A GENERAL AND EFFICIENT NOISE ANALYSIS TECHNIQUE FOR SWITCHED CAPACITOR FILTERS

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Abstract

A general and efficient technique for the analysis of the noise behaviour of switched capacitor (s.c.)-networks is presented. The method incorporates the effects of the folding of white noise from the higher frequency bands to the baseband in a compact way, handles 1/f-noise and multiple clock phases, considers the effects of the continuous I/O-couplings between noise sources and output and is fully automated.
The efficiency of the method is situated in the use of transpose network techniques and the compaction of the noise folding formulas. The method is implemented in the DIANA-SC program.

Introduction

As most of the currently designed s.c.-networks are intended for audio applications, the noise behaviour of these circuits is of prime interest. The realisation of a noise simulator is however obstructed by the so called "aliasing or folding" of white noise from the higher frequency bands to the base band, caused by the sampling nature of the s.c.-network. This aliasing effect is only bandlimited by the finite bandwidths of the analog circuits, which are obtained in each clock phase. These bandwidths are normally orders of magnitude larger than the sampling frequency. This consideration makes the straightforward implementation of a noise simulator very unpractical as it requires an excessive amount of C.P.U.-time : in fact, such a simulator has to sweep over the full frequency axis and to calculate the squared sum of all noise transfer functions until extra contributions are negligible.
Due to the complicated nature of the s.c.-noise behaviour, only a few noise analysis techniques and/or simulators have been published until now. These algorithms however only handle the algebraic evaluation of smaller networks as integrators and biguards [1-2] or are only rough approximations [3-5.] None of the above methods is fully general as they do not handle 1/f-noise, as they do not consider continuous couplings between noise sources and output and as they are based on a number of basic approximations, which limit the accuracy.
The method, presented here, is fully general. It avoids the evaluation of the "infinite aliasing sums" : the total noise is split in three factors, which can all be compacted by an infinite series compression. The whole strategy is based on the framework of [6] for nonideal s.c.-networks, as this mode allows for the introduction of the bandlimiting switch-resistances and opamp-poles, indispensable for accurate noise analysis. The use of adjoint network techniques increases the simulation speed and allows for a simultaneous treatment of all noise sources.

General Formulation of s.c.-noise behaviour

In previous publications (eg [7]), it has been shown that the average output noise spectral density at an output node i is given by (1) :

$$\underset{\sim}{S}_o^{(i)}(\omega)=\sum_{n=-\infty}^{\infty}\left\{\sum_{k=1}^{N}\underset{\sim}{T}_k^{(i)}(\omega,\Omega)\cdot\underset{\sim}{S}_i(\Omega)\cdot\sum_{k=1}^{N}\underset{\sim}{T}_k^{(i)*}(\omega,\Omega)\right\} \quad (1)$$

with $\underset{\sim}{S}_o(\omega)$ and $\underset{\sim}{S}_i(\omega)$ respectively the output and input noise spectral density matrices, $\underset{\sim}{T}_k(\omega,\Omega)$ the frequency domain transmission function for clock-phase k with ω the output pulsation and $\Omega=\omega-n2\pi/T$ the input pulsation (with T the clock period). N denotes the number of clock phases. Note that $\underset{\sim}{S}_o(\omega)$ is an average spectral density function, as the output noise cannot be represented by a stationary stochastic process, due to the time variance. The periodic character of the circuit ascertains however that the process is cyclostationary in the wide sense, which allows for the calculation of an average spectral density function.
A close inspection of (1) reveals that for an ideal (resistorless) s.c.-network the noise $\underset{\sim}{S}_o(\omega)$ is determined by an infinite sum of noise contributions from the higher frequency bands (n) to the baseband. In practical s.c.-networks, this folding is limited by the bandwidth of the analog subcircuits of each clock phase or in other words by the opamp-poles and the switch resistance-capacitance combinations. Another bandlimiting effect is the sin(x)/x-truncation of the continuous I/O-couplings. These arguments show that an accurate noise evaluation requires a simulator, which incorporates the effects of the parasitic resistive time constants on the frequency domain behaviour. Such a simulator was realized in [6] .

Frequency domain transmission functions of resistive s.c. networks

The expression for the frequency domain transmission function $\underset{\sim}{T}_k(\omega,\Omega)$, derived in [6] , can easily be rearranged and split up in a purely discrete term T_d and a second term T_a describing the effects of the continuous I/O-couplings. This is demonstrated in Table 1, where a complete overview of the algorithm is given.
Note that $\eta_k(\omega) = \eta_{kG} + j\omega\,\eta_{kC}$ and $\underset{\sim}{P}_k$ are respectively the complex ac-M.N.A.-matrix and the "extended state transition matrix" of clockphase k. $\Gamma_k(\rho)$ is the sin(x)/x-truncation expression [6] , while $\underset{\sim}{M}(\omega)$ is the z-domain transfer matrix [6,7] . Both $\underset{\sim}{T}_d(\omega,\Omega)$ and $\underset{\sim}{T}_a(\omega,\Omega)$ can be divided in an ω-dependent and an Ω-dependent part. Close inspection of the Ω-factors shows that $\underset{\sim}{T}_d(\omega,\Omega)$ is bandlimited by the bandwidths of the analog subsystem of clockphase k (by $\eta_k^{-1}(\Omega)$), while $\underset{\sim}{T}_a(\omega,\Omega)$ is only bandlimited by the sin(x)/x-truncation effect of $\Gamma_k(\omega,\Omega)$.

Noise algorithms

In a first step, the input noise spectral density can be split up in purely white noise (flat over the full frequency axis) and 1/f-noise :

$$\underset{\sim}{S}_i(\Omega) = \underset{\sim}{S}_i^{1/f}(\Omega) + \underset{\sim}{S}_i^{wh} \qquad (2)$$

The (1/f)-noise is normally limited to the baseband or is negligible with respect to the white noise in the higher frequency bands, so that one (or at most 2) evaluations are normally sufficient to compute $\underset{\sim}{S}_o^{1/f}$.

$$\underset{\sim}{S}_o(\omega) = \underset{\sim}{T}(\omega,\omega)\;\underset{\sim}{S}_i^{1/f}(\omega)\,.\,\underset{\sim}{T}^{*}(\omega,\omega) + \sum_{n=-\infty}^{\infty} \underset{\sim}{T}(\omega,\Omega)\,.\,\underset{\sim}{S}_i^{wh}(\Omega)\,.\,\underset{\sim}{T}^{*}(\omega,\Omega) \qquad (3)$$

As the computation of the (1/f)-noise does not impose any problems, the rest of the text is devoted to the computation of the white noise terms. The superscript wh will be omitted for notional simplicity.

Lemma 1 : The output noise spectral density of a s.c.-system $\mathcal{N}_S$ is composed of three terms, being respectively purely discrete or sample noise, purely analog noise (transferred directly from noise source to output through continuous I/O-couplings) and a mixture of both.

$$\underset{\sim}{S}_o(\omega) = \sum_{n=-\infty}^{\infty} \underset{\sim}{T}_d(\omega,\Omega)\,.\,\underset{\sim}{S}_i\,.\,\underset{\sim}{T}_d^{*}(\omega,\Omega) + \sum_{n=-\infty}^{\infty} \underset{\sim}{T}_a(\omega,\Omega)\,.\,\underset{\sim}{S}_i\,.\,\underset{\sim}{T}_a^{*}(\omega,\Omega) + 2\mathrm{Re}\left|\sum_{n=-\infty}^{\infty} \underset{\sim}{T}_a(\omega,\Omega)\,.\,\underset{\sim}{S}_i\,.\,\underset{\sim}{T}_d^{*}(\omega,\Omega)\right| \qquad (4)$$

where $*$ means complex conjugate.

Proof : Follows directly from a substitution of the top-equation of table 1 in (1).

The interesting feature of (4) is that the factors of (4) can be approximated using infinite series compaction, since all these factors are bandlimited by different but well defined mechanisms :
- The first factor (discrete noise) is limited by the bandwidths of the analog subsystems.
- The analog noise factor is limited by sin(x)/x-truncation.
- The mixed term is limited by a combination of sin(x)/x-truncation and analog bandwidths.

Theorem 1 :Consider a s.c.-network $\mathcal{N}_S$ and suppose that the noise bandwidths of the analog noise transfer functions $\omega_{nk}^{(i)}$ [with k=1 to N and i=1 to m (number of noise sources)]are considerable larger than the sampling frequency ω_s. The duty cycles of the different clockphases k(k=1,...N) are neither too small or too large (say $1/32<\rho_k<31/32$). The noise behaviour of $\mathcal{N}_S$ can be approximated in the compacted form (5).

$$\underset{\sim}{S}_o(\omega) \simeq \sum_{k=1}^{N} \Psi_k(\omega)\left\{\eta_k^{-1}(\omega)\,.\,\dot{\underset{\sim}{S}}_{kk}\,.\,\eta_k^{*-1}(\omega) + \underset{\sim}{P}_k\,.\,\eta_k^{-1}(\omega)\,.\,\dot{\underset{\sim}{S}}_{kk}\,.\,\eta_k^{*-1}(\omega)\,.\,\underset{\sim}{P}_k^{*}\right\}\Psi_k^{*}(\omega) + \eta_k^{-1}(\omega)\,.\,\underset{\sim}{S}_{kk}\,.\,\eta_k^{*-1}(\omega)\,.\,\rho_k + 2\mathrm{R_e}\left\{\Psi_k(\omega)\,.\,\underset{\sim}{T}_k(\omega)\,.\,\underset{\sim}{S}_{kk}\,\eta_k^{-1}(\omega)\right\} \qquad (5)$$

where $\underset{\sim}{S}_{kk}$ is the input spectral density matrix of clock phase k. (Note that all $\underset{\sim}{S}_{ik}(i\neq k)$ drop out of the equations). $\dot{\underset{\sim}{S}}_{kk}$ is the input spectral density matrix, where the entry of noise source i (i=1 to m) is scaled with the factor $2\omega_{nk}^{(i)}/\omega_s$. All other entries are as defined in table 1.
Proof :(5) is obtained from (4) using long but straight forward calculations using infinite series compaction techniques. For full derivations, we refer to [9] . Note that (5) does not contain any infinite summation !

Consequences and remarks

- The noise bandwidth $\omega_{nk}^{(i)}$ is defined in an identical way as in the classical analog theory : it is the bandwidth of a rectangular magnitude lowpass filter, which has the same power output as the original filter. This is illustrated in Fig. 1. The simplest way to compute these $\omega_{nk}^{(i)}$ is to perform a sweep analysis on the analog subsystems with a quadratic step ($\Omega_{next} = \Omega_{prev} \times \Omega_{prev}$). The total noise power is integrated using trapesoidal integration, and division by the base band amplitude results in the noise bandwidth . This computation has to be performed only once !
- Detailed inspection of equation (5) and Table 1 clearly demonstrates that the noise analysis does NOT ask for any additional computation with respect to the frequency domain calculations of Table 1, with exception of the combination of the noise terms and the determination of the noise bandwidths.
- The noise algorithm is made efficient with the use of adjoint or reciprocal network techniques [7] . For resistive s.c.-networks, identical tech-

niques as defined in [7] can be used by taking the transpose of the matrix equations. In this way, the network is excited at the output node and the equations are treated in an opposite direction. The use of these transpose techniques allows for a treatment of all noise sources in one run with conservation of all information concerning the different noise transferfunctions. The adjoint procedure also results in an increase of the computational efficiency.

- The compact noise analysis has been implemented in the DIANA-SC program. When the basic assumptions, being :

- $\omega_{nk}^{(i)} >> \omega_s$, with i=1 to m, where m is the number of noise sources
- $1/32 < \rho_j < 31/32$
- 1/f-breakpoint $< \omega_s/2$

are not fulfilled, the program switches automatically to the slow, full sweep noise computation mode, which calculates the squared sum of all noise aliasing transfer functions.

Example

The output noise spectrum of the third order elliptical LP ladder filter of [10] is examined and plotted in Fig. 2. A close agreement between simulation and measurements can be noted. Fig. 3 contains the contributions of the operational amplifiers (with equivalent input noise of 360 nV/$\sqrt{\text{Hz}}$ and 1/f-break at 10 kHz) to the output noise spectrum (in db). These noise sources prove to be dominant to the switch noise and reduction of the operational amplifier noise would result in a significant improvement in dynamic range. The total C.P.U.-time on VAX/VMS 780/11 for this example equals 25.2 sec (over 40 frequency points).

Conclusions

An efficient, general and rather accurate noise simulator for s.c.-circuits has been developed. The algorithm allows, together with the DIANA-SC distortion analysis mode [8] , for a reasonable predicition of the S/N ratio and the dynamic range of a s.c.-network under design. The efficiency of the method is situated in the use of transpose network techniques, in the compaction of the noise folding formules and in its compatibility with the M.N.A.-based frequency domain analysis mode for resistive s.c.-networks.

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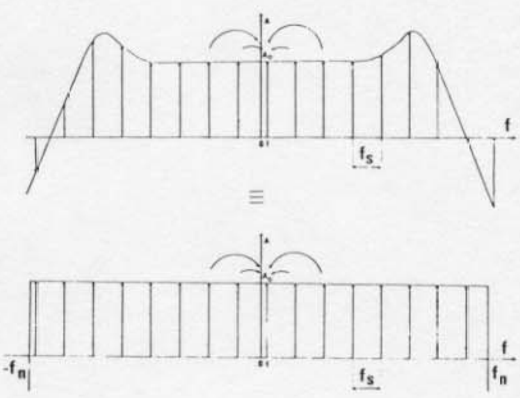

Fig. 1 Definition of noise bandwidth f_n

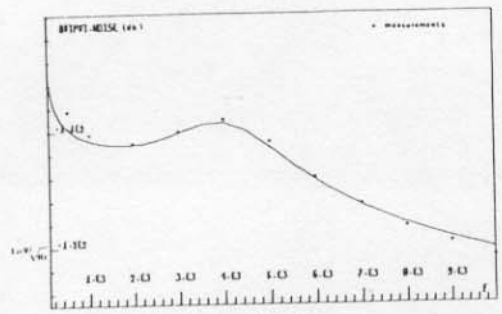

Fig. 2

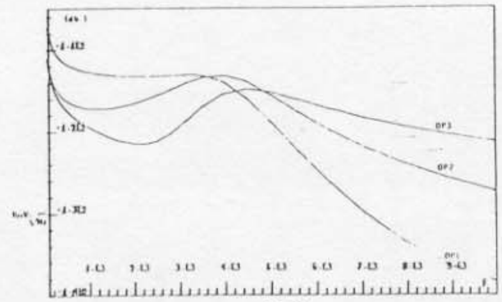

Fig. 3

$$X(\omega) = \sum_{n=-\infty}^{\infty} T(\omega, \omega - \omega_s . n)\ Y(\omega - n.\omega_s)$$

$$= \sum_{n=-\infty}^{\infty} \left[T_d(\omega, \Omega) + T_a(\omega, \Omega) \right] Y(\Omega) \qquad \Omega = \omega - n\,\omega_s$$

$$T_d(\omega,\Omega) = \Psi(\omega) . \Theta(\Omega)$$

$$\Psi(\omega) = \frac{1}{T} \cdot \begin{vmatrix} \eta_1^{-1}(\omega) & & & \\ & \eta_2^{-1}(\omega) & & \\ & & \ddots & \\ & & & \eta_N^{-1}(\omega) \end{vmatrix} \begin{vmatrix} -\eta_{1C} e^{-j\omega t_1} & & & \eta_{1C} e^{-j\omega t_N} \\ \eta_{2C} e^{-j\omega t_1} & -\eta_{2C} e^{-j\omega t_2} & & \\ & \ddots & \ddots & \\ & & \eta_{NC} e^{-j\omega t_{N-1}} & -\eta_{NC} e^{-j\omega t_N} \end{vmatrix} \cdot M(\omega)$$

$$\Theta(\Omega) = \begin{vmatrix} (I e^{j\Omega t_1} - P_1 e^{j\Omega t_0})\, \eta_1^{-1}(\Omega) & & \\ & (I e^{j\Omega t_2} - P_2 e^{j\Omega t_1})\, \eta_2^{-1}(\Omega) & \\ & \cdots & \\ & & (I e^{j\Omega t} - P_N e^{j\Omega t_{N-1}})\, \eta_N^{-1}(\Omega) \end{vmatrix}$$

with $t_0 = t_N - T$

$$T_a(\omega,\Omega) = \Xi(\omega) . \Lambda(\omega - \Omega)$$

$$= \begin{vmatrix} \eta_1^{-1}(\omega) & & & \\ & \eta_2^{-1}(\omega) & & \\ & & \cdots & \\ & & & \eta_N^{-1}(\omega) \end{vmatrix} \begin{vmatrix} I\tau_1(\omega-\Omega) & & & \\ & I\tau_2(\omega-\Omega) & & \\ & & \cdots & \\ & & & I\tau_N(\omega-\Omega) \end{vmatrix}$$

with $\tau_i(\omega-\Omega) = v_i(\omega-\Omega)\, e^{j(\Omega-\omega)t_i}$

TABLE 1