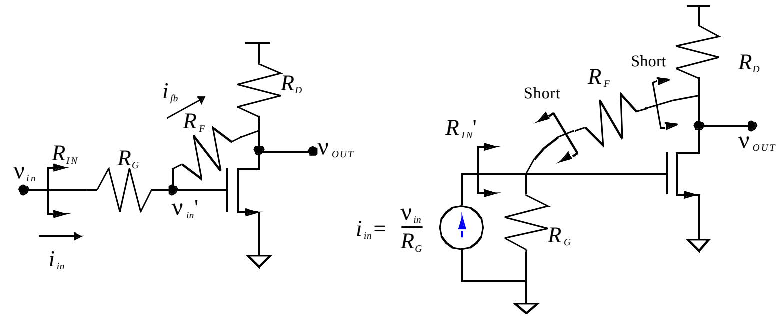
$$= \left(\frac{1}{R_S} \cdot \frac{g_m \cdot \frac{r_o \cdot R_S}{r_o + R_L + R_S}}{1 + \frac{g_m \cdot r_o \cdot R_S}{r_o + R_L + R_S}} = \frac{g_m \cdot r_o}{r_o + R_L + R_S + g_m \cdot r_o \cdot R_S} \right)$$

$$G_M = \frac{g_m \cdot r_o}{R_L + R_S + r_o \cdot (1 + g_m \cdot R_S)} \approx \frac{g_m}{(1 + g_m \cdot R_S)}$$

$$r_o \gg R_L, R_S$$

Shunt - Shunt

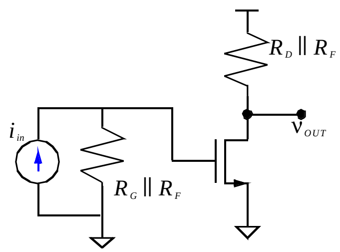
FB-53



Shunt - Shunt (Cont.)

FB-54

Basic amp with loading :



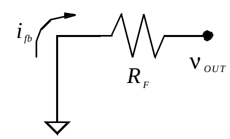
$$v_{OUT} = -i_{in} \cdot (R_G \parallel R_F) \cdot g_m \cdot (R_D \parallel R_F \parallel r_o)$$

$$a_R' = \frac{v_{OUT}}{i_{in}} = -(R_G \parallel R_F) \cdot g_m \cdot (R_D \parallel R_F \parallel r_o)$$

$$r_{in}' = R_G \parallel R_F \qquad r_{OUT}' = R_D \parallel R_F$$

Shunt - Shunt (Cont.)

FB-55



$$f_G = \frac{v_{OUT}}{i_{fb}} = -\frac{1}{R_F}$$

$$T' = a_R \cdot f_G = \frac{1}{R_F} \cdot (R_G \parallel R_F) \cdot g_m \cdot (R_D \parallel R_F \parallel r_o)$$

$$T' \gg 1$$

$$R_{IN}' = \frac{r_{in}'}{1+T'} = \frac{R_G \parallel R_F}{\frac{1}{R_F} \cdot (R_G \parallel R_F) \cdot g_m \cdot (R_D \parallel R_F \parallel r_o)}$$

$$= \frac{R_F}{g_m \cdot (R_D \parallel R_F \parallel r_o)} \approx \frac{R_F}{a_v}$$

Shunt - Shunt (Cont.)

At v_{in} $a_v = \frac{v_{OUT}}{v_{in}'}$

$$R_{IN} = R_G + R_{IN}' = R_G + \frac{R_F}{a_v} \approx R_G$$

$T' \gg 1$

$$R_{OUT} = \frac{r_{OUT}'}{1 + T'} = \frac{(R_D \parallel R_F \parallel r_o)}{\frac{1}{R_F} \cdot (R_G \parallel R_F) \cdot g_m \cdot (R_D \parallel R_F \parallel r_o)}$$

$$= \frac{1}{g_m \cdot \left(\frac{R_G}{R_F + R_G}\right)}$$